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Cambridge:
PRINTED BY J. AND C. F. CLAY,
AT THE UNIVERSITY PRESS.

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CHARLES PENDLEBURY, M.A.,

SENIOR MATHEMATICAL MASTER AT ST PAUL'S SCHOOL, ETC.

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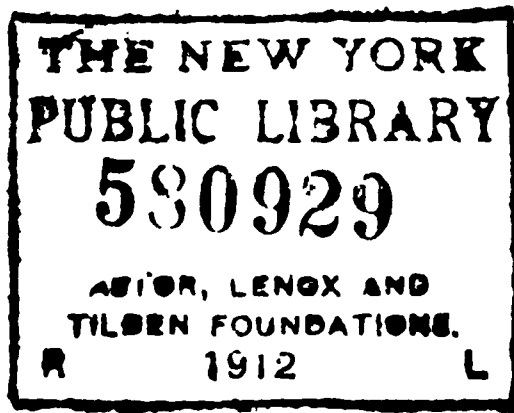
LONDON

GEORGE BELL AND SONS

CAMBRIDGE: DEIGHTON, BELL AND CO.

1902

W. S.



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MADE IN
NEW YORK
BY
W. A. B. S. L.

*First Published, January 1899.
Reprinted, June 1899, 1900, 1901, 1902.*

PREFACE.

THIS book is intended for the lower and middle forms of Secondary Schools. It will be found adapted especially for the use of candidates for the Oxford and Cambridge Junior Local Examinations, the Scotch Universities' Locals, County Council Scholarships, and for examinations conducted by the College of Preceptors, the Board of Intermediate Education for Ireland, &c. &c.

Throughout the book fully worked-out examples have been given to illustrate the theory explained in the text, and to serve as models for the pupil. The examples for exercise, of which there are 4500, are carefully graduated and much trouble has been taken to verify them. The sets of miscellaneous examples are mostly taken from examination papers set at the Examinations referred to above. Questions involving unnecessarily long and tedious calculations have been excluded from the book, as unsuitable for the Junior Students for whom the book is intended.

January, 1899.

WYOM WAM
WAM
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TABLE OF BRITISH WEIGHTS AND MEASURES.

<i>Avoirdupois Weight.</i>		
16 drams (drs.)	make 1 ounce (oz.)	70000 grains
16 ounces 1 pound (lb.)	
28 pounds 1 quarter (qr.)	in 1 lb.
4 quarters } 1 hundredweight (cwt.)	
or 112 pounds }		
20 hundredweight 1 ton.	
<i>Linear Measure.</i>		
12 inches (in.)	make 1 foot (ft.)	
3 feet 1 yard (yd.)	
5½ yards 1 rod, or pole (po.)	
40 poles 1 furlong (fur.)	
8 furlongs } 1 mile (m.)	
or 1760 yards }		
Also 100 links } 1 chain (ch.)	80 chains = 1 mile
or 22 yards }		
<i>Square Measure.</i>		
144 sq. inches	make 1 sq. foot (sq. ft.)	
9 sq. feet 1 sq. yard (sq. yd.)	
30½ sq. yards 1 sq. rod, sq. pole, or perch (per.)	
40 perches 1 rood (ro.)	
4 roods 1 acre (ac.)	
640 acres 1 sq. mile (sq. m.)	
Also 10 sq. chains } 1 acre.	
or 4840 sq. yards }		
<i>Cubic Measure.</i>		
1728 cubic inches	make 1 cubic foot (cub. ft.)	
27 cubic feet 1 cubic yard (cub. yd.).	
<i>Liquid Measure.</i>		
4 gills	make 1 pint (pt.)	
2 pints 1 quart (qt.)	
4 quarts 1 gallon (gall.)	
36 gallons 1 barrel.	
<i>Dry, or Corn Measure.</i>		
4 gills	make 1 pint (pt.)	
2 pints 1 quart (qt.)	
4 quarts 1 gallon (gall.)	
2 gallons 1 peck (pk.)	
4 pecks 1 bushel (bush.)	
8 bushels 1 quarter (qr.)	
<i>Measure of Number.</i>		
12 units	make 1 dozen	
20 units 1 score	
12 dozen 1 gross	
12 gross 1 gt. gross	
also 24 sheets of paper 1 quire	
20 quires 1 ream	
10 reams 1 bale	
<i>Measure of Time.</i>		
60 seconds	make 1 minute (min.)	
60 minutes 1 hour (hr.)	
24 hours 1 day	
7 days 1 week	
28, 29, 30, or 31 days 1 calendar month	
12 calendar months 1 year	
365 days 1 common year	
366 days 1 leap year	

ARITHMETIC.

NOTATION AND NUMERATION.

1. A **unit** is a single thing; as *one boy, one penny, one book.*
2. Quantities *of the same kind, or like* quantities, are such as have the same unit, for example, *five horses and eight horses* are like quantities, the unit, one horse, being the same for both.
3. A **number** is a unit, such as *one girl*, or a collection of units of the same kind, as *six pennies.* In the latter case the *unit* is *one penny.*
4. ARITHMETIC is a part of the SCIENCE of numbers, and of the ART of using them.
5. A number, such as *four* or *five*, not attached to any particular things or units, is called an **abstract number.**
6. A number of particular units, such as *four horses*, or *five men*, is called a **concrete number.**
7. All numbers are written by means of the symbols

0, 1, 2, 3, 4, 5, 6, 7, 8, 9,

which are called **digits**; they are named respectively, *nought* or *zero, one or unity, two, three, four, five, six, seven, eight, nine.*

Each of these digits denotes one unit more than the digit which precedes it. Thus 2 denotes one unit more than 1; 3 denotes one unit more than 2; 8 denotes one unit more than 7, and so on; 0 denoting that there are *none* of the units or things referred to.

8. To express numbers greater than *nine*, two or more of these digits are written side by side.

9. The number next greater than *nine* is called *ten*, and is represented by 10. The numbers that follow *ten* in succession are

<i>eleven,</i>	represented by	11,
<i>twelve,</i>	" "	12,
<i>thirteen,</i>	" "	13,
<i>fourteen,</i>	" "	14,
<i>fifteen,</i>	" "	15,
<i>sixteen,</i>	" "	16,
<i>seventeen,</i>	" "	17,
<i>eighteen,</i>	" "	18,
<i>nineteen,</i>	" "	19,
<i>and twenty,</i>	" "	20;

each of these numbers denoting *one more* unit than the number which precedes it.

Next come

twenty-one, represented by 21,
twenty-two, " " 22,

and so on until we arrive at

twenty-nine, represented by 29;

then follow

thirty, represented by 30,
thirty-one, " " 31,
thirty-two, " " 32,

and so on as before.

After <i>thirty-nine</i>	comes <i>forty</i> ,	represented by 40;
" <i>forty-nine</i>	" <i>fifty</i> ,	" " 50;
" <i>fifty-nine</i>	" <i>sixty</i> ,	" " 60;
" <i>sixty-nine</i>	" <i>seventy</i> ,	" " 70;
" <i>seventy-nine</i>	" <i>eighty</i> ,	" " 80;
" <i>eighty-nine</i>	" <i>ninety</i> ,	" " 90;
" <i>ninety-nine</i>	" <i>one hundred</i> ,	" " 100.

10. The number **ten** plays a most important part in Arithmetic.

If, for example, we have a large number of similar coins and our power of counting does not enable us to count beyond ten, how can we count the coins and represent the number of them by means of the digits, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9?

Let us first take out ten coins and put them together in a pile; then ten more in another pile; then ten more, and so on; and let us do this as often as we can, until there are less than ten coins left over. This remainder, being less than ten, may be denoted by one of the above digits. Suppose it is 3.

Let us next treat the piles of ten in a similar manner. Let us take ten piles and put them together in a group; then ten more in another group; then ten more in a third group, and so on; until, as before, we have less than ten piles left. If this remainder be 7, we will place the digit 7 to the left of the 3, thus:—73. The number of coins in each group of ten piles, each pile containing ten pieces, is called a **hundred**.

After 100, we get *one hundred and one*, represented by 101,
one hundred and two, " " 102,

and so on until we arrive at

one hundred and ninety-nine, represented by 199;

then *two hundred*, " " 200,

two hundred and one, " " 201,

and so on until we reach

three hundred, represented by 300.

Proceeding in this way we arrive in turn at

<i>four hundred,</i>	represented by	400,
<i>five hundred,</i>	"	500,
<i>six hundred,</i>	"	600,
<i>seven hundred,</i>	"	700,
<i>eight hundred,</i>	"	800,
<i>nine hundred,</i>	"	900.

11. We will now put together ten groups of a *hundred* each to form a larger group, the number of coins in which is called a **thousand**, and we will continue the grouping until the number of *hundreds* left over is less than ten. If 3 *hundreds* be left over, we place the digit 3 to the left of the 73, thus getting 373.

The process of putting together ten smaller groups to form one larger must be carried on until at some step the number of larger groups that can be formed is itself less than ten. If this happens when we have formed the groups which contain a *thousand* each, that is, if the number of thousand-groups is only nine, we put the digit 9 to the left of the 373, thus, 9373, and the process is at an end. The expression 9373 will then represent the number of coins.

12. For the number of coins in successive groups, each of which is formed by putting together ten of the next smaller groups, we have the following names:

a unit,
ten,
one hundred,
one thousand,
ten thousand,
one hundred thousand,
one million.

13. Let us return now to the number 9373. Taking the digits one by one, beginning at the right hand, we know that

the first digit 3 denotes	3 coins, or 3 <i>units</i> ;
the next digit 7	" 7 <i>tens</i> , or <i>seventy units</i> ;
the next digit 3	" 3 <i>hundreds</i> ;
and the remaining digit 9	" 9 <i>thousands</i> .

Therefore we may express the number 9373 in the words

nine thousand, three hundred, and seventy-three.

NOTE.—To express a number in words, we begin with the left-hand digit, and we end with the units.

14. Similarly, the number 3765215 denotes

5 *units*,

1 *ten*,

2 *hundreds*,

5 *thousands*,

6 *ten thousands*, or 60 *thousands*,

7 *hundred thousands*,

3 *millions*.

This may be shortened by collecting together all the thousands, and expressing the five units and the one group of ten by the single word *fifteen*.

The number therefore denotes

fifteen,

two hundred,

seven hundred and sixty-five thousand,

three millions;

and

and we express it thus,

three millions, seven hundred and sixty-five thousand,

two hundred and fifteen.

15. If in Art. 11 there had been no hundreds left over, we should have placed the digit 0 instead of 3 to the left of the 7; the number would then have been represented by the digits 9073, and in words by

nine thousand and seventy-three.

If moreover there had been no tens left over as well as no hundreds, the number would have been represented by the digits 9003, and in words by

nine thousand and three.

16. To aid us in reading a large number, it is convenient to place a comma in front of every third digit, counting from the right.

For example, if a number be written thus,

3,254,318,

we can see at a glance that 318 represents the number of *hundreds*, *tens*, and *units*, 254 the number of *thousands*, and 3 the number of *millions*.

We can now read the number much more readily, thus,

three millions,

two hundred and fifty-four thousand,

three hundred, and eighteen.

17. We have seen that all numbers between 10 and 100 are represented by *two* digits. The highest, *ninety-nine*, is represented by 99, and means

9 *tens* + 9 *units*.

By adding 1 to this we get

9 tens + 10 units,
or 10 tens + 0 units,
or 1 hundred + 0 tens + 0 units.

This is represented by 100, and therefore requires *three* digits.

All numbers between 100 and 1000 need *three* digits. The highest is 999; if we add 1 to it we get 1000, which requires four digits.

The greatest number of *four* digits is 9999. If to this we add 1 we get 10000, for which we need *five* digits; and so on.

18. In the number 9373, the digit 3 appears twice; in one place it means 3 units, and in the other three hundreds. Hence, although a digit always has the same *absolute* value, yet in consequence of the difference of position the one 3 here denotes a hundred times as much as the other.

19. The principle of the **value** of a digit depending upon its **position** among the others is a fundamental one in our method of notation, and is called the **PRINCIPLE OF LOCAL VALUE**.

20. Our system of notation is called the **Arabic** system.

21. A few numbers are called also by special names.

For example, 12 units are called a **dozen**, 20 units are called a **score**, and 144 units are called a **gross**.

22. The art of expressing numbers by symbols is called **notation**; and the art of expressing them by words is called **numeration**.

EXERCISE 1.

To be done first orally, then in writing.

State in figures:—

- | | | |
|-----------------------------------|----------------------------------|------------------|
| (1) Eleven. | (2) Fourteen. | (3) Forty-one. |
| (4) Seventeen. | (5) Seventy-one. | (6) Sixty-three. |
| (7) One hundred and one. | (8) One hundred and eleven. | |
| (9) One hundred and ten. | (10) One hundred and twenty-one. | |
| (11) One hundred and eight. | (12) Eight hundred and one. | |
| (13) Eight hundred and eleven. | (14) Two hundred and four. | |
| (15) Two hundred and fourteen. | (16) Two hundred and forty-one. | |
| (17) Seven hundred and seventeen. | (18) Seven hundred and seven. | |
| (19) Seven hundred and seventy. | (20) Seventeen hundred. | |

- (21) Seventeen hundred and seventeen.
 (22) Seventeen hundred and seventy-one.
 (23) Seven thousand and seven. (24) Seven thousand and seventy.
 (25) Seven thousand and seventeen.
 (26) One thousand seven hundred. (27) One thousand and one.
 (28) One thousand and eleven.
 (29) One thousand one hundred and eleven.
 (30) Fifty thousand five hundred and five.
 (31) Seventy thousand and seventy.
 (32) Eighty-eight thousand eight hundred and eighteen.
 (33) One hundred thousand one hundred.
 (34) Three hundred thousand and thirty.
 (35) Nine hundred and nine thousand and nine.
 (36) Eight hundred and eighty thousand and eighty.
 (37) One million and one. (38) One million, one thousand.
 (39) One million, one hundred thousand.
 (40) Ten millions ten thousand and ten.

EXERCISE 2.

To be done first orally, then in writing.

Express in words:—

(1) 11.	(2) 101.	(3) 110.	(4) 12.
(5) 21.	(6) 102.	(7) 201.	(8) 120.
(9) 210.	(10) 23.	(11) 32.	(12) 203.
(13) 29.	(14) 92.	(15) 209.	(16) 902.
(17) 290.	(18) 920.	(19) 9002.	(20) 9020.
(21) 9200.	(22) 9012.	(23) 9021.	(24) 1902.
(25) 9990.	(26) 10001.	(27) 20500.	(28) 70060.
(29) 17007.	(30) 300003.	(31) 300300.	(32) 303013.
(33) 406060.	(34) 401606.	(35) 507005.	(36) 600800.
(37) 900009.	(38) 1020003.	(39) 2002002.	(40) 3000300.

SIMPLE ADDITION.

23. **Addition** is the process of finding a *single* number to represent the same as two or more given numbers together, the numbers being of the same kind.

24. The result of addition is called the **sum**, or **amount**, of the *given numbers*.

25. The mark $+$ placed between two numbers denotes that they are to be added together, and is called the **sign of addition**, or **plus**.

Thus $3 + 4$ denotes that 4 is to be added to 3.

In words, $3 + 4$ is read thus:—three *plus* four.

26. For the word *equals*, or the words *is equal to*, we use the sign $=$, which is called the **sign of equality**.

Thus $3 + 1 = 4$ means that 3 *plus* 1, or 3 with 1 added to it, *is equal* to 4.

27. In the addition of numbers having two or more digits the numbers must be arranged so that units are under units, tens under tens, and so on. To find the sum, we add together the digits in the several columns in turn, beginning with the right-hand column.

Example 1. Add together 328, 129, 293, and 534.

We arrange the numbers as in the margin. The mental steps are as follows:—

- | | |
|---|------|
| 1st. Adding the <i>units</i> , we get 7, 16, 24. | |
| 2nd. We write down 4 units, and carry 2 tens to the tens column. | 328 |
| 3rd. Adding the <i>tens</i> , we get 5, 14, 16, 18. | 129 |
| 4th. We write down 8 tens and carry 1 hundred to the hundreds column. | 293 |
| 5th. Adding the <i>hundreds</i> , we get 6, 8, 9, 12; and we write down 12. | 534 |
| | 1284 |

The sum is therefore 1284.

Example 2. Add together 739, 62604, 85, 91876, and 8753.

We arrange the numbers as in the margin. The mental steps are as follows:—

- | | |
|---|--------|
| 1st. 9, 14, 18, 27; write down 7 and carry 2. | 739 |
| 2nd. 7, 14, 22, 25; write down 5 and carry 2. | 62604 |
| 3rd. 9, 17, 23, 30; write down 0 and carry 3. | 85 |
| 4th. 11, 12, 14; write down 4 and carry 1. | 91876 |
| 5th. 10, 16; write down 16. | 8753 |
| | 164057 |

The sum is therefore 164057.

EXERCISE 3.

Add together

(1) 16	(2) 68	(3) 9	(4) 15	(5) 4	(6) 41
34	10	63	26	59	9
9	9	8	33	7	19
28	8	15	9	26	13
—	—	—	—	—	—

(7)	63 26 17 28 —	(8)	37 26 43 15 —	(9)	42 37 59 16 —	(10)	54 26 78 13 —	(11)	37 59 15 23 —	(12)	49 68 27 16 —
(13)	121 315 247 168 —	(14)	819 287 165 139 —	(15)	672 39 174 17 —	(16)	29 473 287 532 —	(17)	607 198 327 49 —	(18)	183 873 469 8 —
(19)	8694 469 4832 894 —	(20)	137 2986 398 8162 —	(21)	7148 865 1872 654 —	(22)	1276 609 3874 4692 —	(23)	9216 79 1724 657 —	(24)	693 7894 6473 1069 —
(25)	5216 8947 865 8906 —	(26)	1937 4654 712 8893 —	(27)	9872 36 107 2465 —	(28)	7926 647 3825 738 —	(29)	632 9206 387 1231 —	(30)	73 8094 3987 865 —
(31)	56025 3241 15623 105 22345 —	(32)	43203 52062 4501 56324 456 —	(33)	4762 35670 20134 117 23016 —	(34)	121 32003 50201 4867 208 —	(35)	6543 20123 607 80876 65403 —		
(36)	10203 5060 78093 201 98076 —	(37)	98900 99 32062 4308 70506 —	(38)	203 5108 98 76528 32100 —	(39)	301 54987 40308 939 59989 —	(40)	62018 3789 808 99076 50891 —		
(41)	674265 46987 1632 219385 678 —	(42)	182963 6837 54617 298694 82987 —	(43)	820763 692 398201 69839 4625 —	(44)	732465 73 6384 13865 209604 —	(45)	329619 7 65983 602306 86094 —		
(46)	36 169 4625 987654 328765	(47)	465 3083 62198 760708 329065	(48)	2067 46986 618324 367065 98764	(49)	43165 320647 63706 965298 39865	(50)	629865 68307 183065 67894 236085		

(51)	4562	(52)	5681	(53)	6234	(54)	789	(55)	3621
	801		14987		876001		2304		14289
	832302		362		32089		85		7021
	48		184321		2013		91878		847635
	5321		7028		59		321089		62345
	708		362		8001		457		987206
<hr/>									
(56)	906451	(57)	372041	(58)	45210	(59)	3051	(60)	83
	3798		8762		35		842		9
	14587		90308		3703		76531		78034
	357		63		886		849800		529
	80913		562891		970654		7632		989498
	562874		4783		38905		74		8762

Add together

(61) 37420, 56, 89201, 7502, 342, 729865, and 6345.

(62) 8476, 29, 134209, 37561, 421, 587642, and 709.

(63) 56201, 323, 9992, 476500, 1291, 189765, and 18652.

(64) 37651, 843, 920712, 13520, 2159, 234169, and 7452.

(65) 34184, 804, 680824, 6, 3289, 738296, and 896.

(66) 808426, 30180, 561489, 862, 10, 374529, and 18756.

(67) Twenty-nine, two hundred and nineteen, twenty thousand and ninety-two, seven thousand eight hundred and four, eighty thousand and twenty-nine, two thousand and sixteen, twenty thousand and sixty-one, four thousand and ninety-four, eighteen thousand and seventy-five, and thirty thousand two hundred and eight.

(68) Fourteen thousand and nineteen, four hundred thousand nine hundred and nine, eight hundred thousand two hundred and eighty-four, fifty thousand four hundred and sixty, seventy-five thousand and ninety-five, seven hundred thousand five hundred and sixty, two hundred and fifty thousand seven hundred and sixteen, four hundred and twenty-nine thousand eight hundred and ninety-two, three hundred and sixty-four thousand two hundred and seventy, and nine hundred thousand nine hundred and nineteen.

Find the sum of

(69) Eighty-seven thousand and ninety, eight hundred and seventy thousand and nineteen, eight thousand and nine, eight hundred and seventy-eight, eight hundred and seventeen thousand seven hundred and eighty, two hundred and sixteen thousand five hundred and four, forty thousand seven hundred and ninety-two, three hundred and sixty-five thousand and thirty-eight, forty thousand six hundred and eighteen, and three hundred and five.

(70) Three thousand and thirteen, thirty thousand and three, three hundred thousand three hundred, nine hundred and nineteen thousand and ninety-nine, twenty-three thousand and eighty, four hundred thousand and forty-four, five hundred thousand and fifteen, seventy thousand and seventy, seven thousand seven hundred, and two hundred thousand and ninety-seven.

SIMPLE SUBTRACTION.

28. **Subtraction** is the process of finding what remains when one number is taken away from another of the same kind.

29. The number taken away is called the **subtrahend**.

30. The number from which it is taken is called the **minuend**.

31. The result is called the **difference**, or **remainder**.

32. Subtraction is denoted by a short horizontal line — placed between the numbers.

Thus $8 - 5$ means that 5 is to be taken from 8.

In words, $8 - 5$ is read thus:—eight *minus* five, or eight *less* five.

33. The line — is called the **sign of subtraction**, or **minus**.

34. If the minuend and the subtrahend are *equal* numbers, the difference is 0.

35. In the subtraction of large numbers the subtrahend is placed beneath the minuend so that units are under units, tens under tens, and so on.

Example 1. From 9568 take 7133.

We arrange the numbers as in the margin. The mental steps are as follows:—

1st. 3 from 8 leave 5; write down 5 in the units column.

2nd. 3 from 6 leave 3; write down 3 in the tens column. 9568

3rd. 1 from 5 leaves 4; write down 4 in the hundreds column. 7133

4th. 7 from 9 leave 2; write down 2 in the thousands column. 2435

The difference is 2435.

36. In this example each of the digits in the lower line is less than the digit immediately above it, and the process of subtraction is easy. If, however, a digit in the lower line be not less than the digit above it we proceed as in the next example.

37. The general rule for subtraction depends upon the following fact:—

The difference of two numbers is unaltered, when we add the same number to both.

Example 2. Subtract 2784 from 6542.

1st. Since 4 units cannot be taken from 2 units, we add 1 ten (i.e. 10 units) to the 2 units and also to the 8 tens. We then have 6542
2784

6542 = 6 thousands 5 hundreds 4 tens 12 units,
and 2784 = 2 thousands 7 hundreds 9 tens 4 units.

We can now subtract 4 units from 12 units. and we have 8 units left.

2nd. Since 9 tens cannot be taken from 4 tens, we add 1 hundred (i.e. 10 tens) to the 4 tens and 1 hundred to the 7 hundreds. We then have

6542=6 thousands 5 hundreds 14 tens 12 units,
and 2784=2 thousands 8 hundreds 9 tens 4 units.

We can now subtract 9 tens from 14 tens, and we have 5 tens left.

3rd. Again, 8 hundreds cannot be taken away from 5 hundreds, so we add 1 thousand (i.e. 10 hundreds) to the 5 hundreds and also 1 thousand to the 2 thousands. We then have

6542=6 thousands 15 hundreds 14 tens 12 units,
2784=3 thousands 8 hundreds 9 tens 4 units.

We can now subtract 8 hundreds from 15 hundreds, and we have 7 hundreds left.

4th. We take 3 thousands from 6 thousands, and we have 3 thousands left.

The result is 3 thousands 7 hundreds 5 tens 8 units.

The mental steps are as follows:—

1st. 4 from 12 leaves 8; write down 8, and carry 1.

2nd. 8+1=9; 9 from 14 leaves 5; write down 5, and carry 1.

3rd. 7+1=8; 8 from 15 leaves 7; write down 7, and carry 1.

4th. 2+1=3; 3 from 6 leaves 3; write down 3.

The result is 3758.

6542
2784
3758

38. In each of the examples in Arts. 35 and 37 it will be found that the second and third lines of figures, on being added together, give the first line of figures. This will happen in every case. Thus Subtrahend + Remainder = Minuend.

39. This leads us to consider another method of subtraction by which we determine how much must be added to the subtrahend to make up the minuend.

Example. Subtract 2784 from 6542.

The mental steps will be as follows:—

1st. 4 and 8 make 12; put down 8;

2nd. 1 to 8 makes 9, and 5 make 14; put down 5;

3rd. 1 to 7 makes 8, and 7 make 15; put down 7;

4th. 1 to 2 makes 3, and 3 make 6; put down 3.

The result is therefore 3758 as before.

6542	
2784	
<u>3758</u>	
	24
	36
	48
	60
	72
	84
	96
	108
	120
	132
	144

EXERCISE 4.

Find the difference between the following pairs of numbers

(1) 6478 1325	(2) 7256 2123	(3) 5689 3424	(4) 6768 3036	(5) 1794 1360
------------------	------------------	------------------	------------------	------------------

(7) $\begin{array}{r} 3754 \\ 3268 \\ \hline \end{array}$	(8) $\begin{array}{r} 8471 \\ 8394 \\ \hline \end{array}$	(9) $\begin{array}{r} 4723 \\ 3526 \\ \hline \end{array}$	(10) $\begin{array}{r} 5636 \\ 1239 \\ \hline \end{array}$	(11) $\begin{array}{r} 6320 \\ 4568 \\ \hline \end{array}$	(12) $\begin{array}{r} 7562 \\ 6219 \\ \hline \end{array}$
(13) $\begin{array}{r} 9634 \\ 2985 \\ \hline \end{array}$	(14) $\begin{array}{r} 6005 \\ 3008 \\ \hline \end{array}$	(15) $\begin{array}{r} 8307 \\ 2696 \\ \hline \end{array}$	(16) $\begin{array}{r} 7065 \\ 3928 \\ \hline \end{array}$	(17) $\begin{array}{r} 6003 \\ 3090 \\ \hline \end{array}$	(18) $\begin{array}{r} 7060 \\ 6792 \\ \hline \end{array}$
(19) $\begin{array}{r} 32963 \\ 13069 \\ \hline \end{array}$	(20) $\begin{array}{r} 41690 \\ 29731 \\ \hline \end{array}$	(21) $\begin{array}{r} 89012 \\ 74680 \\ \hline \end{array}$	(22) $\begin{array}{r} 93465 \\ 84703 \\ \hline \end{array}$	(23) $\begin{array}{r} 97531 \\ 89427 \\ \hline \end{array}$	
(24) $\begin{array}{r} 70324 \\ 29865 \\ \hline \end{array}$	(25) $\begin{array}{r} 63072 \\ 18094 \\ \hline \end{array}$	(26) $\begin{array}{r} 52706 \\ 19839 \\ \hline \end{array}$	(27) $\begin{array}{r} 39064 \\ 29987 \\ \hline \end{array}$	(28) $\begin{array}{r} 93007 \\ 16090 \\ \hline \end{array}$	
(29) $\begin{array}{r} 874312 \\ 197890 \\ \hline \end{array}$	(30) $\begin{array}{r} 400076 \\ 19832 \\ \hline \end{array}$	(31) $\begin{array}{r} 403298 \\ 117629 \\ \hline \end{array}$	(32) $\begin{array}{r} 741936 \\ 249704 \\ \hline \end{array}$	(33) $\begin{array}{r} 329638 \\ 19806 \\ \hline \end{array}$	
(34) $\begin{array}{r} 420653 \\ 30906 \\ \hline \end{array}$	(35) $\begin{array}{r} 619875 \\ 40699 \\ \hline \end{array}$	(36) $\begin{array}{r} 367432 \\ 50698 \\ \hline \end{array}$	(37) $\begin{array}{r} 427065 \\ 190807 \\ \hline \end{array}$	(38) $\begin{array}{r} 612069 \\ 806085 \\ \hline \end{array}$	

Perform the operations indicated below:—

- (39) $820394 - 123456$. (40) $196740 - 124056$. (41) $700604 - 654829$.
 (42) $260306 - 239084$. (43) $802070 - 743409$. (44) $376004 - 160640$.
 (45) $960304 - 308078$. (46) $464205 - 209087$. (47) $610324 - 309067$.
 (48) $760326 - 107809$. (49) $506047 - 446098$. (50) $807065 - 298798$.
 (51) $490023 - 134067$. (52) $998207 - 307069$. (53) $700604 - 608395$.
 (54) $340607 - 208308$. (55) $807260 - 236087$. (56) $862986 - 769392$.
 (57) $932082 - 369325$. (58) $730602 - 698075$.

(59) By what number is eight thousand and sixteen greater than four thousand and fifty-three?

(60) By what number is three thousand two hundred and five less than seven thousand and eighty?

(61) What is the difference between thirty thousand six hundred and twelve and nine thousand seven hundred and eight?

(62) Find the difference between nine thousand five hundred and seven and twelve thousand three hundred and seventy-one.

(63) What number must be taken from eighty thousand seven hundred and three to leave eighteen hundred and ninety-four?

Ex. What number must be added to one hundred and eighteen to make two hundred and fifty?
 i.e. 100 + three hundred and two?

Subtract twenty thousand seven hundred and eighty-three from one hundred and fifteen thousand and nineteen.

We can now do

(66) What number must be subtracted from eight hundred and sixty thousand and thirty-two to leave ninety-one thousand two hundred and four?

(67) The sum of two numbers is four hundred and seventy thousand and eight, and one of them is nine thousand and eighty-nine; what is the other?

(68) From five hundred thousand two hundred and four take twenty thousand and nineteen.

(69) Take nine thousand two hundred and sixty from one hundred thousand three hundred and sixteen.

(70) The difference between two numbers is fourteen hundred and eight, and the greater number is four hundred thousand five hundred and two, what is the smaller number?

SIMPLE MULTIPLICATION.

40. **Multiplication** is a short way of finding the sum of any number of given equal numbers.

41. The number of given equal numbers is called the **multiplier**.

42. The number to be multiplied, i.e., any one of the given equal numbers, is called the **multiplicand**.

43. The result of the multiplication is called the **product** of the multiplier and the multiplicand; and the multiplier and the multiplicand are called the **factors** of the product.

44. Multiplication is denoted by an *inclined* cross \times , which is called the **sign of multiplication**, and is placed between the numbers.

Forexample, 3 multiplied by 4, which means $3 + 3 + 3 + 3$, is written 3×4 .

In words, 3×4 is read 3 *multiplied by* 4.

45. It will be necessary to commit to memory the following table of results, called the **Multiplication Table**.

	3	4	5	6	7	8	9	10	11	12
Twice times	times	times	times	times	times	times	times	times	times	times
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7	1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 ... 4	2 ... 6	2 ... 8	2 ... 10	2 ... 12	2 ... 14	2 ... 16	2 ... 18	2 ... 20	2 ... 22	2 ... 24
3 ... 6	3 ... 9	3 ... 12	3 ... 15	3 ... 18	3 ... 21	3 ... 24	3 ... 27	3 ... 30	3 ... 33	3 ... 36
4 ... 8	4 ... 12	4 ... 16	4 ... 20	4 ... 24	4 ... 28	4 ... 32	4 ... 36	4 ... 40	4 ... 44	4 ... 48
5 ... 10	5 ... 15	5 ... 20	5 ... 25	5 ... 30	5 ... 35	5 ... 40	5 ... 45	5 ... 50	5 ... 55	5 ... 60
6 ... 12	6 ... 18	6 ... 24	6 ... 30	6 ... 36	6 ... 42	6 ... 48	6 ... 54	6 ... 60	6 ... 66	6 ... 72
7 ... 14	7 ... 21	7 ... 28	7 ... 35	7 ... 42	7 ... 49	7 ... 56	7 ... 63	7 ... 70	7 ... 77	7 ... 84
8 ... 16	8 ... 24	8 ... 32	8 ... 40	8 ... 48	8 ... 56	8 ... 64	8 ... 72	8 ... 80	8 ... 88	8 ... 96
9 ... 18	9 ... 27	9 ... 36	9 ... 45	9 ... 54	9 ... 63	9 ... 72	9 ... 81	9 ... 90	9 ... 99	9 ... 108
10 ... 20	10 ... 30	10 ... 40	10 ... 50	10 ... 60	10 ... 70	10 ... 80	10 ... 90	10 ... 100	10 ... 110	10 ... 120
11 ... 22	11 ... 33	11 ... 44	11 ... 55	11 ... 66	11 ... 77	11 ... 88	11 ... 99	11 ... 110	11 ... 121	11 ... 132
12 ... 24	12 ... 36	12 ... 48	12 ... 60	12 ... 72	12 ... 84	12 ... 96	12 ... 108	12 ... 120	12 ... 132	12 ... 144

46. By the use of this table we may multiply any number whatever by any other number.

Example. Multiply 5324 by 2.

The multiplier is written under the multiplicand, and a line is drawn below.

We see from the table that twice 4=8, twice 2=4, twice 3=6, and twice 5=10. These figures are put down below the line, and directly under the figures from which they come.

$$\begin{array}{r} 5324 \\ \underline{2} \\ 10648 \end{array}$$

47. When the product of two figures is greater than 9 we set down the right-hand digit and carry forward the other, just as we did in Addition.

Example. Multiply 3458 by 6.

From the table we get

$$\begin{array}{r} 6 \text{ times } 8 = 48, \\ 6 \text{ times } 5 = 30, \\ 6 \text{ times } 4 = 24, \\ \text{and } 6 \text{ times } 3 = 18. \end{array} \quad \begin{array}{r} 3458 \\ \underline{6} \\ 20748 \end{array}$$

Then, just as in Addition, we set down the 8 from the 48, and carry on the 4 to increase the 30. The 30 thus becomes 34; and, as before, we set down the 4, carrying on the 3 to increase the 24. The 24 thus becomes 27; and we set down the 7, carrying on the 2 to increase the 18. The required product is thus found to be 20748.

NOTE.—Multiplication, as we have said, is but a short way of finding the sum of equal numbers. If we take the number 3458, and write it down 6 times, and add these six numbers together, we shall find that the result is 20748.

EXERCISE 5.

Perform the following multiplications:—

- | | |
|------------------------------|------------------------------|
| (1) 2845×2 and 3. | (2) 3957×3 and 4. |
| (3) 3864×4 „ 5. | (4) 2579×5 „ 6. |
| (5) 2596×6 „ 7. | (6) 3874×7 „ 8. |
| (7) 3749×8 „ 9. | (8) 2648×8 „ 11. |
| (9) 2986×9 „ 12. | (10) 3785×11 „ 12. |
| (11) 26907×2 „ 3. | (12) 80692×3 „ 4. |
| (13) 37048×4 „ 5. | (14) 94780×5 „ 6. |
| (15) 40259×6 „ 7. | (16) 25806×7 „ 8. |
| (17) 59301×8 „ 9. | (18) 30954×9 „ 11. |
| (19) 61470×11 „ 12. | (20) 47068×5 „ 12. |
| (21) 638475×2 „ 5. | (22) 296548×4 „ 7. |
| (23) 823964×5 „ 11. | (24) 457283×7 „ 9. |
| (25) 398627×9 „ 4. | (26) 746382×6 „ 11. |
| (27) 369457×12 „ 8. | (28) 458736×11 „ 3. |
| (29) 593267×8 „ 9. | (30) 486392×6 „ 5. |

48. To multiply by 10 we move each figure one place to the left and put 0 in the units' place.

Example. Multiply 75492 by 10.

$$\begin{array}{r} 75492 \\ 10 \\ \hline 754920 \end{array}$$

49. Again, since 20 is ten times as great as 2, to multiply by 20 we multiply by 2 and put 0 to the right of the result; in a similar manner we multiply by 30, 40, etc.

Example. Multiply 4392 by 20.

$$\begin{array}{r} 4392 \\ 20 \\ \hline 87840 \end{array}$$

50. To multiply by 100 we put 00 to the right of the number; and to multiply by 200, 300, etc., we multiply by 2, 3, etc., and put 00 to the right of the result.

Example. Multiply 2587 by 100, and by 300.

$$\begin{array}{r} 2587 \\ 100 \\ \hline 258700 \end{array}$$

$$\begin{array}{r} 2587 \\ 300 \\ \hline 776100 \end{array}$$

EXERCISE 6.

Perform the following multiplications:—

(1) 25478×10 and 20 .

(2) 32765×30 and 40 .

(3) 26934×50 „ 60 .

(4) 84987×70 „ 80 .

(5) 53698×90 „ 100 .

(6) 72453×200 „ 300 .

(7) 24579×400 „ 500 .

(8) 82678×600 „ 700 .

(9) 42468×800 „ 900 .

(10) 68345×1100 „ 1200 .

51. If the multiplier be a number which can easily be separated into factors, we can multiply by these factors in succession.

Example. Multiply 78035 by 24.

$24 = 4 \times 6$, or 3×8 , or 2×12 , and we may use any one of these pairs.

$$\begin{array}{r} 78035 \\ 4 \\ \hline 312140 \\ 6 \\ \hline 1872840 \end{array}$$

$$\begin{array}{r} 78035 \\ 8 \\ \hline 284105 \\ 3 \\ \hline 1872840 \end{array}$$

$$\begin{array}{r} 78035 \\ 2 \\ \hline 156070 \\ 12 \\ \hline 1872840 \end{array}$$

EXERCISE 7.

Multiply, by factors,

- | | | |
|---------------------|---------------------|---------------------|
| (1) 358697 by 21. | (2) 673854 by 45. | (3) 420956 by 49. |
| (4) 783473 by 63. | (5) 567432 by 64. | (6) 908736 by 28. |
| (7) 674319 by 84. | (8) 764131 by 96. | (9) 674319 by 32. |
| (10) 780618 by 56. | (11) 498576 by 48. | (12) 291483 by 81. |
| (13) 940625 by 36. | (14) 291483 by 42. | (15) 693700 by 77. |
| (16) 963076 by 88. | (17) 620375 by 108. | (18) 324675 by 121. |
| (19) 632475 by 132. | (20) 396469 by 144. | |

52. The following method may be used in all cases, whatever the multiplier may be.

Example. Multiply 1342 by 324.

The multiplier 324 is equivalent to $4 + 20 + 300$; hence we multiply 1342 successively by 4, 2, and 3, and we arrange the work as in the margin.

Here 5368 is 4 times 1342,
2684 is twice 1342,
and 4026 is 3 times 1342.

The second product, 2684, is written so that its last digit, 4, comes under the multiplying digit, 2.

The third product, 4026, is placed so that its last digit, 6, comes under the multiplying digit, 3.

However many digits there may be in the multiplier, each product must be placed *with its right-hand digit under that multiplying digit which produces it*.

The products are then added as they stand, and as if the vacant spaces at the right hand were filled with 0's.

The result is found to be 434808.

$$\begin{array}{r}
 1342 \\
 324 \\
 \hline
 5368 \\
 2684 \\
 4026 \\
 \hline
 434808
 \end{array}$$

EXERCISE 8.

Perform the following multiplications:—

- | | |
|-------------------------------|-------------------------------|
| (1) 3217×17 and 18. | (2) 4316×19 and 16. |
| (3) 3187×14 „ 15. | (4) 7329×21 „ 34. |
| (5) 8174×45 „ 37. | (6) 9362×54 „ 32. |
| (7) 4567×89 „ 75. | (8) 3892×98 „ 76. |
| (9) 4705×74 „ 56. | (10) 6809×63 „ 47. |
| (11) 6249×28 „ 93. | (12) 3967×39 „ 84. |
| (13) 57304×39 „ 48. | (14) 62975×46 „ 57. |
| (15) 73296×57 „ 64. | (16) 69485×82 „ 73. |
| (17) 63297×45 „ 37. | (18) 73209×38 „ 49. |
| (19) 63214×76 „ 89. | (20) 32987×92 „ 84. |
| (21) 86329×83 „ 49. | (22) 94785×27 „ 38. |
| (23) 609457×65 „ 87. | (24) 329629×76 „ 82. |

- | | |
|---------------------------------|---------------------------------|
| (25) 469395×38 and 46. | (26) 340629×97 and 83. |
| (27) 560178×29 „ 46. | (28) 902329×85 „ 39. |
| (29) 475304×38 „ 59. | (30) 890635×74 „ 28. |
| (31) 380698×47 „ 83. | (32) 706192×83 „ 57. |
| (33) 4628×153 „ 168. | (34) 8729×175 „ 159. |
| (35) 5623×125 „ 137. | (36) 6742×284 „ 567. |
| (37) 8539×265 „ 874. | (38) 6327×894 „ 325. |
| (39) 7458×625 „ 934. | (40) 8329×329 „ 465. |
| (41) 83967×296 „ 348. | (42) 69581×328 „ 475. |
| (43) 96032×568 „ 439. | (44) 80175×139 „ 458. |
| (45) 70327×621 „ 573. | (46) 82076×329 „ 465. |
| (47) 92075×492 „ 576. | (48) 60739×397 „ 285. |
| (49) 79425×396 „ 485. | (50) 96378×472 „ 596. |
| (51) 684258×372 „ 894. | (52) 729346×483 „ 925. |
| (53) 598724×594 „ 276. | (54) 245639×625 „ 387. |
| (55) 482957×736 „ 458. | (56) 426598×547 „ 239. |
| (57) 537986×658 „ 942. | (58) 648275×769 „ 853. |
| (59) 759684×872 „ 394. | (60) 398765×983 „ 675. |

53. If one or more of the figures of the multiplier be 0, these 0's may be neglected in forming the successive products; but the pupil must be careful to place each product in the position explained in Art. 52 so that its last figure may come under the multiplying figure which produces it.

Example. Multiply 9080 by 307 and 7030.

(i)	9080	(ii)	9080
	307		7080
	<hr/> 63560		<hr/> 272400
	27240		63560
	<hr/> 2787560		<hr/> 63832400

EXERCISE 9.

Perform the following multiplications:—

- | | |
|----------------------------------|----------------------------------|
| (1) 70690×408 and 2090. | (2) 68070×350 and 6080. |
| (3) 90560×607 „ 3200. | (4) 25700×905 „ 4030. |
| (5) 80050×820 „ 6500. | (6) 37900×750 „ 3040. |
| (7) 70080×620 „ 5400. | (8) 62900×508 „ 2070. |
| (9) 50670×820 „ 3090. | (10) 20900×730 „ 4080. |
| (11) 130465×607 „ 3005. | (12) 206532×805 „ 4008. |
| (13) 674032×403 „ 5006. | (14) 314708×703 „ 6008. |

(15) 215689×208 and 3007.

(16) 459638×509 and 8008.

(17) 600524×340 „ 6070.

(18) 730086×480 „ 3060.

(19) 324008×507 „ 4600.

(20) 476500×309 „ 5700.

54. The result of multiplying three or more numbers together is called the **continued product** of the numbers.

Thus the continued product of 5, 7, and $9 = 5 \times 7 \times 9$
 $= 35 \times 9$
 $= 315.$

55. If a number be multiplied by itself, the product is called the **square**, or *second power*, of the number.

Thus the square of 8 is 8×8 , that is, 64.

56. If a number be multiplied by itself twice in succession, the product is called the **cube**, or *third power*, of the number.

Thus the cube of 8 is $8 \times 8 \times 8$, that is, 512.

EXERCISE 10.

Find the continued product of

(1) 28, 365 and 1749.

(2) 37, 489 and 1265.

(3) 49, 578 and 1362.

(4) 12, 53 and 546.

(5) 17, 25 and 368.

(6) 19, 67 and 482.

Find the square of

(7) 365.

(8) 578.

(9) 694.

(10) 752.

(11) 869.

(12) 1629.

(13) 2537.

(14) 3648.

(15) 4729.

(16) 5837.

Find the cube of

(17) 25.

(18) 36.

(19) 47.

(20) 58.

(21) 205.

(22) 306.

(23) 407.

(24) 508.

(25) 620.

(26) 730.

SIMPLE DIVISION.

57. **Division** is the process of finding how often one number contains another. The former is said to be divided by the latter.

58. The number to be divided is called the **dividend**.

59. The number by which it is divided is called the **divisor**.

60. The number which tells how many times the divisor is contained in the dividend is called the **quotient**.

61. Division is denoted by the mark \div , which is called the **sign of division**, and is placed between the numbers, *after* the dividend and *before* the divisor.

Thus $8 \div 2$ means 8 *divided by* 2.

62. If the dividend does not contain the divisor an exact number of times, and we take away from the dividend as many times the divisor as we can, what is left is called the **remainder**. When there is no remainder, the division is said to be **exact**.

Thus, 8 contains 2 four times exactly, and there is no remainder. Also 9 contains 2 four times, and leaves a *remainder* 1.

63. When the divisor is not greater than 12, the division can be performed mentally, and the process is called **Short Division**. We proceed as follows:—

Divide the first figure at the left hand of the dividend by the divisor, and write down the quotient underneath that figure.

If there is no remainder, then divide the next figure by the divisor, and set down the quotient underneath it as before, and so on with each figure in succession.

Example 1. Divide 4628 by 2.

Each of the digits in the dividend can itself be divided by 2; hence we can write down the quotient at once, thus:—

$$\begin{array}{r} 2 \overline{) 4628} \\ \underline{2314} \end{array}$$

64. If the divisor does not go exactly into each of the figures of the dividend, we proceed thus:—

Example 2. Divide 63784 by 4.

$$\begin{array}{r} 4 \overline{) 63784} \\ \underline{15946} \end{array}$$

Dividing 6 by 4 the quotient is 1, and the remainder is 2; placing the 2 before the next figure 3, we get 23; dividing 23 by 4, the quotient is 5, and the remainder 3. Now dividing 37 by 4, we get 9 for quotient, and 1 for remainder, and so on. Each quotient digit is put down as before, under that digit which produces it.

65. If the divisor is greater than the first figure of the dividend, we take the first two figures, or more, together.

Example 3. Divide 37232 by 6.

Proceeding as in the former examples, we find that the quotient is 6205, and that there is a remainder 2.

$$\begin{array}{r} 6 \overline{) 37232} \\ \underline{6205} \end{array} \text{ remainder } 2.$$

EXERCISE 11.

Perform the following divisions:—

- | | | | |
|---------------------|---------------------|---------------------|---------------------|
| (1) $46984 \div 2.$ | (2) $87562 \div 2.$ | (3) $69576 \div 3.$ | (4) $97455 \div 3.$ |
| (5) $58268 \div 4.$ | (6) $98756 \div 4.$ | (7) $67896 \div 5.$ | (8) $47835 \div 5.$ |

- (9) $88068 \div 6$. (10) $57498 \div 6$. (11) $87395 \div 7$. (12) $51786 \div 7$.
 (13) $28632 \div 8$. (14) $37496 \div 8$. (15) $31212 \div 9$. (16) $52128 \div 9$.
 (17) $29183 \div 11$. (18) $41679 \div 11$. (19) $30756 \div 12$. (20) $57504 \div 12$.

In the following divisions find both quotient and remainder :—

- (21) $730435 \div 2$. (22) $833785 \div 2$. (23) $761914 \div 3$. (24) $113564 \div 3$.
 (25) $103749 \div 4$. (26) $151579 \div 4$. (27) $284662 \div 5$. (28) $381993 \div 5$.
 (29) $368746 \div 6$. (30) $441169 \div 6$. (31) $345502 \div 7$. (32) $479447 \div 7$.
 (33) $285969 \div 8$. (34) $422987 \div 8$. (35) $287839 \div 9$. (36) $448887 \div 9$.
 (37) $278489 \div 11$. (38) $406398 \div 11$. (39) $358479 \div 12$. (40) $450557 \div 12$.

66. If the divisor be a number which can easily be separated into factors each less than 12, we can divide by each of these factors in succession.

Example. Divide 61075 by 35.

We see that $35 = 5 \times 7$; so we will divide 61075 by 5, and the quotient by 7.

The required quotient is 1745, and is thus obtained by two operations in *short* division.

$$35 \left\{ \begin{array}{r} 5 \overline{) 61075} \\ 7 \overline{) 12215} \\ \hline 1745 \end{array} \right.$$

EXERCISE 12.

Perform the following divisions :—

- | | | |
|-------------------------|-------------------------|-------------------------|
| (1) $67606 \div 14$. | (2) $56070 \div 15$. | (3) $42384 \div 16$. |
| (4) $41904 \div 18$. | (5) $72513 \div 21$. | (6) $88176 \div 24$. |
| (7) $96875 \div 25$. | (8) $80568 \div 27$. | (9) $38612 \div 28$. |
| (10) $58528 \div 32$. | (11) $60690 \div 35$. | (12) $59472 \div 36$. |
| (13) $65688 \div 42$. | (14) $66420 \div 45$. | (15) $66480 \div 48$. |
| (16) $69876 \div 54$. | (17) $65285 \div 55$. | (18) $71456 \div 56$. |
| (19) $86184 \div 63$. | (20) $93376 \div 64$. | (21) $68796 \div 108$. |
| (22) $69454 \div 121$. | (23) $97416 \div 132$. | (24) $93168 \div 144$. |

67. If there should be a remainder after the division by either or both of the factors, the quotient is obtained as in the preceding examples; but to get the true remainder we multiply the second remainder by the first divisor, and add the first remainder.

Example. Divide 83249 by 63.

We see that $63 = 7 \times 9$; so we will divide successively by 7 and 9.

$$63 \left\{ \begin{array}{r} 7 \overline{) 83249} \\ 9 \overline{) 11892-5} \\ \hline 1321-3 \end{array} \right\} 26$$

The first remainder, 5, denotes 5 units; the second remainder, 3, denotes

3 *sevens*; hence, to obtain the true remainder, we multiply the second remainder, 3, by the first divisor, 7, and we add the first remainder, 5, to the result. We thus get 26 for the complete remainder. The quotient is 1321.

EXERCISE 13.

In the following divisions find both quotient and remainder:—

- | | | |
|--------------------------|--------------------------|--------------------------|
| (1) $162347 \div 15$. | (2) $294513 \div 28$. | (3) $343894 \div 45$. |
| (4) $593693 \div 63$. | (5) $237159 \div 16$. | (6) $383624 \div 27$. |
| (7) $452705 \div 42$. | (8) $682549 \div 64$. | (9) $348261 \div 18$. |
| (10) $472705 \div 24$. | (11) $561618 \div 48$. | (12) $791432 \div 66$. |
| (13) $459372 \div 21$. | (14) $561831 \div 32$. | (15) $670526 \div 54$. |
| (16) $870361 \div 72$. | (17) $560483 \div 25$. | (18) $650947 \div 36$. |
| (19) $789437 \div 56$. | (20) $968745 \div 81$. | (21) $729364 \div 108$. |
| (22) $938652 \div 121$. | (23) $638498 \div 132$. | (24) $397683 \div 144$. |

68. When the divisor is 10 or 100, the quotient and remainder can be written down at once. When we divide by 10, the *last* figure of the dividend is the remainder; and when we divide by 100, the last *two* figures of the dividend form the remainder. The other figures of the dividend form the quotient.

Example. $73269 \div 10$ gives 7326 as quotient, and 9 as remainder.
 $73269 \div 100$ gives 732 as quotient, and 69 as remainder.

EXERCISE 14.

Write down the quotient and remainder when each of the following numbers is divided (i) by 10; (ii) by 100.

- | | | |
|--------------|--------------|--------------|
| (1) 6234751. | (2) 7298632. | (3) 4980736. |
| (4) 3293875. | (5) 5016893. | (6) 8564078. |
| (7) 9214657. | (8) 6980079. | (9) 3872058. |

69. To divide by 20, 30, 40, etc., we proceed thus:—

Example. Divide 637825 by 40.

$$\begin{array}{r} 40 \overline{) 63782.5} \\ \underline{15945} - 25 \end{array}$$

We know that the easiest factors of 40 are 10 and 4. To divide by 10 we mark off the 5; that is, $637825 : 10$ gives 63782 *tens* for quotient, and 5 *units* as remainder. Next we divide 63782 by 4, which is the other factor, and we have 15945 for quotient, and 2 as remainder. But it must be remembered that this remainder is 2 *tens*, and when we add to it the first remainder, 5, we get the true remainder, 25.

70. To divide by 200, 300, 400, etc., we proceed in a similar way, but to divide by 100 we mark off the *two* last figures.

EXERCISE 15.

Perform the following divisions.—

- | | | |
|--------------------|---------------------|---------------------|
| (1) 917345 ÷ 20. | (2) 851709 ÷ 30. | (3) 795164 ÷ 40. |
| (4) 639528 ÷ 50. | (5) 828436 ÷ 60. | (6) 762891 ÷ 70. |
| (7) 606255 ÷ 80. | (8) 540619 ÷ 90. | (9) 639527 ÷ 200. |
| (10) 573982 ÷ 300. | (11) 417846 ÷ 400. | (12) 351792 ÷ 500. |
| (13) 740618 ÷ 600. | (14) 684073 ÷ 700. | (15) 528437 ÷ 800. |
| (16) 462863 ÷ 900. | (17) 629392 ÷ 1100. | (18) 983495 ÷ 1200. |

71. When the operations are written in full, as in the following examples, we have what is called **Long Division**.

Example 1. Divide 142817 by 31.

We begin at the left of the dividend and we take as many figures as are required to make up a number greater than the divisor; in this case we must take the three figures 142.

We find that 142 contains 31 four times, with a remainder; hence 4 is the first figure in the quotient. Subtracting 4 times 31 from 142, the remainder is 18.

$$\begin{array}{r}
 31 \overline{) 142817} \quad \underline{4607} \\
 124 \\
 \hline
 188 \\
 186 \\
 \hline
 217 \\
 217 \\
 \hline

 \end{array}$$

Putting at the right of this the next figure 8 of the dividend we get 188, and we find that 188 contains 31 six times, with a remainder; hence 6 is the second figure in the quotient.

Subtracting 6 times 31 from 188, the remainder is 2.

Putting the next figure of the dividend, namely 1, at the right of 2, we get 21. But 31 is not contained in 21; hence we place 0 as the third figure of the quotient.

Next putting 7 at the end of 21 we get 217, and we find that 31 is contained in 217 seven times exactly; hence the fourth figure of the quotient is 7, and the whole quotient is 4607.

CAUTION 1.—At every step the pupil must be careful to have a remainder smaller than the divisor.

CAUTION 2.—For every figure brought down from the dividend a figure must be placed in the quotient.

Example 2. Divide 94276 by 37.

First we take the figures 94 to make up a number greater than the divisor.

For the first figure of the quotient we try 3, but $37 \times 3 = 111$, which is greater than 94; therefore 3 is too great, so we try 2. Now $37 \times 2 = 74$, therefore 2 is the first figure of the quotient.

Subtracting 74 from 94, the remainder is 20; and putting the next figure 2 of the dividend at the end of 20, we get 202. We then proceed as before till all the figures of the dividend are brought down

$$\begin{array}{r}
 37 \overline{) 94276} \quad \underline{2548} \\
 74 \\
 \hline
 202 \\
 185 \\
 \hline
 177 \\
 148 \\
 \hline
 296 \\
 296 \\
 \hline

 \end{array}$$

EXERCISE 16.

Perform the following divisions:—

- | | | |
|-------------------------|-------------------------|-------------------------|
| (1) $10044 \div 31$. | (2) $13284 \div 41$. | (3) $11934 \div 51$. |
| (4) $14274 \div 61$. | (5) $11076 \div 71$. | (6) $12636 \div 61$. |
| (7) $14196 \div 91$. | (8) $10140 \div 52$. | (9) $11408 \div 62$. |
| (10) $12874 \div 82$. | (11) $13708 \div 92$. | (12) $10511 \div 23$. |
| (13) $11567 \div 43$. | (14) $13674 \div 53$. | (15) $18031 \div 73$. |
| (16) $14442 \div 83$. | (17) $17205 \div 93$. | (18) $13430 \div 34$. |
| (19) $10138 \div 74$. | (20) $18912 \div 94$. | (21) $12285 \div 65$. |
| (22) $19875 \div 75$. | (23) $12495 \div 85$. | (24) $16720 \div 95$. |
| (25) $183141 \div 19$. | (26) $247863 \div 29$. | (27) $365352 \div 39$. |
| (28) $506928 \div 59$. | (29) $585948 \div 69$. | (30) $599057 \div 79$. |
| (31) $505431 \div 89$. | (32) $162979 \div 17$. | (33) $330706 \div 37$. |
| (34) $368762 \div 47$. | (35) $494646 \div 57$. | (36) $535129 \div 67$. |

72. We may proceed as in Art. 71, however large the divisor may be.

Example. Divide 562464 by 324.

NOTE.—To help the pupil in finding the successive figures of a quotient he might begin by writing down a table such as follows:—

$324 \times 2 = 648$.
$324 \times 3 = 972$.
$324 \times 4 = 1296$.
$324 \times 5 = 1620$.
$324 \times 6 = 1944$.
$324 \times 7 = 2268$.
$324 \times 8 = 2592$.
$324 \times 9 = 2916$.

$$\begin{array}{r}
 324 \overline{) 562464} \quad (1736 \\
 \underline{324} \\
 2384 \\
 \underline{2268} \\
 1166 \\
 \underline{972} \\
 1944 \\
 \underline{1944} \\
 0
 \end{array}$$

EXERCISE 17.

Perform the following divisions:

- | | | |
|----------------------------|----------------------------|----------------------------|
| (1) $1062765 \div 113$. | (2) $2070966 \div 258$. | (3) $2718258 \div 362$. |
| (4) $2703009 \div 447$. | (5) $2836512 \div 504$. | (6) $2864673 \div 659$. |
| (7) $2924125 \div 745$. | (8) $2199879 \div 891$. | (9) $1760161 \div 923$. |
| (10) $1323735 \div 147$. | (11) $1872720 \div 216$. | (12) $2564250 \div 325$. |
| (13) $2712096 \div 438$. | (14) $3199878 \div 594$. | (15) $2933216 \div 641$. |
| (16) $2603424 \div 752$. | (17) $2431485 \div 837$. | (18) $1761480 \div 932$. |
| (19) $2859912 \div 471$. | (20) $2815058 \div 562$. | (21) $3127500 \div 695$. |
| (22) $2598015 \div 763$. | (23) $2053900 \div 874$. | (24) $1214928 \div 936$. |
| (25) $1018584 \div 1032$. | (26) $1128256 \div 1156$. | (27) $1190810 \div 1234$. |

- (28) $2008773 \div 2301$. (29) $2759988 \div 2987$. (30) $2780765 \div 3107$.
 (31) $3098784 \div 4056$. (32) $3949936 \div 5708$. (33) $3523174 \div 6002$.
 (34) $3423615 \div 7059$. (35) $3544019 \div 8927$. (36) $2568180 \div 9305$.

73. The relation between Divisor, Dividend, and Quotient in **exact** division may be written in either of the two forms

and
$$\begin{aligned} \text{Dividend} \div \text{Divisor} &= \text{Quotient}, \\ \text{Quotient} \times \text{Divisor} &= \text{Dividend}. \end{aligned}$$

74. In **inexact** division

$$\text{Quotient} \times \text{Divisor} + \text{Remainder} = \text{Dividend}.$$

MISCELLANEOUS EXAMPLES.

The Four Simple Rules.

EXERCISE 18.

(1) At the beginning of the year there were 1300 scholars in a school. During the year, 367 were admitted, and 189 left. How many scholars were there at the end of the year?

(2) A boy lost 257 marbles out of 802; then he won 144, after which he lost 224; how many had he left?

(3) Harry has 3 bags of marbles, each containing 260, and Willie has 6 bags, each containing 306; how many marbles have they together?

(4) Tom has 5 lots of marbles, each containing 507, and Bob has 4 lots of 608 marbles each; which has the most, and by how many?

(5) A school contains 165 boys and 123 girls; how many buns would be required to give 3 to each girl and 2 to each boy?

(6) If each of 36 trucks in a goods train contains 18 barrels of beer, and each barrel contains 36 gallons, how many gallons of beer is the train carrying?

(7) How many yards of calico would a factory working 10 hours a day make in a week (6 days), from 305 looms, each making 5 yards an hour?

(8) Twelve trains of 9 carriages each run daily on a railway, and each carriage contains 47 persons; how many people are conveyed in a week (7 days)?

(9) There were 6032 soldiers in a camp, and a quarter of them marched away; how many were left?

(10) A boy had 652 marbles; he kept 28, and divided the rest equally among 8 boys; how many did each boy receive?

(11) Tom and Dick had 462 nuts between them; Dick had a third of them; how many nuts had Tom?

(12) A shopkeeper buys 851 bundles of wood, each containing 264 pieces; how many smaller bundles, each containing 148 pieces, can be made from them?

(13) Each of 155 bags contains 208 nuts. If the nuts are divided among 124 boys, how many will each get?

(14) If 87447 pennies are to be divided equally among 52 men and 51 women, how many will each receive?

(15) If 4281 walnuts, 1135 chestnuts, and 2954 brazil nuts are divided equally among 93 boys, how many nuts does each get?

(16) There are 20 heaps of apples, each consisting of 481. If the apples are divided equally among 37 boys and 37 girls, how many will each receive?

(17) If walnuts are sold at 14 a penny and chestnuts at 20 a penny, how many pence must be paid for 1372 walnuts and 1940 chestnuts?

(18) A merchant bought 56 boxes of eggs, each box holding 120 eggs. 5 eggs in each box were bad. He repacked the good eggs in 40 boxes each holding the same number of eggs; how many did he put into each box?

(19) A merchant bought 3150 dozen oranges. After throwing away 18 score of bad ones he put the rest into boxes, each containing half a gross; how many boxes did he fill?

(20) A newspaper boy sells six dozen papers each day on Monday, Tuesday, Thursday, and Friday, and twice as many each day on Wednesday and Saturday; how many papers will he sell in a year (52 weeks)?

(21) A tram-car carries 1052 people on Monday, 2106 on Tuesday, 1001 on Wednesday, 2093 on Thursday, 1064 on Friday, and 2560 on Saturday; how many more passengers are there during the last three days than during the first three days?

(22) What is the product of 7209 and 29?

(23) The multiplicand is 98760; multiply it by 76.

(24) Find 56 times 74089.

(25) What is the sum of 30 numbers, each of which is 6789?

(26) What number is 39 times as great as 4576?

(27) Add the product of 197 and 506 to their difference.

(28) Find the product of the sum and difference of 584 and 756.

(29) By how much is 200 times 1080 greater than 150 times 1018?

(30) Take the sum of 708 and 590 from their product.

(31) What number must be added to 45 times 57 to make 4000?

(32) Find the difference between 23 times 107 and the 23rd part of 6210.

(33) Divide the difference between 72427 and 90005 by 34.

(34) Divide the product of 376 and 9682 by 47

(35) Divide the sum of 9206, 15052, 163, and 10600 by 68.

(36) How many times is 38 contained in 7866?

(37) How many times does 17155 contain 47?

(38) How many times can 91 be taken from 42770?

- (39) What number must be multiplied by 59 to produce 64310?
- (40) What is the thirteenth part of 41691?
- (41) What number divided by 163 will give 3070 as quotient?
- (42) The product of two numbers is 200410 and the greater of them is 490; find the other.
- (43) What is the dividend, when 287 is the divisor and 20097 the quotient?
- (44) Divide the product of 5093 and 8007 by the sum of 857 and 108; and give the remainder.
- (45) Find the difference between the product of 4083 and 97, and the product of 180 and 569.
- (46) Multiply the sum of three thousand and five, and eighteen hundred and sixty-two, by their difference.
- (47) If 19460 is the dividend and 695 is the quotient, what is the divisor?
- (48) What number subtracted from one hundred thousand would leave seventy thousand and fifty-eight?
- (49) If the quotient is 37, the divisor 35, and the remainder 29, what is the dividend?
- (50) How many times can fifty-nine be subtracted from ten thousand, and what is the remainder?

MEASURES OF MONEY.

75. In the British Isles we have gold coins of two sizes in general circulation. They are called a **sovereign**, and a **half-sovereign**; *two half-sovereigns* together being of exactly the same value as *one sovereign*.

76. To represent values of less than a half-sovereign, we have coins made of silver, and for still smaller values coins made of bronze.

The principal *silver* coin is called a **shilling**, and is such that 20 of them represent the same value as *one* sovereign, and 10 the same value as *one* half-sovereign.

The principal *bronze* coin is called a **penny**, and is such that 12 of them represent the same value as one shilling.

77. The other silver coins in circulation are called

- | | |
|-------------------------|--------------------------------|
| (1) a crown, | which is worth 5 shillings, |
| (2) a half-crown, | 2 shillings and 6 pence, |
| (3) a florin, | 2 shillings, |
| (4) a sixpenny piece, | 6 pence, |
| (5) a threepenny piece, | 3 pence. |

The other bronze coins are

- and*
- | | |
|-------------------|----------------------------|
| (1) a half-penny, | 2 of which make one penny, |
| (2) a farthing, | 4 of which make one penny. |

78. The value of an article is estimated in English money as a certain number of pounds, shillings, and pence.

A sum of money, such as 7 pounds 12 shillings and 5 pence, is written
£7. 12s. 5d.

The symbol £ stands for the Latin word *libra*, the letter *s.* for *solidus*, and the letter *d.* for *denarius*; these words were the names of certain Roman coins or sums of money.

One farthing is represented by placing $\frac{1}{4}$ immediately after the number which denotes pence; two farthings, or one halfpenny, by the symbol $\frac{1}{2}$; and three farthings by $\frac{3}{4}$.

Thus 7 pounds, 12 shillings, 5 pence, and 3 farthings, is written
£7. 12s. 5 $\frac{3}{4}$ d.

The reason for using the symbols $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, will be explained when we consider *fractions*.

79. Of coins which are not now in use, the most important was the **guinea**, which was made of gold and was worth 21 shillings; the word *guinea* is still commonly used in speaking of this amount.

80. The relations between the pound, the shilling, and the penny are stated in the following

BRITISH MONEY TABLE.

4 farthings	make 1 penny.
12 pence 1 shilling.
20 shillings 1 pound.

It should also be remembered that £1 = 4 crowns = 8 half-crowns = 10 florins = 40 sixpences = 80 threepences = 240 pennies = 480 half-pennies = 960 farthings.

REDUCTION OF MONEY.

81. Sums of money expressed in terms of the same unit, for example, 15 shillings and 12 shillings, are said to be of the same **denomination**; and a sum of money expressed in terms of a larger unit, is of *higher* denomination than when expressed in terms of a smaller. For example, 2 pounds is of a higher denomination than 40 shillings, although it is of the same value.

When we find how many farthings there are in a given number of pounds, shillings, and pence, or the number of pounds, shillings, and pence equivalent to a given number of farthings; or, generally, when we convert a quantity from one denomination to another, we are said to **reduce** it, and the process is called **reduction**.

Thus, 20 shillings become 1 pound, when *reduced* to pounds, and 240 pence, when *reduced* to pence.

82. To reduce a sum of money to a lower denomination*Example. Reduce £21. 12s. 6½d. to farthings.*

We multiply the number of pounds by 20, and add to the product the number of odd shillings; we then multiply this result by twelve, and add the number of odd pence; lastly, we multiply the number of pence by 4, and add the odd farthings.

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 21 \quad 12 \quad 6\frac{1}{2} \\
 \underline{20} \\
 439 = \text{number of shillings.} \\
 \underline{12} \\
 5274 = \text{number of pence.} \\
 \underline{4} \\
 21099 = \text{number of farthings.}
 \end{array}$$

EXERCISE 19.

Reduce to shillings

- (1) £4. 15s. (2) £12. 7s. (3) £25. 18s. (4) £50. 4s.

Reduce to sixpences

- (5) £3. 15s. (6) £7. 12s. 6d. (7) £15. 14s. (8) £34. 16s. 6d.

Reduce to threepences

- (9) £5. 16s. (10) £10. 13s. 3d. (11) £21. 17s. 6d. (12) £50. 18s. 9d.

Reduce to crowns

- (13) £36. (14) £10. 5s. (15) £27. 10s. (16) £53. 15s.

Reduce to half-crowns

- (17) £15. (18) £21. (19) £32. 2s. 6d. (20) £45. 5s.
 (21) £50. 7s. 6d. (22) £60. 12s. 6d. (23) £75. 15s. (24) £80. 17s. 6d.

Reduce to florins

- (25) £15. (26) £21. 2s. (27) £32. 4s. (28) £46. 6s.
 (29) £52. 8s. (30) £65. 10s. (31) £70. 12s. (32) £75. 14s.
 (33) £82. 16s. (34) £93. 18s.

Reduce to pence

- (35) £4. 3s. 6d. (36) £8. 17s. 7d. (37) £15. 1s. 7d.
 (38) £19. 11s. 4d. (39) £21. 13s. 5d. (40) £36. 14s. 8d.
 (41) £152. 16s. 9d. (42) £265. 12s. 3d. (43) £375. 5s. 1d.

Reduce to halfpence

- (44) £5. 10s. 8½d. (45) £10. 14s. 6½d. (46) £25. 17s. 7½d.
 (47) £37. 18s. 9d. (48) £150. 13s. 10½d. (49) £265. 10s. 11d.

Reduce to farthings

- (50) £6. 10s. 7¼d. (51) £9. 19s. 7¾d. (52) £12. 13s. 11¼d.
 (53) £16. 17s. 2½d. (54) £18. 11s. 9¾d. (55) £25. 14s. 10¾d.
 (56) £32. 19s. 3½d. (57) £145. 10s. 7½d. (58) £250. 15s. 9½d.
 (59) £375. 12s. 4¾d. (60) £590. 15s. 10¾d.

83. To reduce a sum of money to a higher denomination.

Example. Reduce 21099 farthings to pounds, shillings, pence, etc.

We divide successively by 4, 12 and 20; the successive remainders giving the number of odd farthings, pence, and shillings respectively.

The complete operation is shewn thus:—

$$\begin{array}{r} 4 \overline{) 21099} \\ 12 \overline{) 5274} \text{ pence, and 3 farthings over.} \\ 20 \overline{) 439} \text{ shillings, and 6 pence over.} \\ \quad 21 \text{ pounds, and 19 shillings over.} \end{array}$$

Therefore 21099 farthings = £21 19s. 6½d.

EXERCISE 20.

Express

- (1) 873; 6174; 62328; 613476 farthings in pence and farthings.
- (2) 427; 3297; 69832; 413609 pence in shillings and pence.
- (3) 835; 6174; 32967; 693696 shillings in pounds and shillings.
- (4) 874; 4503; 40501; 518390 halfpence in £. s. d.
- (5) 988; 6852; 13095; 107135 farthings in £. s. d.
- (6) 425; 2067; 10762; 191376 crowns in £. s. d.
- (7) 561; 1363; 29639; 280285 florins in £. s. d.
- (8) 698; 5099; 42735; 379197 half crowns in £. s. d.
- (9) 712; 5560; 29073; 468013 sixpences in £. s. d.
- (10) 832; 1093; 46982; 557923 threepences in £. s. d.

COMPOUND ADDITION—MONEY.

84. A compound quantity is a quantity in the expression of which there are terms of different denominations. For example, £5. 6s. 10d. is a compound quantity with terms of three different denominations.

85. Compound Addition is the addition of compound quantities of the same kind.

86. When we add together several sums of money, we arrange them so that the numbers which denote pence are all one under another in a column, those which denote shillings one under another in a column, and so also those which denote pounds.

We add together the number of pence, carry forward shillings or multiples of 12 pence, and set down the remainder. And we treat the other columns in a similar way.

Example. Add together £37. 6s. 10½d., £42. 12s. 6¼d. and £94. 15s. 4¾d.

£	s.	d.
37	6	10½
42	12	6¼
94	15	4¾
<hr/>		
174	14	9½

We begin at the right hand by adding together $\frac{3}{4}$ which represents 3 farthings, $\frac{1}{4}$ which represents one farthing, and $\frac{1}{2}$ which represents 2 farthings; we get 6 farthings, which is the same as 1 penny and 2 farthings.

We write down $\frac{1}{2}$, the symbol for 2 farthings, under the other symbols for farthings; and we carry the penny to increase the pence-column.

Adding now the pence-column, we find that it amounts to 21 pence. But 21 pence are equivalent to 1 shilling and 9 pence. We therefore write down the 9 pence at the foot of the pence-column, and transfer the 1 shilling to increase the shillings-column.

The shillings-column amounts to 34 shillings. Hence we write down 14 shillings, and transfer 1 pound to increase the pounds-column.

Lastly, adding the pounds-column, we get £174.

The result of the addition is therefore £174. 14s. 9½d.

EXERCISE 21.

Add together

(1)	$d.$ $7\frac{3}{4}$ $2\frac{1}{2}$ $6\frac{1}{4}$ $5\frac{1}{2}$ $6\frac{3}{4}$	(2)	$d.$ $6\frac{1}{2}$ $2\frac{3}{4}$ $7\frac{1}{2}$ $10\frac{1}{4}$ $11\frac{3}{4}$	(3)	$d.$ $6\frac{1}{2}$ $7\frac{1}{2}$ $10\frac{3}{4}$ $8\frac{1}{2}$ $6\frac{3}{4}$	(4)	$d.$ $6\frac{3}{4}$ $2\frac{1}{2}$ $6\frac{1}{4}$ $10\frac{3}{4}$ $7\frac{3}{4}$	(5)	$d.$ $7\frac{1}{2}$ $6\frac{3}{4}$ $7\frac{1}{4}$ $11\frac{3}{4}$ $6\frac{1}{4}$
(6)	$s. \quad d.$ $6 \quad 10\frac{1}{2}$ $2 \quad 3\frac{1}{4}$ $11 \quad 6\frac{1}{2}$ $16 \quad 8\frac{1}{2}$ $17 \quad 2\frac{1}{2}$	(7)	$s. \quad d.$ $10 \quad 6$ $17 \quad 4\frac{1}{2}$ $2 \quad 9\frac{3}{4}$ $6 \quad 8\frac{1}{2}$ $19 \quad 3$	(8)	$s. \quad d.$ $17 \quad 6\frac{1}{4}$ $18 \quad 9$ $12 \quad 3\frac{1}{2}$ $16 \quad 10\frac{3}{4}$ $7 \quad 8\frac{1}{2}$	(9)	$s. \quad d.$ $13 \quad 6\frac{1}{2}$ $12 \quad 3\frac{3}{4}$ $6 \quad 2\frac{1}{2}$ $15 \quad 10\frac{1}{2}$ $13 \quad 7\frac{1}{2}$	(10)	$s. \quad d.$ $13 \quad 6\frac{1}{2}$ $7 \quad 8\frac{3}{4}$ $19 \quad 11\frac{1}{4}$ $6 \quad 8\frac{1}{2}$ $7 \quad 3\frac{1}{4}$
(11)	$£ \quad s. \quad d.$ $2 \quad 13 \quad 5$ $4 \quad 2 \quad 9$ $3 \quad 6 \quad 4$ $5 \quad 10 \quad 8$	(12)	$£ \quad s. \quad d.$ $6 \quad 2 \quad 9$ $7 \quad 3 \quad 1$ $3 \quad 10 \quad 8$ $4 \quad 6 \quad 9$	(13)	$£ \quad s. \quad d.$ $10 \quad 13 \quad 4$ $7 \quad 2 \quad 3$ $4 \quad 6 \quad 3$ $12 \quad 16 \quad 7$	(14)	$£ \quad s. \quad d.$ $11 \quad 11 \quad 6$ $9 \quad 8 \quad 7$ $12 \quad 3 \quad 4$ $2 \quad 9 \quad 10$		
(15)	$£ \quad s. \quad d.$ $4 \quad 8 \quad 9$ $10 \quad 10 \quad 6$ $5 \quad 5 \quad 10$ $12 \quad 11 \quad 3$	(16)	$£ \quad s. \quad d.$ $7 \quad 15 \quad 7\frac{1}{4}$ $10 \quad 13 \quad 10\frac{1}{2}$ $13 \quad 10 \quad 9\frac{3}{4}$ $14 \quad 8 \quad 4\frac{1}{2}$	(17)	$£ \quad s. \quad d.$ $6 \quad 7 \quad 10\frac{1}{2}$ $12 \quad 14 \quad 3\frac{1}{2}$ $15 \quad 16 \quad 8\frac{3}{4}$ $10 \quad 18 \quad 4\frac{1}{2}$	(18)	$£ \quad s. \quad d.$ $7 \quad 8 \quad 6\frac{1}{4}$ $13 \quad 10 \quad 10\frac{1}{4}$ $16 \quad 15 \quad 8\frac{3}{4}$ $8 \quad 4 \quad 9\frac{1}{2}$		

$$\begin{array}{r}
 \text{(19)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 12 \quad 9 \quad 10\frac{1}{2} \\ 15 \quad 10 \quad 9\frac{3}{4} \\ 11 \quad 15 \quad 4\frac{1}{2} \\ 6 \quad 8 \quad 10\frac{1}{4} \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(20)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 7 \quad 10 \quad 7\frac{1}{4} \\ 12 \quad 15 \quad 8\frac{1}{2} \\ 13 \quad 7 \quad 11\frac{3}{4} \\ 9 \quad 16 \quad 10\frac{1}{2} \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(21)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 14 \quad 12 \quad 6\frac{1}{4} \\ 7 \quad 8 \quad 9 \\ 10 \quad 12 \quad 10\frac{1}{2} \\ 7 \quad 18 \quad 3\frac{3}{4} \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(22)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 16 \quad 18 \quad 3\frac{1}{2} \\ 10 \quad 12 \quad 6 \\ 15 \quad 15 \quad 5\frac{1}{2} \\ 6 \quad 9 \quad 7\frac{1}{2} \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(23)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 2 \quad 13 \quad 5\frac{1}{2} \\ 17 \quad 9 \quad 4\frac{1}{4} \\ 5 \quad 15 \quad 4\frac{1}{2} \\ 19 \quad 17 \quad 6\frac{1}{4} \\ 17 \quad 16 \quad 2 \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(24)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 9 \quad 10 \quad 3\frac{1}{2} \\ 16 \quad 14 \quad 3\frac{1}{2} \\ 5 \quad 17 \quad 10\frac{1}{4} \\ 12 \quad 2 \quad 8\frac{1}{2} \\ 19 \quad 17 \quad 4\frac{3}{4} \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(25)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 12 \quad 14 \quad 6\frac{1}{2} \\ 3 \quad 19 \quad 7\frac{1}{4} \\ 6 \quad 2 \quad 9 \\ 17 \quad 14 \quad 6\frac{1}{2} \\ 18 \quad 17 \quad 3\frac{1}{2} \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(26)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 6 \quad 7 \quad 6\frac{1}{2} \\ 10 \quad 12 \quad 9 \\ 13 \quad 16 \quad 4\frac{1}{2} \\ 17 \quad 8 \quad 9\frac{1}{2} \\ 6 \quad 10 \quad 3\frac{3}{4} \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(27)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 15 \quad 10 \quad 9\frac{1}{2} \\ 8 \quad 19 \quad 7 \\ 16 \quad 7 \quad 11\frac{3}{4} \\ 14 \quad 16 \quad 4\frac{3}{4} \\ 7 \quad 8 \quad 3\frac{1}{2} \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(28)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 13 \quad 14 \quad 8\frac{1}{4} \\ 12 \quad 15 \quad 6\frac{1}{2} \\ 7 \quad 18 \quad 10\frac{3}{4} \\ 15 \quad 7 \quad 5\frac{1}{2} \\ 9 \quad 10 \quad 4 \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(29)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 9 \quad 15 \quad 9 \\ 18 \quad 18 \quad 11\frac{1}{2} \\ 7 \quad 19 \quad 10\frac{1}{2} \\ 16 \quad 7 \quad 6\frac{3}{4} \\ 13 \quad 14 \quad 8\frac{3}{4} \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(30)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 10 \quad 7 \quad 9\frac{1}{4} \\ 17 \quad 10 \quad 4\frac{1}{2} \\ 3 \quad 12 \quad 10 \\ 16 \quad 13 \quad 11\frac{3}{4} \\ 19 \quad 9 \quad 6\frac{1}{2} \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(31)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 37 \quad 2 \quad 6\frac{1}{4} \\ 19 \quad 16 \quad 3\frac{1}{2} \\ 7 \quad 13 \quad 6\frac{1}{2} \\ 33 \quad 7 \quad 2\frac{1}{4} \\ 16 \quad 17 \quad 4\frac{1}{2} \\ 69 \quad 12 \quad 3 \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(32)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 63 \quad 19 \quad 6\frac{1}{2} \\ 29 \quad 6 \quad 2 \\ 2 \quad 13 \quad 7\frac{1}{4} \\ 83 \quad 15 \quad 4\frac{1}{2} \\ 36 \quad 10 \quad 6\frac{1}{2} \\ 18 \quad 19 \quad 10 \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(33)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 24 \quad 9 \quad 7\frac{1}{2} \\ 6 \quad 16 \quad 2\frac{3}{4} \\ 92 \quad 9 \quad 10\frac{1}{2} \\ 39 \quad 17 \quad 6 \\ 78 \quad 8 \quad 4\frac{1}{2} \\ 17 \quad 14 \quad 9 \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(34)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 63 \quad 19 \quad 6\frac{1}{2} \\ 20 \quad 14 \quad 6 \\ 7 \quad 8 \quad 2\frac{3}{4} \\ 16 \quad 13 \quad 6\frac{1}{2} \\ 92 \quad 3 \quad 9 \\ 40 \quad 17 \quad 11\frac{1}{4} \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(35)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 62 \quad 14 \quad 10\frac{1}{4} \\ 24 \quad 7 \quad 6 \\ 17 \quad 13 \quad 4\frac{1}{2} \\ 30 \quad 16 \quad 7\frac{1}{4} \\ 17 \quad 2 \quad 3\frac{1}{2} \\ 4 \quad 6 \quad 3\frac{1}{4} \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(36)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 17 \quad 15 \quad 4\frac{1}{2} \\ 32 \quad 14 \quad 8 \\ 16 \quad 2 \quad 3\frac{1}{2} \\ 51 \quad 11 \quad 6\frac{1}{2} \\ 13 \quad 6 \quad 2\frac{3}{4} \\ 24 \quad 13 \quad 6 \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(37)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 21 \quad 16 \quad 11\frac{1}{4} \\ 7 \quad 10 \quad 2 \\ 56 \quad 17 \quad 4\frac{1}{2} \\ 63 \quad 19 \quad 8\frac{1}{2} \\ 4 \quad 9 \quad 7\frac{1}{2} \\ 69 \quad 18 \quad 10\frac{3}{4} \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(38)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 10 \quad 16 \quad 11\frac{1}{2} \\ 9 \quad 7 \quad 8 \\ 63 \quad 16 \quad 3\frac{1}{4} \\ 6 \quad 3 \quad 6\frac{3}{4} \\ 78 \quad 10 \quad 11\frac{1}{2} \\ 63 \quad 12 \quad 8 \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(39)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 619 \quad 13 \quad 6\frac{1}{2} \\ 326 \quad 12 \quad 10\frac{1}{2} \\ 29 \quad 9 \quad 6\frac{3}{4} \\ 862 \quad 19 \quad 6\frac{1}{2} \\ 81 \quad 9 \quad 1 \\ 398 \quad 14 \quad 5\frac{3}{4} \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(40)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 357 \quad 16 \quad 8\frac{1}{2} \\ 135 \quad 13 \quad 2\frac{1}{4} \\ 753 \quad 12 \quad 10\frac{1}{2} \\ 203 \quad 7 \quad 8 \\ 84 \quad 19 \quad 2\frac{3}{4} \\ 541 \quad 16 \quad 7\frac{1}{2} \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(41)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 281 \quad 9 \quad 1\frac{1}{4} \\ 536 \quad 12 \quad 4\frac{1}{2} \\ 78 \quad 5 \quad 9 \\ 372 \quad 18 \quad 4\frac{3}{4} \\ 49 \quad 13 \quad 3\frac{1}{4} \\ 827 \quad 16 \quad 9\frac{1}{2} \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(42)} \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 629 \quad 4 \quad 2\frac{1}{2} \\ 326 \quad 12 \quad 6\frac{3}{4} \\ 96 \quad 3 \quad 2\frac{1}{2} \\ 526 \quad 16 \quad 8\frac{1}{4} \\ 320 \quad 4 \quad 6 \\ 69 \quad 17 \quad 2\frac{1}{2} \end{array} \\
 \hline
 \end{array}$$

(43)	£	s.	d.	(44)	£	s.	d.	(45)	£	s.	d.	(46)	£	s.	d.
	931	12	3½		359	2	4½		702	13	4½		438	17	6½
	40	13	7½		71	13	6½		901	3	7½		183	14	3
	287	12	6½		784	5	7		632	19	11		33	8	3½
	463	8	3		479	12	7½		84	3	7½		234	15	6½
	676	19	10½		93	7	8		9	9	9½		479	12	7½
	535	7	4½		148	6	11½		715	16	4		43	7	4½
	67	11	7½		92	18	4½		123	4	5½		567	8	9
<hr/>				<hr/>				<hr/>				<hr/>			
(47)	£	s.	d.	(48)	£	s.	d.	(49)	£	s.	d.	(50)	£	s.	d.
	132	13	4½		219	16	5½		413	9	7		569	15	8½
	479	12	6½		63	8	4		587	11	6½		53	2	6½
	327	13	7½		514	19	3½		93	4	10½		384	16	5½
	236	11	4½		713	16	9½		432	16	8½		45	7	11½
	67	6	3½		48	9	2½		567	13	4½		297	13	6
	429	12	3½		654	15	6½		83	17	7½		314	4	2½
	363	14	6½		89	8	11½		615	4	5½		93	18	6½
	98	12	7		101	7	4		589	16	9		164	6	10½
<hr/>				<hr/>				<hr/>				<hr/>			

COMPOUND SUBTRACTION—MONEY.

87. Compound Subtraction is the subtraction of one compound quantity from another of the same kind.

88. To subtract one sum of money from another we place the sum to be subtracted below the one from which it is to be taken, pence below pence, shillings below shillings, and pounds below pounds.

Example. Subtract £3. 4s. 6d. from £5. 15s. 10d.

In this example the lower figure in each column is less than the upper one. If therefore we take 6 from 10, 4 from 15, and 3 from 5, the remainders are 4, 11 and 2 respectively; and the result is £2. 11s. 4d.

£	s.	d.
5	15	10
3	4	6
<hr/>		
2	11	4

89. If any term in the lower line be greater than the term of the same name in the upper line, we again proceed as in simple subtraction.

Example. Subtract £6. 17s. 9d. from £9. 10s. 8d.

We cannot take 9 pence from 8 pence, so we increase the 8 pence to 20 pence, and at the same time we increase the 17s. in the lower line to 18s. The remainder from the pence column is then 11 pence. Again, we cannot take 18s. from 10s., so we increase the 10s. to 30s., and we increase the £6 in the lower line to £7. The difference is thus found to be £2. 12s. 11d.

£	s.	d.
9	10	8
6	17	9
<hr/>		
2	12	11

By the method of Art. 39 the mental steps are as follows:

1st. 9d. and 11d. make 1s. 8d.; write down 11d. and carry 1s.;

2nd. 1s. and 17s. make 18s.; and 12s. make £1. 10s.; write down 12s. and carry £1;

3rd. £1 and £6 make £7; and £2 make £9; write down £2.

The result is £2 12s. 11d.

EXERCISE 22.

Perform the following subtractions: -

£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	
(1)	4	10	9	(2)	5	16	8	(3)	10	17	11	(4)	29	15	10
	1	4	6		2	13	5		4	9	8		15	14	3

£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	
(6)	2	4	3	(7)	5	10	2	(8)	8	17	6	(9)	7	9	1
	1	2	6		2	7	4		4	10	8		5	10	6

£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
(11)	21	10	5½	(12)	35	12	9½	(13)	42	18	8
	15	9	7½		16	7	10½		18	14	2½

£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
(15)	60	8	10½	(16)	70	7	6	(17)	80	12	3½
	32	14	6½		45	10	3½		16	16	6½

£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
(19)	14	16	10	(20)	10	6	2½	(21)	16	17	2½
	8	17	0½		0	19	8½		8	17	9½

£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
(23)	45	0	7	(24)	67	13	2½	(25)	63	16	2½
	33	10	10½		39	15	10½		18	19	6½

£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
(27)	91	1	1½	(28)	67	18	4½	(29)	52	9	1½
	9	18	8		10	14	7½		21	18	7½

£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
(31)	300	0	0½	(32)	900	13	10½	(33)	804	13	10½
	10	19	10½		89	10	11½		710	10	6

£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
(35)	903	1	1	(36)	700	0	10	(37)	904	10	10½
	99	1	10½		5	12	7½		93	19	11½

- (39) Take £309. 17s. 6½d. from £532. 5s. 2½d.
 (40) Take £607. 14s. 7½d. from £730. 12s. 6½d.
 (41) Take £85. 16s. 2½d. from £108. 11s. 2d.
 (42) Subtract £624. 16s. 10½d. from £720. 14s. 7½d.
 (43) Subtract £60. 12s. 6½d. from £630. 9s. 5d.
 (44) Subtract £96. 16s. 2½d. from £103. 15s. 1d.
 (45) Find the difference between £791. 1s. 1½d. and £309. 18s. 8d.
 (46) By how much is £120. 16s. 3d. greater than £92. 17s. 4d ?
 (47) By how much does £250. 17s. 3d. fall short of £500 ?
 (48) By how much is £500. 16s. 5d. less than £750. 5s. 3d. ?
 (49) How much must be added to £85. 12s. 9d. to make £200 ?
 (50) How much must be taken from £200. 19s. 5d. to leave £105. 17s. 10d. ?

COMPOUND MULTIPLICATION—MONEY.

90. **Compound Multiplication** is a short way of finding the sum of a number of equal compound quantities.

91. In multiplying a sum of money, we write the multiplier immediately underneath the term of lowest denomination. We then multiply, and carry forward multiples of 12 pence and multiples of 20 shillings in the same way as in addition.

Example. Multiply £65. 13s. 7½d. by 7.

The work is stated thus:—

£	s.	d.
65	13	7½
		7
459	15	6½

We know that 7 times 3 farthings = 21 farthings = 5½d.; therefore we write down ½d., and carry forward 5 pence.

Again, 7 times 7 pence = 49 pence, which become 54 pence when we add the 5 pence brought forward. Also 54 pence are equivalent to 4 shillings and 6 pence. We therefore write down 6 pence, and carry forward 4 shillings.

Next, 7 times 13 shillings = 91 shillings; which become 95 shillings when we add the 4 shillings brought forward. And we know that 95 shillings are the same as 4 pounds and 15 shillings. Therefore we write down 15 shillings and carry forward 4 pounds.

Lastly, 7 times £65 = £455, which becomes £459 when we add the £4. Hence the result is £459. 15s. 6½d.

NOTE.—It may help the pupil to realize that multiplication is, as we have said, but a simplified method of adding equal quantities, if he work out the above example as an example in addition. He would also thereby verify the result just obtained.

EXERCISE 23.

Perform the following multiplications:—

	£	s.	d.		£	s.	d.	
(1)	3	7	6	$\times 2$ and 3.	(2)	4	5	9 $\times 3$ and 4.
(3)	5	9	4	$\times 4$ „ 5.	(4)	6	8	5 $\times 5$ „ 6.
(5)	7	4	3	$\times 6$ „ 7.	(6)	8	10	$2\frac{1}{2} \times 7$ „ 8.
(7)	9	12	$10\frac{1}{2}$	$\times 8$ „ 9.	(8)	8	15	$6\frac{3}{4} \times 9$ „ 10.
(9)	7	13	$4\frac{1}{2}$	$\times 10$ „ 11.	(10)	18	10	$5\frac{1}{2} \times 7$ „ 12.
(11)	11	11	$11\frac{1}{4}$	$\times 5$ „ 9.	(12)	14	18	$4\frac{1}{2} \times 4$ „ 11.
(13)	17	16	$7\frac{1}{2}$	$\times 3$ „ 10.	(14)	19	17	$2\frac{1}{2} \times 6$ „ 9.
(15)	51	18	$4\frac{1}{2}$	$\times 3$ „ 10.	(16)	64	13	$6\frac{1}{2} \times 4$ „ 9.
(17)	76	15	$8\frac{3}{4}$	$\times 5$ „ 8.	(18)	82	17	$10\frac{3}{4} \times 6$ „ 11.
(19)	85	19	$9\frac{1}{2}$	$\times 7$ „ 12.	(20)	93	11	$7\frac{1}{2} \times 8$ „ 10.

92. When the multiplier is greater than 12, and can easily be separated into factors each less than 12, it is best to proceed as in the following example.

Example. Multiply £84. 17s. 6½d. by 24.

The multiplier 24 is the same as 4 × 6; therefore we multiply £84. 17s. 6½d. by 4, and the product by 6.

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 84 \quad 17 \quad 6\frac{1}{2} \\
 \hline
 4 \\
 \hline
 339 \quad 10 \quad 1 = \text{£}84 \quad 17\text{s.} \quad 6\frac{1}{2}\text{d.} \times 4. \\
 \hline
 6 \\
 \hline
 2037 \quad 0 \quad 6 = \text{£}84 \quad 17\text{s.} \quad 6\frac{1}{2}\text{d.} \times 24.
 \end{array}$$

EXERCISE 24.

Perform the following multiplications:—

£	s.	d.		£	s.	d.	
(1)	15	16	$8\frac{1}{2} \times 14$ and 25.	(2)	26	17	$9\frac{1}{2} \times 16$ and 30.
(3)	37	15	$7\frac{3}{4} \times 18$ „ 32.	(4)	48	14	$6\frac{1}{2} \times 20$ „ 42.
(5)	54	13	$4\frac{3}{4} \times 21$ „ 40.	(6)	69	12	$3\frac{1}{2} \times 24$ „ 50.
(7)	39	16	$11\frac{1}{4} \times 15$ „ 22.	(8)	42	15	$7\frac{3}{4} \times 35$ „ 81.
(9)	21	10	$10\frac{1}{2} \times 28$ „ 54.	(10)	23	9	$11\frac{1}{4} \times 36$ „ 56.
(11)	32	8	$10\frac{1}{2} \times 44$ „ 60.	(12)	43	15	$8\frac{3}{4} \times 48$ „ 66.
(13)	49	17	$2\frac{1}{2} \times 64$ „ 70.	(14)	50	13	$2\frac{1}{2} \times 72$ „ 77.
(15)	65	14	$10\frac{1}{2} \times 80$ „ 84.	(16)	73	15	$6\frac{3}{4} \times 88$ „ 90.
(17)	49	12	$9\frac{1}{2} \times 96$ „ 100.	(18)	56	10	$7\frac{3}{4} \times 108$ „ 110.
(19)	34	11	$4\frac{1}{2} \times 120$ „ 132.	(20)	42	14	$11\frac{1}{2} \times 124$ „ 144.

93. When the multiplier is a number which cannot be easily separated into simple factors, it is best to proceed as in the following example.

Example. Multiply £12. 4s. 9½d. by 365.

The multiplier is 365, or 300 + 60 + 5; we therefore multiply £12. 4s. 9½d. by 300, 60, and 5, and add the results together.

To multiply by 300, we multiply by the factors, 10, 10, and 3 in succession.

To multiply by 60, we multiply by 10 and 6.

The work can be arranged thus:—

£	s.	d.		
12	4	9½	× 5 (1) =	£12. 4s. 9½d.
		10		
122	7	11	× 6 (2) =	10 times £12. 4s. 9½d.
		10		
1223	19	2	(3) =	100 times £12. 4s. 9½d.
		3		
3671	17	6	(4) =	300 times £12. 4s. 9½d.
734	7	6	(5) =	60 times £12. 4s. 9½d.
61	3	11½	(6) =	5 times £12. 4s. 9½d.
4467	8	11½	(7) =	365 times £12. 4s. 9½d.

To get (5) we multiply (2) by 6.

To get (6) „ „ (1) by 5.

To get (7) we add (4), (5), and (6).

EXERCISE 25.

Perform the following multiplications:—

£	s.	d.			£	s.	d.		
(1)	6	14	3	× 13 and 23.	(2)	7	15	4	× 17 and 29.
(3)	8	17	8½	× 19 „ 29.	(4)	9	13	10½	× 37 „ 65.
(5)	10	12	6½	× 41 „ 87.	(6)	12	11	2¾	× 43 „ 95.
(7)	15	15	10½	× 47 „ 93.	(8)	17	18	7½	× 52 „ 89.
(9)	18	19	4¾	× 67 „ 75.	(10)	20	12	11½	× 69 „ 73.
(11)	69	13	4½	× 13 „ 31.	(12)	70	14	6¾	× 17 „ 71.
(13)	25	16	8½	× 19 „ 91.	(14)	53	10	10½	× 23 „ 65.
(15)	71	12	11½	× 29 „ 78.	(16)	89	17	11½	× 34 „ 85.
(17)	45	12	6½	× 37 „ 79.	(18)	37	15	7¾	× 39 „ 97.
(19)	92	11	3½	× 41 „ 95.	(20)	63	10	9½	× 43 „ 89.
(21)	10	6	5	× 150 „ 205.	(22)	11	13	4	× 115 „ 251.
(23)	12	17	8	× 171 „ 317.	(24)	13	18	6	× 182 „ 328.
(25)	14	15	9	× 156 „ 365.	(26)	15	12	10	× 164 „ 473.
(27)	16	9	11½	× 129 „ 582.	(28)	17	7	8½	× 159 „ 645.

	£	s.	d.	
(29)	18	8	$7\frac{3}{4} \times 158$	and 724.
(31)	34	7	$9\frac{1}{2} \times 375$	„ 537.
(33)	44	11	$3\frac{1}{4} \times 716$	„ 878.
(35)	85	18	$4\frac{1}{2} \times 456$	„ 652.
(37)	72	19	$5\frac{1}{2} \times 936$	„ 493.
(39)	57	17	$10\frac{1}{4} \times 358$	„ 857.

	£	s.	d.	
(30)	20	16	$10\frac{3}{4} \times 167$	and 836.
(32)	73	16	$2\frac{1}{4} \times 216$	„ 365.
(34)	97	15	$8\frac{3}{4} \times 954$	„ 495.
(36)	49	14	$3\frac{3}{4} \times 768$	„ 876.
(38)	65	19	$10\frac{1}{4} \times 573$	„ 537.
(40)	38	13	$11\frac{3}{4} \times 216$	„ 612.

COMPOUND DIVISION—MONEY.

94. Compound Division is of two kinds: (1) the process of separating a compound quantity into a given number of equal parts, and finding the value of each part; (2) a short way of finding how many times one compound quantity is contained in another of the same kind.

95. When the divisor is not greater than 12, we proceed as in the following example.

Example. Divide £164. 16s. $10\frac{1}{2}$ d. by 3.

In simple division we begin at the left hand: so in compound division we begin at the left hand. If we divide 164 by 3, the quotient is 54 and the remainder is 2. Hence, if we divide £164 by 3, the quotient is £54, and the remainder is £2.

We change the remainder £2 into 40 shillings, and add to it the 16s. in the dividend: we thus get 56s. altogether. If now we divide 56s. by 3, the quotient is 18s., and the remainder 2s.

Changing the 2s. into 24 pence, and adding the 10 pence from the dividend, we get 34 pence altogether. And if we divide 34 pence by 3, we get 11 pence for the quotient, and 1 penny over.

Finally, changing the 1 penny into 4 farthings, and adding the 2 farthings from the dividend, we get 6 farthings. Dividing this by 3, the quotient is 2 farthings, and there is no remainder.

The result is therefore £54. 18s. $11\frac{1}{2}$ d.

The work may be stated briefly thus:

$$\begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 3 \overline{) 164 \, 16 \, 10\frac{1}{2}} \\ \underline{54 \, 18 \, 11\frac{1}{2}} \end{array}$$

EXERCISE 26.

Perform the following divisions:—

	£	s.	d.		£	s.	d.		£	s.	d.
(1)	1	5	$4\frac{1}{2} \div 2$	(2)	2	10	$4\frac{1}{2} \div 3$	(3)	2	15	$7 \div 4$
(4)	10	18	$4 \div 5$	(5)	16	8	$6 \div 6$	(6)	26	10	$10 \div 7$
(7)	29	0	$4 \div 8$	(8)	41	4	$5\frac{1}{4} \div 9$	(9)	46	17	$3\frac{1}{2} \div 10$
(10)	60	14	$1\frac{1}{2} \div 11$	(11)	62	19	$3\frac{1}{4} \div 11$	(12)	82	13	$6 \div 12$

£	s.	d.		£	s.	d.		£	s.	d.	
(13)	154	3	3 ÷ 6.	(14)	118	4	8½ ÷ 7.	(15)	215	6	4 ÷ 8.
(16)	339	7	8½ ÷ 9.	(17)	397	16	10½ ÷ 10.	(18)	410	3	8½ ÷ 11.
(19)	1472	19	4 ÷ 2.	(20)	1216	8	9 ÷ 3.	(21)	2201	17	8 ÷ 4.
(22)	1037	11	4½ ÷ 5.	(23)	1023	14	1½ ÷ 6.	(24)	2160	0	0¾ ÷ 7.
(25)	3621	8	4 ÷ 8.	(26)	4533	8	9¾ ÷ 9.	(27)	3334	17	10½ ÷ 9.
(28)	3757	16	0½ ÷ 10.	(29)	3099	18	0¾ ÷ 11.	(30)	3526	15	3 ÷ 12.

96. When the divisor is greater than 12, and can easily be separated into factors each less than 12, it is best to proceed as in the following example.

Example. Divide £178. 3s. by 56.

Knowing that $56 = 7 \times 8$, we first divide the given sum of money by 7, and then we divide the quotient by 8; for, if we divide a quantity into 7 equal parts, and each of these parts again into 8 equal parts, the latter parts must be such that 56 of them will make up the original quantity.

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 56 \left\{ \begin{array}{l} 7) 178 \quad 3 \quad 0 \\ 8) 25 \quad 9 \quad 0 \\ \hline 3 \quad 3 \quad 7\frac{1}{2} \end{array} \right.
 \end{array}$$

EXERCISE 27.

Perform the following divisions:—

£	s.	d.		£	s.	d.	
(1)	46	13	4 ÷ 14.	(2)	69	12	6 ÷ 15.
(3)	132	18	0 ÷ 24.	(4)	167	8	5½ ÷ 25.
(5)	247	5	4 ÷ 32.	(6)	369	7	7½ ÷ 42.
(7)	457	6	0 ÷ 48.	(8)	531	3	11½ ÷ 50.
(9)	741	2	8 ÷ 64.	(10)	501	19	3¾ ÷ 63.
(11)	318	4	10½ ÷ 66.	(12)	412	12	8½ ÷ 70.
(13)	336	7	0 ÷ 16.	(14)	581	9	1½ ÷ 18.
(15)	1453	11	6 ÷ 36.	(16)	1562	2	9 ÷ 44.
(17)	2402	12	6 ÷ 60.	(18)	1189	16	2½ ÷ 77.
(19)	3426	11	9 ÷ 84.	(20)	3120	2	0 ÷ 96.
(21)	1109	16	3 ÷ 108.	(22)	1159	16	3 ÷ 110.
(23)	1403	6	11½ ÷ 121.	(24)	1818	3	0 ÷ 144.
(25)	24562	7	2 ÷ 56.	(26)	17902	15	3 ÷ 28.
(27)	69788	13	5½ ÷ 81.	(28)	34663	16	6 ÷ 84.
(29)	25078	7	11 ÷ 22.	(30)	14028	12	9 ÷ 108.

97. When the divisors are large and cannot easily be separated into small factors, the work may be stated in full as below.

Example. Divide £1756. 19s. 3½d. by 127.

	£	s.	d.	£	s.	d.
127)	1756	19	3½	(13	16 8½
	127					
	486					
	381					
	105					
	20					
	2119	<i>shillings</i>				
	127					
	849					
	762					
	87					
	12					
	1047	<i>pence</i>				
	1016					
	31					
	4					
	127	<i>farthings</i>				
	127					

The quotient = £13. 16s. 8½d.

EXERCISE 28.

Perform the following divisions :—

- | | £ | s. | d. | |
|------|-------|----|-----------|--|
| (1) | 10 | 18 | 8½ ÷ 29. | |
| (3) | 40 | 13 | 9½ ÷ 53. | |
| (5) | 94 | 9 | 5¾ ÷ 97. | |
| (7) | 275 | 5 | 3½ ÷ 37. | |
| (9) | 326 | 8 | 1 ÷ 38. | |
| (11) | 153 | 1 | 3¾ ÷ 29. | |
| (13) | 1422 | 17 | 3 ÷ 76. | |
| (15) | 1279 | 13 | 8¾ ÷ 31. | |
| (17) | 6304 | 15 | 11½ ÷ 97. | |
| (19) | 15028 | 18 | 0¼ ÷ 19. | |
| (21) | 25210 | 9 | 0¼ ÷ 37. | |
| (23) | 11653 | 12 | 5½ ÷ 89. | |

- | | £ | s. | d. | |
|------|-------|----|-----------|--|
| (2) | 19 | 13 | 10¾ ÷ 37. | |
| (4) | 51 | 9 | 4½ ÷ 61. | |
| (6) | 182 | 5 | 6½ ÷ 29. | |
| (8) | 462 | 12 | 9¼ ÷ 83. | |
| (10) | 146 | 14 | 10½ ÷ 53. | |
| (12) | 1040 | 19 | 7½ ÷ 59. | |
| (14) | 2065 | 19 | 6 ÷ 23. | |
| (16) | 1368 | 2 | 9¾ ÷ 47. | |
| (18) | 1148 | 5 | 0¾ ÷ 83. | |
| (20) | 22912 | 9 | 0½ ÷ 53. | |
| (22) | 55631 | 15 | 3 ÷ 76. | |
| (24) | 90248 | 5 | 2½ ÷ 82. | |

	£	s.	d.	
(25)	16501	8	4½	÷197.
(27)	16196	10	2½	÷325.
(29)	13960	12	11½	÷401.
(31)	20766	0	3½	÷613.
(33)	19282	7	4¾	÷811.
(35)	514237	8	3¾	÷237.
(37)	632210	19	0¾	÷705.
(39)	862287	5	7½	÷465.

	£	s.	d.	
(26)	89133	7	0	÷456.
(28)	41614	14	1½	÷702.
(30)	46680	6	6½	÷719.
(32)	44415	8	11½	÷761.
(34)	16395	17	0½	÷329.
(36)	307016	16	0	÷768.
(38)	123612	9	0	÷936.
(40)	167454	12	0¾	÷417.

98. To find how many times one compound quantity is contained in another of the same kind, we must express both as simple quantities of the same denomination, and then divide one result by the other.

Example. How many times is £2. 17s. 4½d. contained in £2194. 11s. 10½d.?

In this case we reduce both quantities to halfpence.

£	s.	d.
2	17	4½
20		
57	shillings	
12		
688	pence	
2		
1377	halfpence	

£	s.	d.
2194	11	10½
20		
43891	shillings	
12		
526702	pence	
2		
1377)	1053405 (765
		9639
		8950
		8262
		6885
		6885

Hence the quotient = 765.

NOTE.—The quantities should not be reduced to a lower denomination than is necessary. In this example it is not necessary to reduce them to farthings; halfpence are low enough.

EXERCISE 29.

Find how often the first amount is contained in the second:—

- | | |
|----------------------------------|-----------------------------------|
| (1) 1s. 7d.; 17s. 5d. | (2) 2s. 9½d.; 8s. 3¾d. |
| (3) 16s. 3d.; £9. 15s. | (4) 5s. 4d.; £62. 8s. |
| (5) 8s. 1½d.; £5. 13s. 9d. | (6) £2. 4s. 6d.; £80. 2s. |
| (7) 19s. 4½d.; £38. 14s. 2d. | (8) £3. 6s. 8d.; £43. 6s. 8d. |
| (9) £2. 17s. 6d.; £163. 17s. 6d. | (10) £34. 7s. 2d.; £652. 16s. 2d. |

MISCELLANEOUS EXAMPLES.

Compound Rules (Money).

EXERCISE 30.

- (1) How many more halfpenny stamps could I buy with £2. 4s. 3d. than penny stamps with £1. 12s. 8d.?
- (2) If oranges are sold at three for a penny, how many could be bought for £3. 16s. 4d.?
- (3) How many more farthings are there in 305 half-crowns than in the same number of florins?
- (4) In a collection £2. 7s. 6d. was taken in threepenny pieces, £1. 18s. 6d. in sixpences, £5. 10s. in shillings, and £2. 12s. 6d. in half-crowns; how many coins were there?
- (5) How many £5 notes are equal in value to 80400 penny stamps?
- (6) A box contained 20000 oranges, if the oranges were sold at the rate of two for three halfpence, how much money did the whole box fetch?
- (7) The distance from London to Birmingham is 112 miles, and the third-class railway fare is one penny per mile; what sum would the booking clerk receive for 59 third class tickets?
- (8) Three thousand workmen strike because an increase of a half-penny per hour is not granted; find the additional sum that would be required weekly if their wages were increased, taking the working-day at nine hours.
- (9) I took £6. 17s. 10½d. with me to pay a bill, but found I had too little by 7s. 8d.; what was the amount of the bill?
- (10) I have 5s. left after paying £16. 10s. for a horse, £2. 18s. for a cart, £1. 17s. 6d. for harness, and £1. 15s. 9d. for hay; what money had I at first?
- (11) Tom has 6s. 4d., Bob has 2s. 6d. more than Tom, and George 1s. 9d. more than Bob; how much have they altogether?
- (12) I paid a butcher £3. 10s. 6d., a grocer 15s. 7d. more than the butcher, and a baker £1. 12s. 5d. more than the grocer; what money did I pay altogether?
- (13) A man put in the bank in January £37. 17s., in February £15. 16s. 10d., in March £3. 19s. 9d., in April £50. 16s. 4d., in May £9. 7s. 11d., in June £12. 17s. 6d.; how much did he put in the bank during the six months?
- (14) A man bought a piece of land for £130. 15s., and he paid an architect £50. 17s. 6d., bricklayers £350. 19s. 4d., carpenters £197. 14s. 3d., painters and paperhangers £97. 15s. 10d., ironmongers £86. 16s. 4d., and other workmen £69. 14s. 11d.; what did his house and land cost altogether?
- (15) A farmer sold off his stock: oxen for £673. 16s. 8d., horses for £803. 19s. 9d., sheep for £500. 13s. 7d., farm produce for £989. 17s. 6d., and farming implements for £365. 19s. 3d.; find the total amount received from the sale.

(16) In a town the workpeople subscribe £152. 18s. 3d. towards the hospital, the churches and chapels give £466 17s. 10d., the mayor gives £105, and other subscriptions amount to £315. 10s. 8d.; what is the total amount received by the hospital?

(17) I owe Smith £4. 7s. 4d., and he owes me £3. 17s. 5½d.; how much money must I give him to settle?

(18) What money would be left out of a £10 note after paying a bill of £8. 6s. 5½d.?

(19) In copying down his sum a boy wrote 13s. 6d. instead of £13. 6s.; how much was his error?

(20) A tradesman allowed me 17s. off a bill for £17. 16s. 8d.; how much did I pay?

(21) I wished to pay a bill for £25. 10s. 6d., but found I had not enough by £3. 19s. 11d.; how much money had I?

(22) A pair of horses and a carriage cost £150. 12s. 6d.; the carriage cost £32. 17s. 10d.; what did the horses cost?

(23) A man has £250. 13s. 3d. in the bank; if he draws out £176. 18s. 10d., how much will be left in the bank?

(24) A clerk adds up a cash book and makes the amount come to £250. 17s. 3d.; another makes the amount £249. 18s. 4d.; what is the difference in their results?

(25) A cow and a calf cost £21. 15s.; the calf cost £3. 17s. 6d.; how much more than the calf does the cow cost?

(26) A horse and carriage cost £150. 16s. 2d.; if the carriage cost £47. 19s. 8d., how much more did the horse cost than the carriage?

(27) I paid £100. 3s. 4d. into the bank, and afterwards drew out £41. 17s. 10d., £21. 6s. 5d., £9. 3s. 6d., and £7. 10s. 5d., how much have I now in the bank?

(28) A horse, carriage, and harness cost £100. 16s. 8d.; the horse cost £50. 15s., and the carriage £38. 17s. 9d.; find the cost of the harness.

(29) A clerk copied down in an account 7s. 6d. for £7. 6s. and £10. 9s. for 10s. 9d.; by what amount was the account wrong?

(30) I owed £6. 13s. 3d., and received £3. 16s. 9d. out of a £10 note; was this the correct change, and if not, was it too much or too little, and by how much?

(31) Jones pays me £16. 11s. 6½d., Robinson pays me £57. 15s. 9½d., and Smith pays me £25. 12s. 8½d. If I pay Brown £23. 14s. 7½d., how much money shall I have left?

(32) A man borrowed £200, of which he repaid at Lady Day £40. 10s. 9d., at Midsummer £68. 10s. 11d., and at Michaelmas £25. 10s. 6d.; how much of the £200 does he still owe?

(33) A man having £100. 3s. 4d. owes his butcher £17. 8s. 6d., his tailor £25 14s. 6d., and his baker £12. 19s. 11d.; find what he will have in hand after paying these bills.

(34) What change shall I have out of two £5 notes after paying £6. 15s. for rent, 16s. 2d. for water rate, and 17s. 9d. for gas?

(35) I have £3. 16s. 8d., my brother has 8s. 9d. more, and my sister 18s. 9d. less; how much have we all three together?

(36) What must be paid for 6 tons of Wallsend coal at 29s. per ton, 3 tons of Silkstone at 25s. per ton, and 7 tons of Derbyshire at 22s. 6d.?

(37) A housekeeper spends daily (excluding Sundays) 2s. 3d. on bread, 16s. 6d. on meat, 4s. 4½d. on groceries, and 1s. 8d. on vegetables; what is the weekly amount paid for these articles?

(38) What sum of money must I have in order to give 10 men, 8 women, and 7 boys, respectively 17s. 6d., 15s. 3d., and 8s. 9d. each?

(39) A labourer's wages are 12s. 3d. per week for one half of the year, and 10s. 4d. per week for the other half; what is the total amount of his wages in a year (52 weeks)?

(40) What is the whole cost of 5 pairs of gloves at 2s. 11½d. a pair, 24 yards of muslin at 1s. 9½d. a yard, 17 yards of ribbon at 8½d. a yard, and 35 yards of flannel at 1s. 7¾d. a yard?

(41) What is the total value of the following groceries:—56 lb. of soap at 2½d. a lb., 12 lb. of sugar at 2¾d. a lb., and 21 lb. of flour at 1s. 1½d. per bag of 7 lb.?

(42) If I spend 1s. 9d. a day for a return railway ticket, travelling six days in the week, how much should I save in a year (52 weeks) by paying £25 for a season ticket?

(43) A woman goes shopping with a £5 note. She buys 10 lb. of tea at 2s. 4d. per lb., 8 lb. of coffee at 1s. 7d. per lb., 5 lb. of cheese at 10½d. per lb., and 7 lb. of bacon at 9¾d. per lb.; how much change did she receive?

(44) A builder pays his men wages as follows:—Four carpenters at £1. 5s. 9d. each, eight painters at £1. 10s. 8d. each, seven bricklayers at £1. 6s. 10d. each, and nine labourers at 19s. 6d. each; how much did he pay away altogether?

(45) How much must I have in my pocket to pay 110 men 3s. 6d. each and 123 women 2s. 9d. each?

(46) A man earns 4s. 6d. a day, and his two sons 3s. 4d. each; how much do they all earn in a year (313 working days)?

(47) A carriage and a pair of horses are worth £150; the carriage is worth 35 guineas; what is the value of each horse?

(48) Sixty four yards of damask at 4s. 3d. per yard are made into six curtains. Find the cost of each curtain.

(49) If £63. 10s. 6d. be divided equally among 4 men, 7 boys, and 3 women, find the share of each.

(50) A farmer sold 76 sheep at £4. 16s. 10d. each, and spent the sum received in buying 19 cows; how much did each cow cost?

WEIGHTS AND MEASURES.

99. The weights of all substances, except gold, silver, and jewellery (and medicines under certain circumstances), are measured by **Avoirdupois weight**.

AVOIRDUPOIS WEIGHT.

16 drams (dra.)	make 1 ounce (oz.).
16 ounces 1 pound (lb.).
14 pounds 1 stone (st.).
28 pounds 1 quarter (qr.).
4 quarters, or } 1 hundredweight (cwt.).
112 pounds }	
20 hundredweight, or } 1 ton.
2240 pounds }	

100. The weights of gold, silver, and jewellery are measured by **Troy weight**.

TROY WEIGHT.

24 grains (grs.)	make 1 pennyweight (dwt.).
20 pennyweights 1 ounce (oz. tr.).
12 ounces 1 pound (lb. tr.).

NOTE.—The ounce is also divided into 24 parts, called *carats*. Consequently one carat is equal to 20 grains.

101. In compounding medicines, apothecaries use a different division of the Troy ounce.

APOTHECARIES' WEIGHT.

20 grains	make 1 scruple (scr.).
3 scruples 1 dram (dr.).
8 drams 1 ounce (oz. tr.).
12 ounces 1 pound (lb. tr.).

102. The pound Avoirdupois is the *standard* pound, and is defined by Act of Parliament as containing 7000 grains.

A *grain* denotes the same weight in all cases.

NOTE.—The **Pound Troy** must be carefully distinguished from the **Pound Avoirdupois**.

For	1 lb. Troy = 5760 grains;
but	1 lb. Avoirdupois = 7000 grains.

103. **Lengths** are measured with reference to a unit, which is called a **yard**.

The length of the line *AB* is called an **inch**, and is such
 £40. 36 lines, each an inch long, be placed end to end in a
 £25. 10s. they will make up exactly one yard.

(33) A B.
 tailor £25. 14s. Other measures of length and their relations to
 in hand after paying in the following table:—

A

B

LINEAR MEASURE.

12 inches (in.)	make	1 foot (ft.).
3 feet	1 yard (yd.).
5½ yards	1 rod or pole (po.).
40 poles	1 furlong (fur.).
8 furlongs } or 1760 yards }	1 mile (m.).

NOTE 1. In measuring land, surveyors use a **chain**, (called *Gunter's chain*), which is 22 yards long and is subdivided into 100 equal parts, each of which is called a **link**.

Thus 100 links = 1 chain = 22 yards,
and 10 chains = 220 yards = 1 furlong.

NOTE 2.—The following are also sometimes used:—4 inches make 1 hand (for measuring the height of horses); 6 feet make 1 fathom (for measuring ocean depths); 6080 feet make 1 knot (a nautical term).

105. The principal measure of **area** is a square which has each of its sides equal to the principal measure of length, one yard. It is therefore called a **square yard**.

In the margin is drawn a square which has each of its sides an inch in length; it is therefore called a **square inch**.

If we place 12 square inches side by side, we form an area one inch wide, and 12 inches or one foot long.

Again, if we place 12 such rows of 12 square inches one above another, we form an area 12 inches long and 12 inches wide. It will therefore be a **square foot**, and will obviously be made up of 12 times 12, or 144, square inches.

Hence 1 square foot = 144 square inches.

Similarly, 1 square yard = 9 square feet.

It also follows that 1 square rod } = 30¼ square yards.
or 1 square pole }

A square rod or square pole is also called a **perch**.

106. These and certain other measures of area are arranged in the following table:—

SQUARE MEASURE.

144 sq. inches	make	1 sq. foot.
9 sq. feet	1 sq. yard.
30¼ sq. yards	1 sq. rod, sq. pole, or perch.
40 perches	1 rood (ro.).
4 roods } or 4840 sq. yards }	1 acre (ac.).
640 acres	1 sq. mile.

NOTE.—The area of land is also estimated in **square links** and **square chains**, which are connected with the **square yard** through their linear relations.

Thus

$$\begin{aligned} 1 \text{ chain} &= 22 \text{ yards;} \\ \therefore 1 \text{ square chain} &= 22 \times 22 \text{ square yards;} \\ &= 484 \text{ square yards;} \\ \therefore 10 \text{ sq. chains} &= 4840 \text{ square yards;} \\ \therefore 10 \text{ sq. chains} &= 1 \text{ acre.} \end{aligned}$$

Also

$$\begin{aligned} 1 \text{ sq. chain} &= 100 \times 100 \text{ sq. links;} \\ \therefore 100000 \text{ sq. links} &= 1 \text{ acre.} \end{aligned}$$

107. The principal measure of **volume** is a solid similar to that described in the margin. Its length, breadth, and height are each *one yard*, and the solid is called a **cubic yard**.

The drawing in the margin represents an exact **cubic inch**, its length, breadth, and height being each *one inch*.

If we arrange 144 *cubic inches* in 12 rows, each containing 12 cubes, we form a solid, one inch high, with a top surface whose area is a **square foot**.

If again we form 12 such solids, and place them one above another, we build up a cube whose length, breadth, and height are each one foot. It will therefore be a *cubic foot*; and since we have used 12×144 cubes, each of whose sides is one inch, in making it, it is clear that 1728 *cubic inches* = 1 *cubic foot*.

Similarly it may be shown that 27 **cubic feet** = 1 **cubic yard**.



108. These measures are arranged in the following table.—

CUBIC MEASURE.

1728 cu. inches make 1 cu. foot.
27 cu. feet 1 cu. yard.

109. The following measures are used for certain *dry goods* such as grain and peas, and for some kinds of fruit and vegetables. But the measuring of such things by *volume* is being gradually superseded by their measure by *weight*.

MEASURE OF CAPACITY.

4 gills make 1 pint (pt.).
2 pints 1 quart (qt.).
4 quarts 1 gallon (gall.).
2 gallons 1 peck (pk.).
4 pecks 1 bushel (bush.).
8 bushels 1 quarter (qr.)
5 quarters 1 load.

NOTE.—The *gallon* is the principal unit in this mode of measurement. According to its legal definition, it is such that a gallon of pure water weighs 10 pounds; so

“A pint of pure water weighs a pound and a quarter.”

In measuring beer the following terms are used:—

1 firkin = 9 gallons.	1 kilderkin = 18 gallons.
1 barrel = 36 gallons.	1 hogshead = 54 gallons.
2 hogsheads = 1 butt.	

The terms hogshead and butt are used also in measuring wine, but they vary with the kind of wine.

110. The chief measure of time is a **day**, which is the average time in which the earth turns round on its axis.

The time in which the earth travels round the sun is called a **solar year**, and is equal to about 365 days 6 hours.

The **common year** consists of 365 days. We take account of the difference between a common year and a solar year by adding a whole day to every *fourth* common year. Consequently in every 4th year there are 366 days.

But by adding a whole day to every 4th year, we add too much; we therefore omit the extra day three times every 400 years.

The years which have the extra day are called **Leap Years**. The day is inserted at the end of February, and is called the 29th of February.

The years in which the day is omitted are the last year of each century, except those which are divisible by 400; for example 2000 will be a leap year, but 1900 will not.

It can generally be ascertained whether a year is a leap year or not by dividing the last two digits by 4, except when they are both zeros. If there is no remainder, it is a leap year; for example, 1896 was a leap year, but 1894 was not.

111. The divisions and multiples of the day are given in the following table:—

TIME MEASURE.

60 seconds (sec.)	make 1 minute (min.).
60 minutes 1 hour.
24 hours 1 day.
7 days 1 week.
28, 29, 30, or 31 days 1 calendar month.
12 calendar months 1 year.
365 days 1 common year.
366 days 1 leap year.
100 years 1 century.

Of the calendar months, February has 28 days in a common year, and 29 days in a leap year; April, June, September, and November have each 30 days; the others have each 31 days.

A **lunar month**, or the time in which the moon travels round the earth, is equal to 28 days nearly.

Reduction of Weights and Measures.

112. In the reduction of Weights and Measures the principle is the same as in the reduction of Money.

Example 1. Reduce to drams
5 tons 13 cwt. 2 qrs. 9 lb. 13 oz. 10 drs.

tons	cwt.	qrs.	lb.	oz.	drs.
5	13	2	9	13	10
20					
113	cwt.				
4					
454	qrs.				
28					
3641					
908					
12721	lb.				
16					
76339					
12721					
203549	oz.				
16					
1221304					
203549					
3256794	drs.				

To reduce tons to cwt. we multiply by 20; to reduce cwt. to qrs. we multiply by 4; to reduce qrs. to lb. we multiply by 28, and so on. The odd cwt., qrs., lb., etc., in the given weight must be added to the products successively in the same way as the odd shillings, pence, etc., were added in the reduction of money.

Example 2. Reduce to inches
3 m. 1 fur. 17 po. 3 yds. 1 ft. 8 in.

m.	fur.	po.	yds.	ft.	in.
3	1	17	3	1	8
8					
25	furlongs				
40					
1017	poles				
5½					
5088					
508½					
5596½	yards				
8					
16790½	feet				
12					
201494	inches				

NOTE.—To multiply by 5½, multiply by 5 and then divide the same line by 2 and add. When the units' figure is divided by 2 the result is a units' figure, and must be placed under the other units' figure.

EXERCISE 31.

Reduce

Avoirdupois.

- | | |
|--|--|
| (1) 18 lb. 3 oz. to drs. | (2) 10 lb. 11 oz. 10 drs. to drs. |
| (3) 15 cwt. 2 qrs. 27 lb. to lb. | (4) 2 cwt. 1 qr. 13 lb. to oz. |
| (5) 17 cwt. 3 qrs. 15 lb. to drs. | (6) 6 tons 7 cwt. to oz. |
| (7) 7 tons 12 cwt. 3 qr. to lb. | (8) 25 tons 10 cwt. 3 qrs. 19 lb. to lb. ✓ |
| (9) 4 tons 5 cwt. 3 qrs. 27 lb. to oz. | (10) 3 tons 19 cwt. 3 qrs. 27 lb. to dra. |

Troy.

- (11) 8 oz. 4 dwt. to grains. (12) 14 lb. 8 oz. to grains.
 (13) 7 lb. 8 oz. 17 dwt. to grains. (14) 2 lb. 5 oz. 9 dwt. 3 grs. to grains.

Linear Measure.

- (15) 3 yds. 1 ft. to inches. (16) 18 miles 5 fur. 16 yds. to yards.
 (17) 5 fur. 171 yds. 2 ft. to inches. (18) 2 miles 6 fur. 4 yds. to feet.
 (19) 2 miles 3 fur. 17 yds. to inches. (20) 21 po. 2 yds. 2 ft. 3 in. to inches.
 (21) 3 miles 5 fur. 36 po. 3 yds. 2 ft. to inches.

Square Measure.

- (22) 8 ac. 1 ro. 9 per. to perches.
 (23) 17 sq. yds. 8 sq. ft. 71 sq. in. to sq. inches.
 (24) 5 ac. 2 ro. 16 per. to sq. yards.
 (25) 8 ac. 2 ro. 12 per. 8 sq. yds. to sq. feet.
 (26) 3 ac. 2 ro. 4 per. 7 sq. yds. 3 sq. ft. to sq. inches.

Cubic Measure.

- (27) 101 cub. ft. 678 cub. in. to cub. inches.
 (28) 714 cub. yds. 26 cub. ft. to cub. feet.
 (29) 13 cub. yds. 5 cub. ft. 19 cub. in. to cub. inches.

Capacity.

- (30) 38 qrs. 4 bush. to pecks. (31) 19 qrs. 6 bush. 3 pks. to gallons.
 (32) 121 qrs. 1 bush. 1 pk. to quarts.
 (33) 3 qrs. 3 bush. 1 pk. 1 gall. to pints.

Time.

- (34) 2 weeks 3 hrs. 6 min. to minutes.
 (35) 6 months 3 weeks 2 days to hours.
 (36) 3 weeks 4 days 5 hrs. 54 min. to seconds. ✓

Example 3. Reduce 3256794 drams to tons, cwt., etc.

$$\begin{array}{r}
 16 \overline{) 3256794} \\
 \underline{4 } 10 \text{ drams.} \\
 16 \overline{) 814198} \\
 \underline{4 } 13 \text{ ounces.} \\
 16 \overline{) 203549} \\
 \underline{1 } 9 \text{ pounds.} \\
 28 \overline{) 50887} \\
 \underline{1 } 2 \text{ qrs.} \\
 28 \overline{) 12721} \\
 \underline{7 } \\
 4 \overline{) 3180} \\
 \underline{4 } \\
 20 \overline{) 454} \\
 \underline{2 }
 \end{array}$$

5 tons 13 cwt. 2 qrs. 9 lb. 13 oz 10 drs.

It will be seen that 203549 in the third line denotes *ounces*; 12721 in the fifth denotes *pounds*; 454 in the seventh denotes *quarters*, and 113 in the eighth denotes *hundredweight*.

113. In Linear Measure we know that $5\frac{1}{2}$ yards, or 11 *half-yards*, make 1 pole; hence, to reduce yards to poles, we multiply the number of yards by 2 to reduce them to half-yards, and we then divide the number of half-yards by 11 to reduce them to poles. The remainder after the division by 11 denotes *half-yards*, and must be expressed in yards, feet, and inches.

Example 4. Reduce 201494 inches to miles, furlongs, etc.

$$\begin{array}{r}
 12 \overline{) 201494} \\
 \underline{3 \overline{) 16791} \text{ ft. 2 in.}} \\
 \quad 5597 \text{ yds. 2 in.} \\
 \quad \quad 2 \\
 11 \overline{) 11194} \text{ half-yds. 2 in.} \\
 \underline{40 \overline{) 1017} \text{ poles 7 half-yds. 2 in.}} \\
 \quad 8 \overline{) 25} \text{ fur. 17 po. 3 yds. 1 ft. 8 in.} \\
 \quad \quad 3 \text{ m. 1 fur. 17 po. 3 yds. 1 ft. 8 in.}
 \end{array}$$

NOTE.—If in a result the yds., ft., and inches exceed 5 yds. 1 ft. 6 in., we substitute 1 *pole* for this, and leave the *remainder only* in yds., ft., and inches.

In Square Measure we know that $30\frac{1}{4}$ square yards, or 121 *quarter-yards*, make 1 perch; hence, to reduce square yards to perches, we multiply the number of square yards by 4 to reduce them to quarter-yards, and we then divide the number of quarter-yards by 121 to reduce them to perches.

Example 5. Reduce 33631207 sq. in. to acres, roods, etc.

$$\begin{array}{r}
 144 \left\{ \begin{array}{l} 12 \overline{) 33631207} \\ 12 \overline{) 2802600} - 7 \\ 9 \overline{) 233550} - 0 \end{array} \right\} 7 \text{ sq. inches.} \\
 \quad 25950 \text{ sq. yds.} \\
 \quad \quad 4 \\
 121 \left\{ \begin{array}{l} 11 \overline{) 103800} \\ 11 \overline{) 9436} - 4 \\ 40 \overline{) 857} - 9 \end{array} \right\} 103 \text{ quarter sq. yds.} \\
 \quad 4 \overline{) 21} - 17 \text{ perches.} \\
 \quad \quad 5 \text{ ac. 1 rood.}
 \end{array}$$

Hence the result = 5 ac. 1 ro. 17 per. $25\frac{3}{4}$ sq. yds. 7 sq. in.

= 5 ac. 1 ro. 17 per. 25 sq. yds. 6 sq. ft. 115 sq. in.;

(1) $\frac{3}{4}$ sq. yd. = 6 sq. ft. 108 sq. inches.

(3) 1.

(5) 17 $\frac{3}{4}$.—If in a result the sq. yds., ft., and inches exceed 30 sq. yds. 2 ft. substitute 1 *perch* for this.

(7) 7 tons

(9) 4 tons 5.

EXERCISE 32.

Reduce

Avoirdupois.

- (1) 36540 drams to lb., etc. (2) 2749712 lb. to cwt.
 (3) 32391 oz. to cwt., etc. (4) 10729 lb. to tons, etc.
 (5) 935583 lb. to tons, etc. (6) 648327 oz. to tons, etc.
 (7) 1956742 oz. to tons, etc. (8) 9367875 oz. to tons, etc.
 (9) 6435920 drams to tons, etc. (10) 93015296 drams to tons, etc.

Troy.

- (11) 878162 grs. to lb., etc. (12) 3251763 grs. to lb., etc.
 (13) 144 lb. " lb. troy. (14) 109375 lb. troy to lb. avo.

Linear Measure

- ards, etc. (16) 58739 yds. to miles, etc.
 miles, etc. (18) 182214 in. to miles, etc.

Square Measure.

- sq. in. to sq. yards, etc. (20) 107826 sq. yards to acres.
 sq. ft. to acres, etc. (22) 41000917 sq. in. to acres, etc.

Cubic Measure.

- 632 cub. ft. to cub. yds., etc. (24) 138297 cub. in. to cub. yds., etc.
 33850 cub. in. to cub. yds., etc.

Capacity.

- (27) 560 half pints to gallons. (28) 4617 quarts to pecks, etc.
 (29) 1198592 pecks to quarters. (30) 10000 gallons to quarters, etc.
 (31) 19050 pints to bushels, etc. (32) 3659712 pints to quarters, etc.

Time.

- (33) 12928 seconds to hours, etc. (34) 1554297 seconds to days, etc.
 (35) 2250000 minutes to weeks, etc. (36) 3484274 seconds to weeks, etc.

Addition of Weights and Measures.

114. In the addition of weights and measures the principle is the same as in the addition of money, i.e., we arrange the quantities so that all terms of the same name are in a column, and so that each column is of a lower denomination than the column to the left of it.

Example. Add together 3 tons 4 cwt. 2 qrs. 12 lb., 5 tons 17 cwt. 3 qrs. 10 lb., and 4 tons 12 cwt. 1 qr. 8 lb.

The sum of the pounds column is 30 lb., or 1 qr. 2 lb. We write down 2 lb. at the foot, and transfer the 1 qr. to increase the next column.

The sum of the quarters column is now 7 qrs., or 1 cwt. 3 qrs. We therefore write down 3 qrs. at the foot, and transfer the 1 cwt. as before.

The sum of the cwt. column is now 1 ton 14 cwt., and the sum of the tons column becomes 13 tons.

tons	cwt.	qrs	lb.
3	4	2	12
5	17	3	10
4	12	1	8
13	14	3	2

EXERCISE 33.

Add together

Avoirdupois.

(1) 16 lb. 12 oz. 4 drs., 7 lb. 8 oz. 6 drs., 10 lb. 14 oz. 12 drs., 5 lb. 9 oz. 13 drs.

(2) 14 lb. 15 oz. 12 drs., 21 lb. 14 oz. 12 drs., 4 lb. 5 oz. 11 drs., 16 lb. 12 oz. 3 drs., 9 lb. 7 oz. 13 drs., 18 lb. 10 oz. 5 drs.

(3) 8 cwt. 1 qr. 17 lb., 16 cwt. 3 lb., 2 cwt. 2 qrs. 21 lb., 14 cwt. 1 qr. 13 lb., 10 cwt. 23 lb., 5 cwt. 2 qrs. 17 lb.

(4) 13 tons 19 cwt. 27 lb., 6 tons 16 cwt. 3 qrs. 2 lb., 29 tons 10 cwt. 2 qrs. 24 lb., 17 tons 8 cwt. 1 qr. 6 lb., 14 tons 6 cwt. 3 qrs. 26 lb., 13 cwt. 15 lb., 36 tons 19 cwt. 2 qrs. 10 lb.

(5) 7 tons 10 cwt. 2 qrs. 18 lb., 25 tons 15 cwt. 1 qr. 14 lb., 9 tons 4 cwt. 3 qrs. 6 lb., 33 tons 19 cwt. 2 qrs. 15 lb., 16 tons 13 cwt. 19 lb., 24 tons 3 qrs. 10 lb., 9 cwt. 3 qrs. 9 lb.

(6) 7 tons 3 cwt. 2 qrs. 27 lb., 21 tons 19 cwt. 3 qrs. 23 lb., 13 tons 17 cwt. 1 qr. 22 lb., 14 tons 6 cwt. 19 lb., 6 tons 10 cwt. 2 qrs. 18 lb., 29 tons 18 cwt. 21 lb., 10 cwt. 2 qrs. 6 lb.

Troy.

(7) 5 oz. 16 dwt. 12 grs., 2 oz. 17 dwt. 13 grs., 10 oz. 5 dwt. 23 grs., 5 oz. 19 dwt. 18 grs., 11 oz. 14 dwt. 19 grs.

(8) 11 oz. 12 dwt. 21 grs., 9 oz. 19 dwt. 19 grs., 7 oz. 6 dwt. 22 grs., 8 oz. 17 dwt. 18 grs., 10 oz. 18 dwt. 23 grs.

(9) 2 lb. 10 oz. 9 dwt. 22 grs., 3 lb. 11 oz. 2 dwt. 15 grs., 5 lb. 9 oz. 18 dwt. 20 grs., 7 lb. 7 oz. 15 dwt. 16 grs., 8 lb. 8 oz. 19 dwt. 21 grs., 10 oz. 14 dwt. 23 grs.

(10) 12 lb. 11 oz. 19 dwt. 19 grs., 42 lb. 6 oz. 16 dwt. 5 grs., 28 lb. 5 oz. 14 dwt. 23 grs., 9 lb. 3 oz. 7 dwt. 16 grs., 74 lb. 10 oz. 18 dwt. 19 grs., 18 lb. 7 oz. 12 dwt. 21 grs.

Linear Measure.

(11) 73 yds. 2 ft. 7 in., 16 yds. 5 ft. 10 in., 7 ft. 11 in., 1 ft. 4 in.

(12) 41 miles 3 fur. 29 po., 16 miles 4 fur. 19 po., 87 miles 5 fur. 39 po., 42 miles 3 fur. 16 po.

Square Measure.

(13) 19 ac. 2 ro. 4 per., 3 ac. 1 ro. 16 per., 1 ac. 21 per., 11 ac. 3 ro. 18 per.

(14) 67 ac. 3 ro. 30 per. 23 sq. yds., 1 ac. 2 ro. 17 per. 19 sq. yds., 17 ac. 3 ro. 39 per. 29 sq. yds., 18 ac. 2 ro. 19 per. 17 sq. yds., 19 ac. 3 ro. 36 per. 20 sq. yds.

Cubic Measure.

(15) 9 cub. yds. 26 cub. ft. 1727 cub. in., 8 cub. yds. 21 cub. ft. 1642 cub. in., 7 cub. yds. 22 cub. ft. 1504 cub. in., 8 cub. yds. 12 cub. ft. 1379 cub. in.

(16) 8 cub. yds. 19 cub. ft. 72 cub. in., 47 cub. yds. 13 cub. ft. 24 cub. in., 51 cub. yds. 21 cub. ft. 1002 cub. in., 19 cub. yds. 13 cub. ft. 1278 cub. in.

Capacity.

(17) 29 gall. 3 qts. 1 pt., 16 gall. 2 qts. 1 pt., 94 gall. 3 qts., 102 gall. 2 qts. 1 pt.

(18) 16 qrs. 3 bush. 2 pks. 1 gall., 7 bush. 2 pks., 41 qrs. 7 bush. 2 pks. 1 gall., 124 qrs. 6 bush. 1 pk. 1 gall.

Time.

(19) 91 days 16 hrs. 48 min. 43 sec., 48 days 12 hrs. 27 min. 29 sec., 29 days 13 hrs. 14 min. 17 sec., 89 days 17 hrs. 18 min. 29 sec.

(20) 6 weeks 6 days 12 hrs. 18 min. 42 sec., 1 day 20 hrs. 37 min. 25 sec., 2 weeks 6 days 14 hrs. 42 min. 21 sec., 3 weeks 5 days 18 hrs. 17 min. 40 sec.

Subtraction of Weights and Measures.

115. To subtract one weight or measure from another, we place the weight or measure to be subtracted below the one from which it is to be taken, as in the case of subtraction of money.

Example. Subtract 12 cwt. 3 qrs. 21 lb. from 18 cwt. 1 qr. 16 lb.

cwt.	qrs.	lb.
18	1	16
12	3	21
5	1	23

By the method of Art. 39 the mental steps are as follows:—

1st. 21 lb. and 23 lb. make 1 qr. 16 lb.; write down 23 lb. and carry 1 qr.

2nd. 4 qrs. and 1 qr. 1 cwt. 1 qr.; 1 qr. and carry 1 cwt.

3rd. 13 cwt. and 5 cwt. ... 18 cwt.; ... 5 cwt.

The result is 5 cwt. 1 qr. 23 lb.

EXERCISE 34.**Subtract***Avoirdupois.*

(1) 15 cwt. 3 qrs. 17 lb from 19 cwt. 2 qrs. 4 lb.

(2) 7 cwt. 3 qrs. 15 lb. 9 oz. from 18 cwt. 2 qrs. 14 lb. 6 oz.

(3) 12 tons 16 cwt. 3 qrs 12 lb. from 16 tons 14 cwt. 1 qr. 9 lb.

(4) 19 tons 12 cwt. 3 qrs. 21 lb. 15 oz. from 34 tons 10 cwt. 2 qrs. 13 lb. 1 oz.

(5) 18 tons 13 cwt. 2 qrs. 19 lb. 12 oz. from 20 tons 10 cwt. 1 qr. 17 lb. 4 oz.

(6) 4 lb. 13 oz. 7 drs. from 15 lb. 8 oz. 2 drs.

Troy.

- (7) 15 dwt. 12 grs. from 1 oz.
 (8) 13 dwt. 18 grs. from 1 oz. 11 dwt. 3 grs.
 (9) 6 oz. 15 dwt. 17 grs. from 1 lb. 4 oz. 10 dwt. 16 grs.

Linear Measure.

- (10) 13 yds. 2 ft. 9 in. from 31 yds. 1 ft. 3 in.
 (11) 156 miles 4 fur. 216 yds. from 191 miles 2 fur. 2 yds.
 (12) 14 miles 1345 yds. 2 ft. from 17 miles 125 yds. 1 ft.

Square Measure.

- (13) 6 ac. 3 ro. 27 per. from 13 ac. 2 ro. 25 per.
 (14) 10 sq. yds. 8 sq. ft. 100 sq. in. from 13 sq. yds. 7 sq. ft. 70 sq. in.

Cubic Measure.

- (15) 156 cub. yds. 21 cub. ft. 1132 cub. in. from 347 cub. yds. 10 cub. ft. 830 cub. in.
 (16) 83 cub. yds. 19 cub. ft. 1109 cub. in. from 150 cub. yds. 7 cub. ft. 140 cub. in.

Capacity.

- (17) 6 qrs. 5 bush. 3 pks. 1 gall. from 7 qrs. 7 bush. 2 pks.
 (18) 7 bush. 3 pks. 1 gall. 3 qts. from 1 qr. 1 bush. 3 pks. 1 gall. 1 qt.

Time.

- (19) 4 days 21 hrs. 10 min. 52 sec. from 24 days 14 hrs. 46 min. 31 sec.
 (20) 10 weeks 4 days 11 hrs. 7 min. 20 sec. from 12 weeks 3 days 15 hrs. 17 sec.

Multiplication of Weights and Measures.

116. The following is an example of multiplication.

Example. Multiply 5 cwt. 3 qrs. 24 lb. by 8.

tons	cwt.	qrs.	lb.
	5	3	24
			8
<hr/>			
2	7	2	24
<hr/>			

We see that 8 times 24 lb. = 192 lb. = 6 qrs. 24 lb. Hence we write down **24** under the lb. and carry 6 qrs. forward.

Again, 8 times 3 qrs. = 24 qrs., which become 30 qrs. when we add the 6 qrs.; and 30 qrs. = 7 cwt. 2 qrs. Hence we write down **2** under the qrs. and carry forward 7 cwt.

Lastly, 8 times 5 cwt. = 40 cwt., which become 47 cwt. when we add the 7 cwt.; and 47 cwt. = 2 tons 7 cwt.

EXERCISE 35.

Multiply

Avoirdupois.

- (1) 5 lb. 3 oz. 14 drs. by 3. (2) 18 cwt. 2 qrs. 14 lb. by 8.
(3) 2 tons 13 cwt. 2 qrs. 17 lb. by 36.
(4) 1 ton 9 cwt. 16 lb. by 24.
(5) 2 tons 9 cwt. 15 lb. 9 oz. by 27.
(6) 1 cwt. 27 lb. 15 oz. by 17.
(7) 11 cwt. 3 qrs. 15 lb. 4 oz. by 46.
(8) 3 tons 14 cwt. 2 qrs. 14 lb. 12 oz. by 38.
(9) 12 cwt. 2 qrs. 17 lb. by 176.
(10) 4 tons 15 cwt. 2 qrs. 27 lb. by 195.

Troy.

- (11) 7 lb. 8 oz. 16 dwt. 13 gra. by 9.
(12) 3 lb. 7 oz. 15 dwt. 9 gra. by 32.
(13) 2 lb. 1 oz. 13 dwt. 15 gra. by 13.

Linear Measure.

- (14) 5 yds. 2 ft. 7 in. by 12. (15) 25 yds. 1 ft. 11 in. by 29.
(16) 1 mile 3 fur. 100 yds. 1 ft. by 24.
(17) 53 miles 5 fur. 23 po. 4 yds. by 19.

Square Measure.

- (18) 13 ac. 3 ro. 15 per. by 7. (19) 385 ac. 3 ro. 18 per. by 85.
(20) 25 sq. yds. 7 sq. ft. 132 sq. in. by 36.
(21) 1 ac. 3 ro. 7 per. 4 sq. yds. by 17.

Cubic Measure.

- (22) 76 cub. yds. 8 cub. ft. 1650 cub. in. by 9.
(23) 7 cub. yds. 24 cub. ft. 5 cub. in. by 72.
(24) 3 cub. yds. 4 cub. ft. 32 cub. in. by 34.

Capacity.

- (25) 3 bush. 3 pks. 1 gall. 3 qts. by 5.
(26) 7 bush. 1 gall. 3 qts. by 18. (27) 8 qrs. 4 bush. 1 pk. by 17.

Time.

- (28) 1 month 1 week 3 days 21 hrs. 36 min. by 10.
(29) 6 days 14 hrs. 5 min. 9 sec. by 16.
(30) 4 days 22 hrs. 19 min. 5 sec. by 46.

Division of Weights and Measures.

117. The following is an example of the first kind of division (Art. 94).

Example. Divide 15 tons 10 cwt. 3 qrs. by 8.

$$\begin{array}{r}
 \text{tons cwt. qrs. lb.} \\
 8 \overline{) 15 \quad 10 \quad 3 \quad 4} \\
 \underline{1 \quad 18 \quad 3 \quad 11}
 \end{array}$$

If we divide 15 tons by 8, the quotient is 1 ton and the remainder is 7 tons. We put down the 1 ton.

We now change the 7 tons into 140 cwt., and add the 10 cwt. to it: we thus get 150 cwt. altogether. If now we divide this by 8, the quotient is 18 cwt., which we put down, and the remainder is 6 cwt.

We next change the 6 cwt. into 24 qrs., and add the 3 qrs.: we thus get 27 qrs. altogether. If we now divide this by 8, the quotient is 3 qrs., which we put down, and the remainder is 3 qrs.

Lastly, we change the remainder 3 qrs. into 84 lb., and add the 4 lb.: we thus get 88 lb. altogether. If we now divide this by 8, the quotient is 11 lb., which we put down, and there is no remainder.

The operation is now at an end, and the result is

1 ton 18 cwt. 3 qrs. 11 lb.

NOTE.—When the divisor is large the work is stated in full as in the example worked out in the case of Money.

EXERCISE 36.

Divide

Avoirdupois.

- (1) 7 tons 7 cwt. 2 qrs. 16 lb. by 6.
- (2) 15 tons 10 cwt. 3 qrs. 4 lb. by 8.
- (3) 41 tons 4 cwt. 4 lb. 12 oz. by 11.
- (4) 57 tons 7 cwt. 3 qrs. 17 lb. 12 oz. by 12.
- (5) 16 cwt. 3 qrs. 14 lb. 8 oz. by 64.
- (6) 74 tons 5 cwt. 3 qrs. 21 lb. 7 oz. by 49.
- (7) 12 cwt. 1 qr. 3 lb. 11 oz. by 23.
- (8) 12 tons 3 cwt. 1 qr. 25 lb. by 37.
- (9) 40 tons 19 cwt. 13 lb. 5 oz. by 59.

Troy.

- (10) 23 lb. 2 oz. 12 dwt. by 6.
- (11) 116 lb. 8 oz. 12 dwt. by 32.
- (12) 3 lb. 11 oz. 16 dwt. 16 grs. by 41.

Linear Measure.

- (13) 44 yds. 2 ft. 3 in. by 9.
- (14) 168 yds. 1 ft. 4 in. by 16.
- (15) 743 yds. 1 ft. 7 in. by 29.
- (16) 10 miles 1 fur. 6 po. by 22.
- (17) 75 miles 1 fur. 39 po. 3 yds. by 75.
- (18) 12 miles 2 fur. 20 po. 4 yds. 2 ft. 6 in. by 47.

Square Measure.

- (19) 9 ac. 3 ro. 15 per. by 7. (20) 42 ac. 3 ro. 2 per. by 12.
 (21) 29 ac. 3 ro. 1 per. 21 sq. yds. by 71.

Cubic Measure.

- (22) 69 cub. yds. 10 cub. ft. 576 cub. in. by 96.
 (23) 944 cub. yds. 1 cub. ft. 1104 cub. in. by 59.
 (24) 194 cub. yds. 23 cub. ft. 1557 cub. in. by 127.

Capacity.

- (25) 3 qrs. 5 bush. 2 pks. 1 qt. by 35.
 (26) 154 qrs. 3 bush. 2 pks. 1 qt. by 69.
 (27) 53 qrs. 6 bush. 2 pks. 3 qts. 1 pt. by 217.

Time.

- (28) 7 weeks 6 days 4 hrs. 14 min. by 12.
 (29) 23 weeks 5 days 21 hrs. 30 min. by 40.
 (30) 317 days 13 hrs. 31 min. 57 sec. by 53.

118. The following is an example of the second kind of division (Art. 94).

Example. How often is 1 cwt. 8 lb. contained in 2 tons 2 cwt. 3 qrs. 12 lb.?

We must first reduce both quantities to lb.

cwt. lb.
 1 8
 112
 120 lb.

tons. cwt. qrs. lb.
 2 2 3 12
 20
 42 cwt.
 4
 171 qrs.
 28
 1380
 342
 120)4800 lb.
 40

Hence the quotient = 40.

EXERCISE 37.

Find how often the first quantity is contained in the second :—

- (1) 8 oz. 4 drs.; 15 lb. 7 oz. 8 drs.
 (2) 3 cwt. 3 qrs. 15 lb.; 1 ton 7 cwt. 21 lb.
 (3) 2 lb. 4 oz.; 4 cwt. 3 qrs. 1 lb. 4 oz.
 (4) $137\frac{1}{2}$ yards; 5 miles. (5) 55 inches; $32\frac{1}{2}$ miles.
 (6) 2 yds. 2 ft. 3 in.; $12\frac{1}{2}$ miles. (7) 1 ro. 14 per.; 18 ac. 36 per.

- (8) 4 ac. 3 ro. 27 per.; 1416 ac. 2 ro. 16 per.
 (9) 121 sq. yds.; 10 ac.
 (10) 3 cub. ft. 576 cub. in.; 3 cub. yds. 12 cub. ft. 576 cub. in.
 (11) 3 bush. 1 pk.; 145 qrs. 3 bush. 2 pks.
 (12) 14 days 13 hrs. 27 min.; 684 days 8 hrs. 9 min.

MISCELLANEOUS EXAMPLES.

Compound Rules (Weights and Measures).

EXERCISE 38.

- (1) Four prize oxen were said to have weighed, respectively, 23 cwt. 2 qrs. 3 lb., 22 cwt., 18 cwt. 3 qrs. 8 lb., and 22 cwt. 2 qrs. 15 lb.; what was their total weight?
- (2) A grocer bought a quantity of sugar weighing 16 cwt. 20 lb.; he sold to one customer 1 cwt. 3 qrs. 4 lb., to another 2 cwt. 3 qrs. 24 lb., to a third 2 cwt. 1 qr. 17 lb., and to a fourth 6 cwt. 3 qrs. 19 lb.; what quantity remained unsold?
- (3) If a bushel of barley weighs 54 lb., find the weight of 39 bushels in cwt., qrs., and lb.
- (4) A truck when full of coal weighs 45 cwt. 1 qr. 13 lb., and the truck itself weighs 13 cwt. 2 qrs. 16 lb.; find the amount of coal in seven such trucks.
- (5) A truck contains 120 sacks; each sack weighs 7 lb. 10 oz. and contains 84 lb. of grain; find the weight of the whole load in tons, cwt., and lb.
- (6) If 12 cartloads of coal, each weighing 19 cwt. 3 qrs., and 8 cartloads, each weighing 1 ton 3 cwt. 21 lb., are divided equally among 28 poor people, what weight of coal will each receive?
- (7) If 1 ton 5 cwt. 19 lb. of tea is to be made up as far as possible into 10 lb. parcels, how many parcels will there be, and how much tea will be left?
- (8) A tank holds 5360 gallons of water; find the weight of the water (in tons, etc.), assuming that a gallon of water weighs 10 lb.
- (9) A sovereign contains 123 grains; how many can be coined out of a bar of gold weighing 3 lb. 10 oz. 2 dwt. 12 grs.?
- (10) If six dozen table-spoons weigh 11 lb. 6 oz. 12 dwt., and the same number of tea-spoons weigh 3 lb. 7 oz. 10 dwt., what is the difference in weight between a table-spoon and a tea-spoon?
- (11) A road, 1500 yards long, has trees planted on *both* sides and $2\frac{1}{2}$ yards apart; how many trees were required?
- (12) A bicycle wheel is 55 inches round, how often will it turn round *in a distance of $32\frac{1}{2}$ miles?*

(13) Thirty-three telegraph posts, placed at equal distances, extend a mile, find the distance between any two posts.

(14) Along 20 miles 4 fur. 20 po. of railway are evenly placed 330 telegraph poles, there being one at the beginning and one at the end; how many yards are they apart?

(15) A flash of lightning was seen 9 seconds before the thunder was heard; how many miles distant was the thunder-cloud, sound travelling at the rate of 1130 feet per second?

(16) A farmer rents six fields, each containing 7 ac. 3 ro. 35 per.; five fields, each containing 3 ac. 2 ro. 17 per.; and nine fields, each containing 5 ac. 1 ro. 25 per., how much land has he altogether?

(17) From an estate of 1700 acres, two farms were sold, the one of 125 ac. 2 ro. 25 per., the other of 237 ac. 1 ro. 30 per.; how much of the estate is left?

(18) After deducting 3 roods 2 perches $29\frac{1}{2}$ sq. yds. for making a road, a field containing 31 acres is divided into 40 allotments of equal area, find the size of each piece.

(19) How many cubic feet are there in a cistern which contains 28 gallons of water, if each gallon occupies $277\frac{1}{4}$ cubic inches?

(20) How many half-pint bottles can be filled from 5 barrels of ale?

(21) From a cask containing 26 gall. 1 qt. of ale there is drawn off daily 3 qts. 1 pt.; for how many days will the ale last?

(22) If an acre of land produces 3 qrs. 6 bush. 3 pks. of corn, what amount of corn will 313 acres yield at this rate?

(23) How many hours are there between midnight on March 20th and midnight on September 29th?

(24) How many days are there from January 19th to June 24th in a common year, excluding the first date?

(25) How many weeks are there from August 1st to October 31st?

(26) If the sun rises at 3.51 a.m. and sets at 8.17 p.m., for how many minutes does it shine?

(27) The moon revolves round the earth once in 2360591 seconds; express this time in days, hours, minutes, and seconds.

(28) If it were possible to travel from the earth to the moon, how many days would it take, at the rate of 81 miles per hour, the moon being 237615 miles from the earth?

(29) A traveller arrived at a railway station at 20 min. 45 sec. past 4 o'clock, but found that the train did not start until 6 o'clock; how long had he to wait?

(30) If a large wheel revolves once in 1 min. 15 sec., how many times will it revolve in 24 hours?

FACTORS AND PRIME NUMBERS.

119. Any number which divides another without a remainder is called a **measure** of that number.

Thus, 6 is a measure of 48.

120. Any number which contains another an exact number of times is called a **multiple** of that number.

Thus, 48 is a multiple of 6.

121. All numbers are multiples of unity, and each contains itself once. Any number can therefore be divided by itself and by unity, but if it cannot be divided without remainder by *any other* number it is said to be **prime**, and is called a **prime number**.

We may therefore define a prime number as *a number which has no factor or divisor but itself and unity*.

Numbers which are not prime are called **composite**.

It will be seen that 2, 3, 5, 7, 11, 13, 17, etc., are prime numbers, and that 4, 6, 8, 9, 10, 12, 14, 15, etc., are composite.

122. Such a number as 12 is the product of 6 and 2, also of 4 and 3, and also of 2, 2 and 3; but the last of these sets, namely 2, 2 and 3, has a property which the others have not, for 2, 2 and 3 are all *prime*.

No number can be separated into *prime* factors in more than one way. When therefore we speak of separating a number into factors, we shall generally mean *prime* factors.

Thus, $12 = 2 \times 2 \times 3 = 2^2 \times 3$.

123. The prime factors of a number can be found by trying the prime numbers in succession, beginning with 2. When we have found one factor, we divide the number by it, and then treat the quotient in a similar way; and we continue this process until we arrive at a divisor which gives a quotient less than itself.

124. What numbers to try will readily be seen after a little practice, but it will be well to remember the following **rules of divisibility** :—

(1) 2 is a factor of all numbers whose last digit can be divided by 2.

(2) 4 is a factor if the number composed of the last 2 digits can be divided by 4.

(3) 8 is a factor if the number composed of the last 3 digits can be divided by 8.

(4) 5 is a factor if the last digit be either 0 or 5.

(5) 10 is a factor if the last digit is 0.

(6) 3 is a factor if the sum of the digits can be divided by 3.

(7) 9 is a factor if the sum of the digits can be divided by 9.

(8) 6 is a factor if both 2 and 3 are factors.

(9) **12** is a factor if both **3** and **4** are factors.

(10) **11** is a factor if, when the 1st, 3rd, 5th, etc., digits are added together, and also the 2nd, 4th, 6th, etc., the difference between these sums is 0, or can be divided by 11.

(11) Divisibility by **7** can be determined only by actual division.

The prime numbers up to 100 are 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

125. We will now apply these rules.

Example. Find the prime factors of 3498.

1st. 2 is a factor because the last digit 8 can be divided by 2; thus

$$3498 = 2 \times 1749.$$

2nd. 3 is a factor of 1749, because it divides the sum of the digits, viz. 21; thus

$$3498 = 2 \times 3 \times 583.$$

3rd. 11 is a factor of 583, because the sum of the 1st and 3rd digits is 8, and the 2nd digit is also 8; the difference of these sums being 0, it follows that 583 is divisible by 11. Hence

$$3498 = 2 \times 3 \times 11 \times 53.$$

The work shown above illustrates the application of the rules, but the process of separating a number into factors may be stated briefly thus:-

$$2 \mid 3498$$

$$3 \mid 1749$$

$$11 \mid 583$$

$$53$$

$$\therefore 3498 = 2 \times 3 \times 11 \times 53.$$

EXERCISE 39.

Separate each of the following numbers into its prime factors:-

- | | | | | |
|------------|------------|------------|------------|------------|
| (1) 60. | (2) 78. | (3) 102. | (4) 210. | (5) 252. |
| (6) 315. | (7) 525. | (8) 612. | (9) 715. | (10) 854. |
| (11) 1331. | (12) 1430. | (13) 1456. | (14) 3465. | (15) 3675. |
| (16) 4536. | (17) 4620. | (18) 4851. | (19) 5250. | (20) 7623. |

GREATEST COMMON MEASURE.

126. A number which is a factor of two or more numbers is said to be a **common factor**, or **common measure**, of the numbers. Thus, 2 is a common measure of 4 and 6.

The *greatest* number which will divide each of two or more numbers is called their **Highest Common Factor**, or **Greatest Common Measure**, and is denoted by the letters **H.C.F.** or **G.C.M.**

127. In many cases the G.C.M. of two or more numbers may readily be determined by separating each number into its prime factors.

Example. Find the G.C.M. of 36 and 60.

We find that $36 = 2 \times 2 \times 3 \times 3$,
and that $60 = 2 \times 2 \times 3 \times 5$.

The factor 2 occurs *twice* in each number, and the factor 3 occurs *once*; hence the G.C.M. $= 2 \times 2 \times 3 = 12$, for this number is the product of all the prime factors common to both.

128. To find the G.C.M. of two numbers whose prime factors cannot readily be found: divide the greater number by the smaller, and then divide the first divisor by the remainder, and so on, using the remainder at each stage to divide the previous divisor; the last divisor is the G.C.M. required.

Example. Find the G.C.M. of 6802 and 1253.

$$\begin{array}{r}
 1253 \overline{) 6802} \text{ (5)} \\
 \underline{6265} \\
 537 \overline{) 1253} \text{ (2)} \\
 \underline{1074} \\
 179 \overline{) 537} \text{ (3)} \\
 \underline{537}
 \end{array}
 \quad \therefore \text{G.C.M.} = \underline{179}.$$

The process is based upon the principle that any number which divides each of two numbers divides also (1) their sum, (2) their difference, and (3) the sum and difference of any multiples of the numbers.

Hence it follows that

the G.C.M. of 1253 and 6802
 $=$ G.C.M. of 537 and 1253
 $=$ G.C.M. of 179 and 537
 $= 179$.

Our object being to find the greatest *common* factor, we may simplify the divisor and the dividend at any stage by dividing either of them by any factor which is *not* common to both.

The work may be arranged more concisely thus:—

$$\begin{array}{r|l}
 2 \overline{) 1253 \ 6802} & 5 \\
 \underline{1074 \ 6265} & \\
 179 \overline{) 537} & 3 \\
 \underline{537} &
 \end{array}$$

EXERCISE 40.

Find the G.C.M. of

- | | | |
|------------------------|-----------------------|------------------------|
| (1) 45 and 72. | (2) 45 and 81. | (3) 72 and 162. |
| (4) 132 and 165. | (5) 72 and 480. | (6) 45 and 162. |
| (7) 216 and 258. | (8) 720 and 960. | (9) 936 and 2925. |
| (10) 429 and 6942. | (11) 813 and 1062. | (12) 42336 and 53088. |
| (13) 10353 and 14877. | (14) 2145 and 3471. | (15) 4929 and 43672. |
| (16) 94248 and 105336. | (17) 61655 and 70092. | (18) 66429 and 169037. |

129. To find the G.C.M. of three or more numbers.

(1) When the numbers can easily be separated into prime factors, this should be done.

(2) When the numbers cannot be easily separated, find the G.C.M. of the first two numbers; then the G.C.M. of this result and the third number; and so on.

EXERCISE 41

Find the g.c.m. of

- | | |
|--------------------------|---------------------------|
| (1) 45, 72 and 480. | (2) 162, 729 and 4374. |
| (3) 372, 994 and 3132. | (4) 218, 484 and 73170. |
| (5) 492, 744 and 1044. | (6) 975, 3150 and 146250. |
| (7) 1326, 3094 and 4420. | (8) 1288, 1736 and 104. |

LEAST COMMON MULTIPLE.

130. A number which can be divided without remainder by two others, or more, is a **common multiple** of them. Thus, 12 is a common multiple of 2, 3, 4, and 6.

The *least* number which can be so divided is called their **Least Common Multiple**, and is denoted by the letters **L.C.M.**

131. The L.C.M. may be determined very readily when the numbers can be separated into prime factors.

Example. Find the L.C.M. of 36 and 60.

We find that $36 = 2 \times 2 \times 3 \times 3$,
and that $60 = 2 \times 2 \times 3 \times 5$.

Now the L.C.M. must contain all the prime factors in 36 and 60, and each of these factors must occur at least as often in the L.C.M. as it occurs in either of the numbers themselves. Now, 2 occurs *twice* in both, and must therefore occur *twice* in the L.C.M. Also, as 3 occurs *once* in 60, and *twice* in 36, it must occur *twice* in the L.C.M. Again, as 5 occurs in 60, it must occur in the L.C.M. ;

$$\therefore \text{L.C.M.} = 2 \times 2 \times 3 \times 3 \times 5 = \underline{180}.$$

132. To find the L.C.M. of three or more numbers we proceed in exactly the same way.

Example. Find the L.C.M. of 9, 10, 15, 16, 20.

$$9 = 3 \times 3$$

$$10 = 2 \times 5$$

$$15 = 3 \times 5$$

$$16 = 2 \times 2 \times 2 \times 2$$

$$20 = 2 \times 2 \times 5$$

Here the L.C.M. must contain the prime factors 2, 3 and 5. Now, 2 occurs *four* times in one number, 3 occurs *twice*, and 5 *once*.

$$\therefore \text{L.C.M.} = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 = \underline{720}.$$

133. We will now consider the form in which the work can be best arranged when the numbers are numerous and small. Having written the numbers in a row, we see that 10 is a factor of 20; hence all numbers divisible by 20 must also be divisible by 10. We may consequently cross out 10 thus: —

$$9, \quad 10, \quad 15, \quad 16, \quad 20;$$

and we state the work as follows : -

2	9,	10,	15,	16,	20
2	9,		15,	8,	10
3	9,		15,	4,	5
	3,		5,	4	

The divisors 2, 2 and 3 are prime factors which are common to two or more of the numbers. In any case we divide by such prime factors as will divide two or more of the numbers ; we begin with the lowest, and we take them in order. We write down the quotients below such numbers as can be divided, and bring down unaltered the numbers which cannot be divided. In the third line we strike out 5, because it is a factor of 15, and may therefore be neglected ; in the fourth line there are three numbers, no two of which have a common factor.

Hence the L.C.M. $= 2 \times 2 \times 3 \times 3 \times 5 \times 4 = \underline{720}$.

EXERCISE 42.

Find the L.C.M. of

- | | |
|--------------------------------|-------------------------------|
| (1) 8, 12 and 20. | (2) 9, 12 and 21. |
| (3) 18, 20 and 30. | (4) 18, 36, 60 and 72. |
| (5) 18, 36, 45 and 81. | (6) 18, 56, 50 and 72. |
| (7) 12, 36, 72 and 120. | (8) 16, 28, 35 and 63. |
| (9) 26, 33, 39 and 44. | (10) 32, 48, 40 and 14. |
| (11) 12, 23, 55 and 40. | (12) 16, 21, 49 and 72. |
| (13) 7, 17, 68 and 238. | (14) 8, 12, 20, 30 and 80. |
| (15) 3, 5, 12, 36 and 45. | (16) 9, 12, 16, 18 and 28. |
| (17) 21, 45, 60, 84 and 90. | (18) 12, 18, 27, 30 and 40. |
| (19) 8, 12, 18, 24, 28 and 30. | (20) 4, 11, 18, 20, 48 and 72 |

FRACTIONS.

134. If a quantity be divided into any number of equal parts, one or more of these parts is called a **fraction** of the quantity.

If, for example, we have the amount of a shilling in penny pieces, we have 12 pennies altogether ; and if we divide it into 3 equal parts we shall have 4 pennies in each part.

Each of these equal parts is called **one-third** of the whole shilling, and its relation to the shilling is expressed by the symbol $\frac{1}{3}$. Thus 4 pence $= \frac{1}{3}$ of 1 shilling. If we take 2 parts together there will be 8 pennies, and their relation to the whole shilling is described as **two-thirds**, and is expressed by the symbol $\frac{2}{3}$.

135. It will be seen that the number *below* the line gives a name to the parts ; for this reason it is called the **denominator**.

Also, since the number *above* the line denotes the *number* of equal parts that are taken, it is called the **numerator**.

Thus, in the symbol $\frac{2}{3}$, 3 is the *denominator* and 2 is the *numerator*.

136. Fractions such as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, etc. are called **vulgar fractions**, that is, common fractions.

137. When both the numerator and the denominator are integers the fraction is called a **simple fraction**.

EXERCISE 43.

Find the value of

- | | |
|---|---|
| (1) $\frac{1}{2}$ of £17. 11s. 8d. | (2) $\frac{1}{3}$ of £17. 5s. 6d. |
| (3) $\frac{1}{4}$ of £28. 15s. 6d. | (4) $\frac{2}{3}$ of £12. 3s. 6d. |
| (5) $\frac{3}{4}$ of £28. 15s. 9d. | (6) $\frac{1}{4}$ of £27. 13s. 4d. |
| (7) $\frac{1}{7}$ of £12. 4s. 6 $\frac{1}{2}$ d. | (8) $\frac{3}{4}$ of £32. 7s. 6d. |
| (9) $\frac{1}{10}$ of £71. 16s. 10 $\frac{1}{2}$ d. | (10) $\frac{1}{12}$ of £52. 12s. 6d. |
| (11) $\frac{1}{4}$ of 2 lb. 3 oz. | (12) $\frac{1}{4}$ of 3 cwt. 1 qr. 14 lb. |
| (13) $\frac{1}{4}$ of 5 cwt. 1 qr. 20 lb. | (14) $\frac{1}{4}$ of 3 cwt. 3 qrs. 12 lb. |
| (15) $\frac{3}{4}$ of 7 cwt. 3 qrs. 7 lb. | (16) $\frac{1}{8}$ of 1 ton 2 cwt. 3 qrs. 12 lb. |
| (17) $\frac{1}{4}$ of 13 lb. 2 oz. | (18) $\frac{1}{4}$ of 12 cwt. 3 qrs. 21 lb. |
| (19) $\frac{1}{4}$ of 3 tons 5 cwt. 10 lb. | (20) $\frac{1}{4}$ of 31 tons 10 cwt. 3 qrs. 24 lb. |

Reduction of Fractions to their Lowest Terms.

138. We can pay a debt of 10s. by paying over either 1 half-sovereign, or 4 half-crowns, or 5 florins, or 10 shilling pieces, or 20 sixpenny pieces, or 40 threepenny pieces, and in many other ways too.

But	1 half-crown = $\frac{1}{2}$ of £1,
	\therefore 4 half-crowns = $\frac{4}{2}$ of £1.
Similarly	5 florins = $\frac{5}{10}$ of £1,
	10 shillings = $\frac{10}{20}$ of £1,
	20 sixpences = $\frac{20}{40}$ of £1.

Hence $\frac{1}{2}$ of £1, $\frac{5}{10}$ of £1, $\frac{10}{20}$ of £1, $\frac{20}{40}$ of £1, are each equal to $\frac{1}{2}$ of £1 ; and therefore the symbols $\frac{1}{2}$, $\frac{5}{10}$, $\frac{10}{20}$, $\frac{20}{40}$, are equivalent to one another. Thus the *same* fraction may be represented by many *different* symbols.

139. The matter may be considered in another way. Suppose we divide an orange into 12 equal parts, we may group the parts in 3 equal sets of 4, 4, and 4 ; and each set is $\frac{1}{3}$ of the whole orange. But each set contains 4 *twelfth*-parts, and its relation to the whole may therefore be represented by $\frac{4}{12}$.

Hence $\frac{4}{12}$, and similarly $\frac{5}{15}$, $\frac{6}{18}$, are each equivalent to $\frac{1}{3}$.

140. From these illustrations we see that the numerator and the denominator of a fraction may be divided, or multiplied, by the same number without altering the value of the fraction.

141. When there are no factors common to numerator and denominator, the fraction is said to be in its **lowest terms**.

142. When there are factors common to numerator and denominator, a fraction may be **reduced to its lowest terms** by dividing both numerator and denominator by their G.C.M.

Example 1. Reduce $\frac{56}{112}$ to its lowest terms.

$$\begin{aligned}\frac{56}{112} &= \frac{7 \times 8}{16 \times 8} \\ &= \frac{7}{16}.\end{aligned}$$

Example 2. Reduce $\frac{10812}{22800}$ to its lowest terms.

The G.C.M. of 10812 and 22800 is 12;

$$\begin{aligned}\therefore \frac{10812}{22800} &= \frac{10812 \div 12}{22800 \div 12} \\ &= \frac{901}{1900}.\end{aligned}$$

EXERCISE 44a.

Reduce each of the following fractions to its lowest terms:

- | | | | | |
|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| (1) $\frac{1}{2}$. | (2) $\frac{1}{4}$. | (3) $\frac{3}{8}$. | (4) $\frac{1}{8}$. | (5) $\frac{1}{16}$. |
| (6) $\frac{1}{32}$. | (7) $\frac{3}{16}$. | (8) $\frac{1}{8}$. | (9) $\frac{3}{16}$. | (10) $\frac{1}{32}$. |
| (11) $\frac{3}{16}$. | (12) $\frac{1}{8}$. | (13) $\frac{3}{16}$. | (14) $\frac{1}{8}$. | (15) $\frac{1}{16}$. |
| (16) $\frac{1}{32}$. | (17) $\frac{1}{16}$. | (18) $\frac{1}{16}$. | (19) $\frac{1}{16}$. | (20) $\frac{1}{16}$. |

EXERCISE 44b.

Reduce each of the following fractions to its lowest terms:—

- | | | | |
|-----------------------|-----------------------|-----------------------|-----------------------|
| (1) $\frac{1}{16}$. | (2) $\frac{1}{16}$. | (3) $\frac{1}{16}$. | (4) $\frac{1}{16}$. |
| (5) $\frac{1}{16}$. | (6) $\frac{1}{16}$. | (7) $\frac{1}{16}$. | (8) $\frac{1}{16}$. |
| (9) $\frac{1}{16}$. | (10) $\frac{1}{16}$. | (11) $\frac{1}{16}$. | (12) $\frac{1}{16}$. |
| (13) $\frac{1}{16}$. | (14) $\frac{1}{16}$. | (15) $\frac{1}{16}$. | (16) $\frac{1}{16}$. |
| (17) $\frac{1}{16}$. | (18) $\frac{1}{16}$. | (19) $\frac{1}{16}$. | (20) $\frac{1}{16}$. |
| (21) $\frac{1}{16}$. | (22) $\frac{1}{16}$. | (23) $\frac{1}{16}$. | (24) $\frac{1}{16}$. |
| (25) $\frac{1}{16}$. | (26) $\frac{1}{16}$. | (27) $\frac{1}{16}$. | (28) $\frac{1}{16}$. |

Comparison of Fractions.

143. To compare one fraction with another we first substitute for them equivalent fractions in which the equal parts have the *same name*, that is, we determine equivalent fractions with equal *denominators*.

For example, in the case of $\frac{2}{3}$ the equal parts are *sixths*, and in the case of $\frac{1}{4}$ they are *ninths*. But

$$\frac{2}{3} = \frac{2 \times 3}{3 \times 3} = \frac{2}{9}, \text{ and } \frac{1}{4} = \frac{1 \times 2}{4 \times 2} = \frac{1}{8}.$$

The parts are now *18ths* in both cases, and we see at once that the first fraction is the greater.

The common denominator may be *any* common multiple of the original denominators, but we ought to take the *least*.

EXERCISE 45.

Determine which is the greater in each of the following pairs of fractions. —

- | | | |
|--|--|--|
| (1) $\frac{1}{2}$ and $\frac{3}{4}$. | (2) $\frac{2}{3}$ and $\frac{3}{4}$. | (3) $\frac{3}{4}$ and $\frac{5}{8}$. |
| (4) $\frac{1}{3}$ and $\frac{1}{5}$. | (5) $\frac{2}{7}$ and $\frac{3}{8}$. | (6) $\frac{3}{10}$ and $\frac{1}{4}$. |
| (7) $\frac{3}{8}$ and $\frac{1}{17}$. | (8) $\frac{1}{6}$ and $\frac{1}{12}$. | (9) $\frac{1}{11}$ and $\frac{2}{33}$. |
| (10) $\frac{1}{14}$ and $\frac{2}{21}$. | (11) $\frac{1}{15}$ and $\frac{1}{17}$. | (12) $\frac{1}{12}$ and $\frac{1}{13}$. |
| (13) $\frac{1}{6}$ and $\frac{1}{8}$. | (14) $\frac{2}{9}$ and $\frac{3}{11}$. | (15) $\frac{3}{10}$ and $\frac{5}{11}$. |
| (16) $\frac{1}{8}$ and $\frac{1}{10}$. | (17) $\frac{2}{11}$ and $\frac{3}{13}$. | (18) $\frac{1}{11}$ and $\frac{2}{10}$. |
| (19) $\frac{1}{11}$ and $\frac{1}{13}$. | (20) $\frac{3}{14}$ and $\frac{2}{15}$. | (21) $\frac{1}{11}$ and $\frac{1}{13}$. |
| (22) $\frac{1}{14}$ and $\frac{1}{15}$. | (23) $\frac{1}{16}$ and $\frac{1}{17}$. | (24) $\frac{1}{17}$ and $\frac{1}{18}$. |

144. To compare and arrange in order of magnitude three or more fractions, we find the L.C.M. of the denominators, and replace each symbol by one which has for its denominator this L.C.M.

Example. Reduce $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{2}{7}$ to equivalent fractions having the least common denominator, and arrange them in ascending order of magnitude.

We find that the L.C.M. of the denominators = 105; and that

$$\frac{1}{3} = \frac{35}{105}, \quad \frac{1}{5} = \frac{21}{105}, \quad \frac{2}{7} = \frac{30}{105};$$

hence their ascending order of magnitude is

$$\frac{1}{5}, \quad \frac{1}{3}, \quad \frac{2}{7},$$

i.e. $\frac{1}{5}$, $\frac{1}{3}$, $\frac{2}{7}$.

EXERCISE 46.

Express by equivalent fractions having the least common denominator, and arrange in ascending order of magnitude:—

- | | | |
|--|--|--|
| (1) $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{1}{5}$. | (2) $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{1}{5}$. | (3) $\frac{3}{4}$, $\frac{1}{5}$ and $\frac{2}{3}$. |
| (4) $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$ and $\frac{1}{7}$. | (5) $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{5}$ and $\frac{1}{6}$. | (6) $\frac{1}{7}$, $\frac{2}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$. |
| (7) $\frac{1}{11}$, $\frac{1}{12}$, $\frac{2}{13}$ and $\frac{3}{14}$. | (8) $\frac{1}{15}$, $\frac{1}{16}$, $\frac{2}{17}$ and $\frac{3}{18}$. | (9) $\frac{1}{19}$, $\frac{2}{20}$, $\frac{3}{21}$ and $\frac{4}{22}$. |
| (10) $\frac{1}{21}$, $\frac{2}{22}$, $\frac{3}{23}$ and $\frac{4}{24}$. | (11) $\frac{1}{25}$, $\frac{2}{26}$, $\frac{3}{27}$ and $\frac{4}{28}$. | (12) $\frac{1}{29}$, $\frac{2}{30}$, $\frac{3}{31}$ and $\frac{4}{32}$. |

Express by equivalent fractions having the least common denominator, and arrange in descending order of magnitude:

- | | | |
|---|---|---|
| (13) $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. | (14) $\frac{1}{5}$, $\frac{1}{6}$ and $\frac{1}{7}$. | (15) $\frac{1}{8}$, $\frac{1}{9}$ and $\frac{1}{10}$. |
| (16) $\frac{1}{11}$, $\frac{1}{12}$ and $\frac{1}{13}$. | (17) $\frac{1}{14}$, $\frac{1}{15}$ and $\frac{1}{16}$. | (18) $\frac{1}{17}$, $\frac{1}{18}$ and $\frac{1}{19}$. |
| (19) $\frac{1}{20}$, $\frac{1}{21}$ and $\frac{1}{22}$. | (20) $\frac{1}{23}$, $\frac{1}{24}$ and $\frac{1}{25}$. | (21) $\frac{1}{26}$, $\frac{1}{27}$ and $\frac{1}{28}$. |
| (22) $\frac{1}{29}$, $\frac{1}{30}$ and $\frac{1}{31}$. | (23) $\frac{1}{32}$, $\frac{1}{33}$ and $\frac{1}{34}$. | (24) $\frac{1}{35}$, $\frac{1}{36}$ and $\frac{1}{37}$. |

Improper Fractions and Mixed Numbers.

145. When the numerator of a fraction is *less than* the denominator, the fraction is called a **proper** fraction; when the numerator is *greater than* the denominator, the fraction is called an **improper** fraction. For example, $\frac{1}{3}$ is a *proper* fraction; and $\frac{4}{3}$ is an *improper* fraction.

146. A number made up of a whole number and a proper fraction is called a **mixed number**. For example, $3\frac{1}{4}$, which means $3 + \frac{1}{4}$, is a mixed number.

147. In the following examples we will show how to express an *improper fraction* in the form of a *mixed number*.

Example 1. Express $\frac{25}{3}$ as a mixed number.

$$\begin{aligned}\frac{25}{3} &= 25 \text{ third-parts} \\ &= 24 \text{ third-parts} + 1 \text{ third-part} \\ &= 8 \text{ units} + 1 \text{ third-part} \\ &= 8\frac{1}{3}.\end{aligned}$$

Hence, if we divide the numerator of the improper fraction by the denominator, the quotient obtained will be the integral part of the mixed number; and the fractional part will have for its numerator the remainder after the division, and for its denominator the original denominator.

NOTE.—If there is *no remainder* after the division, the improper fraction represents a whole number.

Example 2. Express $\frac{9}{2}$ as a mixed number.

The numerator can be divided by the denominator without remainder, and the quotient is 5.

Hence $\frac{9}{2} = 5.$

EXERCISE 47.

Express each of the following improper fractions as a mixed number, or as a whole number:—

- | | | | | |
|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|
| (1) $\frac{11}{8}$. | (2) $\frac{42}{5}$. | (3) $\frac{57}{8}$. | (4) $\frac{83}{9}$. | (5) $\frac{72}{7}$. |
| (6) $\frac{23}{4}$. | (7) $\frac{108}{12}$. | (8) $\frac{125}{3}$. | (9) $\frac{120}{7}$. | (10) $\frac{120}{11}$. |
| (11) $\frac{130}{8}$. | (12) $\frac{7314}{7}$. | (13) $\frac{1003}{9}$. | (14) $\frac{5206}{11}$. | (15) $\frac{1405}{12}$. |
| (16) $\frac{540}{20}$. | (17) $\frac{563}{34}$. | (18) $\frac{641}{42}$. | (19) $\frac{700}{63}$. | (20) $\frac{257}{91}$. |
| (21) $\frac{1706}{103}$. | (22) $\frac{1942}{106}$. | (23) $\frac{4523}{241}$. | (24) $\frac{6084}{481}$. | (25) $\frac{5627}{373}$. |

148. To express a mixed number in the form of an improper fraction is the converse of the preceding.

Example. Express $3\frac{4}{5}$ as an improper fraction.

Each unit = 5 fifth-parts,

$\therefore 3 \text{ units} = 15 \dots\dots\dots,$

$\therefore 3 \text{ units and } 4 \text{ fifth-parts} = 19 \dots\dots\dots,$

$\therefore 3\frac{4}{5} = \frac{19}{5}.$

Hence, if we multiply the integral part of a mixed number by the denominator of the fractional part, and add to the product the numerator of the fractional part, the result will be the numerator of the improper fraction; and the original denominator will be its denominator.

EXERCISE 48.

Express each of the following mixed numbers as an improper fraction:—

- | | | | | |
|-----------------------|------------------------|------------------------|-----------------------|------------------------|
| (1) $6\frac{7}{8}.$ | (2) $5\frac{8}{9}.$ | (3) $4\frac{9}{10}.$ | (4) $51\frac{1}{2}.$ | (5) $10\frac{4}{5}.$ |
| (6) $12\frac{7}{10}.$ | (7) $15\frac{3}{4}.$ | (8) $17\frac{2}{3}.$ | (9) $19\frac{5}{8}.$ | (10) $21\frac{2}{3}.$ |
| (11) $32\frac{3}{8}.$ | (12) $43\frac{4}{5}.$ | (13) $54\frac{3}{10}.$ | (14) $65\frac{7}{8}.$ | (15) $78\frac{9}{11}.$ |
| (16) $12\frac{1}{2}.$ | (17) $23\frac{1}{4}.$ | (18) $13\frac{3}{4}.$ | (19) $14\frac{3}{4}.$ | (20) $45\frac{2}{3}.$ |
| (21) $14\frac{2}{5}.$ | (22) $13\frac{1}{11}.$ | (23) $36\frac{1}{8}.$ | (24) $67\frac{1}{8}.$ | (25) $71\frac{1}{4}.$ |

Express the whole numbers

- (26) 3, 5, 7, 9 and 11 as fractions having 7 for their denominators.
 (27) 4, 6, 8, 10 and 12 11
 (28) 5, 8, 11, 13 and 15 13
 (29) 3, 7, 9, 11 and 13 15
 (30) 4, 8, 10, 12 and 15 17

Addition of Fractions.

149. We will now consider how two or more fractions may be added together, and the result expressed by a *single* fraction.

Example. Add together $\frac{2}{3}$ and $\frac{2}{5}$.

We must first express them as parts having the *same name*, i.e., as fractions with the same denominator.

The L.C.M. of the denominators is 15, and we find that $\frac{2}{3} = \frac{10}{15}$, and $\frac{2}{5} = \frac{6}{15}$.

Hence the fractions to be added are equivalent to 10 *fifteenth*-parts and 6 *fifteenth*-parts, and the result of addition is clearly 16 *fifteenth*-parts.

The process may be stated briefly as follows:—

$$\frac{2}{3} + \frac{2}{5} = \frac{10}{15} + \frac{6}{15} = \frac{16}{15} = 1\frac{1}{15}.$$

In the same manner we add together three or more fractions.

EXERCISE 49.

Add together the following fractions:—

- | | | |
|--|--|--|
| (1) $\frac{1}{8} + \frac{2}{8} + \frac{3}{8}$. | (2) $\frac{2}{7} + \frac{2}{7} + \frac{6}{7}$. | (3) $\frac{6}{11} + \frac{7}{11} + \frac{3}{11}$. |
| (4) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$. | (5) $\frac{1}{2} + \frac{1}{8} + \frac{1}{12}$. | (6) $\frac{7}{10} + \frac{1}{12} + \frac{4}{15} + \frac{2}{3}$. |
| (7) $\frac{5}{12} + \frac{7}{6} + \frac{2}{3} + \frac{3}{10}$. | (8) $\frac{2}{3} + \frac{2}{4} + \frac{4}{5} + \frac{7}{12}$. | (9) $\frac{4}{5} + \frac{4}{5} + \frac{3}{10} + \frac{5}{6}$. |
| (10) $\frac{2}{3} + \frac{2}{3} + \frac{7}{10} + \frac{2}{4}$. | (11) $\frac{2}{5} + \frac{6}{5} + \frac{2}{5} + \frac{3}{5}$. | (12) $\frac{5}{18} + \frac{6}{7} + \frac{5}{36} + \frac{2}{21}$. |
| (13) $\frac{5}{11} + \frac{2}{15} + \frac{1}{33} + \frac{1}{60}$. | (14) $\frac{4}{7} + \frac{5}{18} + \frac{9}{14} + \frac{2}{34}$. | (15) $\frac{1}{24} + \frac{5}{72} + \frac{1}{63} + \frac{7}{24}$. |
| (16) $\frac{2}{7} + \frac{8}{21} + \frac{3}{42} + \frac{1}{2}$. | (17) $\frac{8}{11} + \frac{5}{6} + \frac{1}{2} + \frac{2}{22}$. | (18) $\frac{7}{18} + \frac{7}{30} + \frac{1}{6} + \frac{2}{3}$. |
| (19) $\frac{1}{8} + \frac{1}{24} + \frac{1}{6} + \frac{1}{20}$. | (20) $\frac{7}{13} + \frac{3}{39} + \frac{5}{12} + \frac{1}{62}$. | (21) $\frac{1}{7} + \frac{5}{12} + \frac{1}{3} + \frac{1}{42}$. |
| (22) $\frac{1}{12} + \frac{4}{27} + \frac{5}{36} + \frac{1}{28}$. | (23) $\frac{7}{11} + \frac{1}{18} + \frac{3}{22} + \frac{1}{36}$. | (24) $\frac{2}{11} + \frac{1}{33} + \frac{4}{15} + \frac{5}{22}$. |
| (25) $\frac{5}{17} + \frac{1}{4} + \frac{7}{8} + \frac{1}{61}$. | (26) $\frac{1}{27} + \frac{5}{18} + \frac{4}{81} + \frac{1}{36}$. | (27) $\frac{4}{7} + \frac{1}{10} + \frac{2}{28} + \frac{1}{18}$. |
| (28) $\frac{9}{18} + \frac{5}{11} + \frac{8}{33} + \frac{1}{4}$. | (29) $\frac{1}{11} + \frac{1}{66} + \frac{1}{13} + \frac{1}{33}$. | (30) $\frac{7}{12} + \frac{1}{36} + \frac{2}{18} + \frac{1}{60}$. |

150. If some of the fractions are *mixed numbers*, we add together the integral parts at once, and treat the fractional parts as in the last section.

Example. Add together $\frac{3}{8} + 2\frac{1}{8} + 3\frac{3}{4} + 1\frac{7}{8}$.

$$\begin{aligned}
 \frac{3}{8} + 2\frac{1}{8} + 3\frac{3}{4} + 1\frac{7}{8} &= 6 + \frac{72+20+90+108}{120} \\
 &= 6 + \frac{287}{120} \\
 &= 6 + 2\frac{47}{60} \\
 &= 8\frac{47}{60}.
 \end{aligned}$$

EXERCISE 50.

Find the value of each of the following expressions:—

- | | | |
|--|--|--|
| (1) $\frac{4}{5} + 7\frac{2}{5} + 6 + 4\frac{4}{5}$. | (2) $7 + \frac{1}{3} + 3\frac{1}{2} + 6\frac{2}{3}$. | (3) $15\frac{2}{3} + 8 + 12\frac{7}{8} + 11\frac{2}{3}$. |
| (4) $6\frac{1}{5} + \frac{7}{5} + 2\frac{2}{3} + 1\frac{1}{4}$. | (5) $3\frac{7}{8} + 4 + 7\frac{1}{2} + 6\frac{1}{8}$. | (6) $5\frac{1}{8} + 3\frac{1}{2} + 4\frac{1}{8} + 1\frac{1}{4}$. |
| (7) $\frac{5}{8} + 4\frac{7}{10} + 2\frac{1}{5} + 10\frac{2}{5}$. | (8) $\frac{3}{4} + 2\frac{1}{2} + \frac{3}{8} + 8\frac{1}{8}$. | (9) $4\frac{2}{15} + 1\frac{7}{20} + \frac{1}{12} + 2\frac{5}{15}$. |
| (10) $\frac{3}{4} + 3\frac{5}{8} + \frac{9}{14} + 1\frac{1}{24}$. | (11) $7\frac{2}{10} + 12\frac{1}{15} + 4\frac{7}{12} + \frac{9}{40}$. | |
| (12) $1\frac{1}{4} + 2\frac{5}{12} + \frac{4}{15} + \frac{7}{6}$. | (13) $9\frac{7}{8} + 4\frac{8}{15} + \frac{1}{6} + 3\frac{1}{30}$. | |
| (14) $11\frac{1}{8} + 16\frac{3}{11} + 19\frac{4}{33} + 7\frac{1}{16}$. | (15) $11\frac{3}{19} + 14\frac{5}{12} + 17\frac{1}{6} + 19\frac{5}{6}$. ✓ | |
| (16) $2\frac{1}{5} + 3\frac{1}{15} + 1\frac{1}{18} + \frac{8}{45}$. | (17) $8\frac{1}{7} + \frac{9}{14} + 4\frac{5}{21} + 2\frac{1}{7}$. | |
| (18) $8\frac{5}{8} + \frac{7}{11} + \frac{1}{22} + 5\frac{4}{66}$. | (19) $3\frac{5}{24} + 14\frac{3}{82} + 25\frac{1}{16} + 18\frac{3}{16}$. | |
| (20) $4\frac{5}{8} + 11\frac{9}{88} + 2\frac{5}{84} + \frac{1}{36}$. | (21) $16\frac{1}{7} + 2\frac{5}{8} + \frac{1}{8} + \frac{1}{36}$. | |
| (22) $4\frac{5}{11} + \frac{1}{12} + 15\frac{1}{32} + 9\frac{1}{16}$. | (23) $5\frac{1}{36} + \frac{7}{6} + 11\frac{9}{40} + 16\frac{1}{12}$. | |
| (24) $9\frac{1}{36} + 4\frac{2}{14} + 8\frac{4}{45} + 5\frac{2}{36}$. | (25) $9\frac{7}{8} + 4\frac{8}{15} + \frac{1}{6} + 3\frac{1}{30}$. | |
| (26) $7\frac{3}{8} + 1\frac{1}{4} + \frac{3}{8} + 13\frac{1}{8}$. | (27) $11\frac{3}{9} + 4\frac{5}{12} + 7\frac{7}{8} + 9\frac{5}{8}$. | |
| (28) $5\frac{3}{8} + 7\frac{1}{12} + 4\frac{1}{6} + 11\frac{7}{12}$. | (29) $10\frac{2}{12} + 2\frac{1}{3} + \frac{5}{6} + 3\frac{2}{11}$. | |
| (30) $7\frac{1}{12} + \frac{1}{12} + 5\frac{4}{6} + 11\frac{1}{12}$. | (31) $4\frac{1}{12} + 7\frac{2}{12}$. | |

- (32) $4\frac{1}{2} + 7\frac{1}{2}$. (33) $10\frac{1}{2} + 1\frac{1}{2}$. (34) $11\frac{1}{2} + 7\frac{1}{2}$. (35) $5\frac{1}{2} + 6\frac{1}{2}$.
 (36) $7\frac{1}{2} + 4\frac{1}{2}$. (37) $11\frac{1}{2} + 13\frac{1}{2}$. (38) $5\frac{1}{2} + 7\frac{1}{2}$. (39) $10\frac{1}{2} + 11\frac{1}{2}$.
 (40) $5\frac{1}{2} + 8\frac{1}{2}$. (41) $15\frac{1}{2} + 18\frac{1}{2}$. (42) $11\frac{1}{2} + 10\frac{1}{2}$. (43) $4\frac{1}{2} + 7\frac{1}{2}$.
 (44) $7\frac{1}{2} + 11\frac{1}{2}$. (45) $4\frac{1}{2} + 8\frac{1}{2}$. (46) $3\frac{1}{2} + 5\frac{1}{2}$. (47) $7\frac{1}{2} + 8\frac{1}{2}$.
 (48) $10\frac{1}{2} + 6\frac{1}{2}$. (49) $8\frac{1}{2} + 11\frac{1}{2}$. (50) $17\frac{1}{2} + 18\frac{1}{2}$.

Subtraction of Fractions.

151. As in addition, we express the given fractions as fractions having the same denominator; their difference will then be found by subtracting the smaller numerator from the greater.

Example. Find the difference between $\frac{2}{3}$ and $\frac{1}{5}$.

$$\frac{2}{3} - \frac{1}{5} = \frac{10}{15} - \frac{3}{15} = \frac{7}{15}.$$

152. If the fractions are *mixed numbers*, we subtract the whole numbers and the fractional parts separately.

Example. Find the difference between $16\frac{1}{2}$ and $12\frac{1}{2}$.

$$16\frac{1}{2} - 12\frac{1}{2} = 16 - 12 + \frac{1}{2} - \frac{1}{2} = 4 + \frac{0}{2} = 4.$$

EXERCISE 51.

Find the value of each of the following expressions:—

- (1) $\frac{2}{3} - \frac{1}{5}$. (2) $\frac{3}{4} - \frac{1}{6}$. (3) $\frac{5}{8} - \frac{2}{3}$. (4) $\frac{7}{9} - \frac{4}{5}$.
 (5) $1\frac{1}{2} - \frac{3}{4}$. (6) $\frac{1}{2} - \frac{1}{10}$. (7) $\frac{1}{3} - \frac{1}{15}$. (8) $\frac{2}{3} - \frac{1}{15}$.
 (9) $\frac{1}{2} - \frac{1}{10}$. (10) $2\frac{1}{2} - 1\frac{1}{2}$. (11) $5\frac{1}{2} - 2\frac{1}{2}$. (12) $3\frac{1}{2} - 2\frac{1}{2}$.
 (13) $5\frac{1}{2} - 3\frac{1}{2}$. (14) $13\frac{1}{2} - 3\frac{1}{2}$. (15) $5\frac{1}{2} - \frac{1}{10}$. (16) $8\frac{1}{2} - 3\frac{1}{2}$.

153. If the fractional part in the larger mixed number is less than in the smaller, we proceed as in the following example.

Example. Find the difference between $4\frac{1}{2}$ and $2\frac{3}{4}$.

$$4\frac{1}{2} - 2\frac{3}{4} = 2 + \frac{1}{2} - \frac{3}{4} = 2 - \frac{1}{4} = 1\frac{3}{4}.$$

EXERCISE 52.

Find the value of each of the following expressions:—

- (1) $15\frac{1}{2} - 3\frac{1}{2}$. (2) $16\frac{1}{2} - 7\frac{1}{2}$. (3) $12\frac{1}{2} - 11\frac{1}{2}$. (4) $10\frac{1}{2} - 8\frac{1}{2}$.
 (5) $14\frac{1}{2} - 9\frac{1}{2}$. (6) $8\frac{1}{2} - 6\frac{1}{2}$. (7) $15\frac{1}{2} - 11\frac{1}{2}$. (8) $12\frac{1}{2} - 7\frac{1}{2}$.
 (9) $19\frac{1}{2} - 6\frac{1}{2}$. (10) $13\frac{1}{2} - 3\frac{1}{2}$. (11) $12\frac{1}{2} - 7\frac{1}{2}$. (12) $15\frac{1}{2} - 7\frac{1}{2}$.
 (13) $16\frac{1}{2} - 6\frac{1}{2}$. (14) $18\frac{1}{2} - 6\frac{1}{2}$. (15) $23\frac{1}{2} - 10\frac{1}{2}$. (16) $20\frac{1}{2} - 13\frac{1}{2}$.
 (17) $4\frac{1}{2} - 3\frac{1}{2}$. (18) $5\frac{1}{2} - 2\frac{1}{2}$. (19) $7\frac{1}{2} - 6\frac{1}{2}$. (20) $10\frac{1}{2} - 5\frac{1}{2}$.
 (21) $23\frac{1}{2} - 14\frac{1}{2}$. (22) $36\frac{1}{2} - 25\frac{1}{2}$. (23) $19\frac{1}{2} - 15\frac{1}{2}$.
 (24) $7\frac{1}{2} - 3\frac{1}{2}$. (25) $50\frac{1}{2} - 32\frac{1}{2}$. (26) $37\frac{1}{2} - 31\frac{1}{2}$.
 (27) $11\frac{1}{2} - 6\frac{1}{2}$. (28) $30\frac{1}{2} - 18\frac{1}{2}$. (29) $31\frac{1}{2} - 17\frac{1}{2}$.
 (30) $62\frac{1}{2} - 45\frac{1}{2}$. (31) $22\frac{1}{2} - 14\frac{1}{2}$. (32) $10\frac{1}{2} - 3\frac{1}{2}$.
 (33) $24\frac{1}{2} - 17\frac{1}{2}$. (34) $65\frac{1}{2} - 19\frac{1}{2}$. (35) $11\frac{1}{2} - 7\frac{1}{2}$.
 (36) $36\frac{1}{2} - 19\frac{1}{2}$. (37) $11\frac{1}{2} - 9\frac{1}{2}$. (38) $20\frac{1}{2} - 17\frac{1}{2}$.
 (39) $101\frac{1}{2} - 83\frac{1}{2}$. (40) $58\frac{1}{2} - 43\frac{1}{2}$.

154. We will now consider some examples which involve both addition and subtraction.

Example 1. Express $\frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \frac{1}{5} - \frac{2}{8}$ by a single fraction.

We see, by inspection, that the L.C.M. of the denominators is 24,

Hence $\frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \frac{1}{5} - \frac{2}{8} = \frac{12+16-18+4-6}{24}.$

The sum of the terms in the numerator which are preceded by + is 32, and the sum of those preceded by - is 27;

\therefore the given expression $= \frac{32-27}{24} = \frac{5}{24}.$

Example 2. Simplify

$$2\frac{1}{2} + 3\frac{1}{3} - 4\frac{1}{4} - 1\frac{2}{5} + 6\frac{3}{6}.$$

$$\begin{aligned}\text{Expression} &= 6 + \frac{15+10-16-20+24}{30} \\ &= 6 + \frac{13}{30} \\ &= 6\frac{13}{30} \\ &= 6\frac{1}{2}.\end{aligned}$$

Example 3. Simplify

$$5\frac{1}{4} - 1\frac{1}{2} - 1\frac{7}{8} - \frac{5}{8}.$$

$$\begin{aligned}\text{Expression} &= 4 + \frac{5-4-2-1-5}{8} \\ &= 4 + \frac{-7}{8} \\ &= 4 - \frac{7}{8} \\ &= 4 - 2\frac{7}{8} \\ &= 1\frac{1}{8}.\end{aligned}$$

EXERCISE 53.

Find the value of each of the following expressions:—

- (1) $6\frac{1}{2} - 2\frac{3}{4} + \frac{5}{8} - 1\frac{7}{8}.$ (2) $3\frac{2}{3} - 1\frac{3}{10} + 7\frac{1}{2} - 2\frac{5}{6}.$ (3) $10\frac{2}{5} - 6\frac{7}{10} - 4\frac{3}{5} + 3\frac{1}{5}.$
- (4) $8\frac{7}{8} - 4\frac{1}{2} + 7\frac{1}{8} - 5\frac{3}{4}.$ (5) $8\frac{2}{3} - 2\frac{5}{6} - \frac{7}{12} - 3\frac{1}{6}.$ (6) $9\frac{1}{12} - \frac{7}{6} - 2\frac{5}{6} - \frac{1}{2}.$
- (7) $3\frac{3}{18} - \frac{6}{18} - 1\frac{2}{18} + \frac{5}{18}.$ (8) $4 - 2\frac{5}{8} + 18\frac{3}{8} - 19.$ (9) $3\frac{1}{8} - 2\frac{4}{8} - 4\frac{1}{20} + 43\frac{1}{20}.$
- (10) $3\frac{1}{2} - 1\frac{7}{8} + 4\frac{5}{8} - 2\frac{1}{10}.$ (11) $9\frac{7}{8} + 16\frac{1}{8} - 10\frac{1}{2} - 1\frac{1}{8}.$
- (12) $3\frac{2}{3} + 2\frac{1}{2} - 6\frac{1}{6} + 5\frac{1}{12} - 4\frac{3}{12}.$ (13) $2\frac{3}{4} + 1\frac{5}{8} - 3\frac{5}{8} + 2\frac{3}{10} + 1\frac{3}{10}.$
- (14) $7\frac{2}{3} + 6\frac{2}{3} - 3\frac{4}{6} - 2\frac{1}{6} + \frac{1}{2}.$ (15) $7\frac{2}{3} - \frac{5}{6} - 3\frac{5}{6} + 3\frac{7}{10} - 1\frac{1}{10}.$
- (16) $\frac{1}{6} + 2\frac{1}{3} + 13\frac{2}{10} + \frac{2}{5} - 4\frac{3}{5}.$ (17) $3\frac{1}{12} + 5\frac{7}{12} - 2\frac{2}{12} - 4\frac{2}{10} + 3\frac{1}{10}.$
- (18) $3\frac{1}{8} - 4\frac{3}{8} + 5\frac{1}{8} - 7\frac{5}{8} + 8\frac{7}{8}.$ (19) $8\frac{2}{3} - 7\frac{2}{3} + 5\frac{2}{3} - 4\frac{1}{3} + 2\frac{1}{3}.$
- (20) $47\frac{5}{12} - 3\frac{2}{3} - 31\frac{5}{12} - 2\frac{5}{6} + \frac{4}{12}.$ (21) $8\frac{5}{6} - 5\frac{4}{6} + 7\frac{1}{6} - \frac{1}{6} + 1\frac{2}{6}.$
- (22) $13 - 11\frac{1}{10} + 10\frac{6}{10} - 9\frac{7}{10} + 1\frac{3}{10}.$ (23) $\frac{1}{2} - \frac{1}{6} + \frac{2}{3} - \frac{2}{4} + \frac{1}{10} - \frac{1}{10}.$
- (24) $\frac{2}{4} - \frac{1}{6} + \frac{2}{10} - \frac{1}{6} + \frac{5}{6} - \frac{2}{6} + \frac{1}{10}.$ (25) $\frac{1}{2} + \frac{2}{3} + \frac{2}{3} + \frac{7}{6} + \frac{5}{6} - \frac{2}{3} - \frac{1}{6} - \frac{5}{12}.$

Multiplication by a Whole Number.

155. Just as 4 shillings $\times 3 = 12$ shillings,
so also 4 fifth-parts $\times 3 = 12$ fifth-parts;
that is, $\frac{4}{5} \times 3 = 1\frac{2}{5}.$

Again,

$$\frac{4}{15} \times 3 = \frac{3 \times 4}{15} = \frac{3 \times 4}{3 \times 5} = \frac{4}{5}.$$

Hence, to multiply a fraction by a whole number, we multiply the numerator, or (when possible) divide the denominator by the number.

156. The resulting fraction must always be expressed in its lowest terms, and an improper fraction must be expressed as a mixed number.

Example. Multiply $\frac{4}{15}$ by 6.

$$\frac{4}{15} \times 6 = \frac{4 \times 6}{15} = \frac{4 \times 2 \times 3}{15} = \frac{4 \times 2}{5} = \frac{8}{5} = 1\frac{3}{5}.$$

Hence, if the multiplier consists of factors, some of which are also factors of the denominator and others not, the result may be obtained at once by removing common factors from the multiplier and the denominator, and multiplying the numerator by the factors of the multiplier that remain.

This process is called **cancelling** common factors.

157. To multiply a mixed number, we multiply the whole number and the fractional part separately, and add the products.

Example. Multiply $2\frac{1}{2}$ by 3.

$$2\frac{1}{2} \times 3 = 6 + \frac{3}{2} = 6 + 1\frac{1}{2} = 7\frac{1}{2}.$$

EXERCISE 54.

Multiply

- | | | |
|-----------------------------------|-----------------------------------|------------------------------------|
| (1) $\frac{1}{11}$ by 2 and 3. | (2) $\frac{1}{8}$ by 5 and 9. | (3) $\frac{1}{2}$ by 11 and 5. |
| (4) $\frac{1}{11}$ by 9 and 12. | (5) $\frac{2}{3}$ by 3 and 11. | (6) $\frac{2}{9}$ by 7 and 14. |
| (7) $\frac{1}{2}$ by 6 and 12. | (8) $\frac{1}{11}$ by 9 and 21. | (9) $\frac{6}{11}$ by 48 and 49. |
| (10) $2\frac{1}{2}$ by 4 and 5. | (11) $4\frac{1}{2}$ by 7 and 11. | (12) $6\frac{1}{2}$ by 8 and 12. |
| (13) $7\frac{1}{2}$ by 4 and 15. | (14) $8\frac{1}{4}$ by 13 and 19. | (15) $3\frac{1}{2}$ by 2 and 11. |
| (16) $10\frac{1}{4}$ by 2 and 17. | (17) $5\frac{1}{2}$ by 7 and 8. | (18) $2\frac{3}{4}$ by 39 and 117. |

Division by a Whole Number.

158. Just as $12 \text{ shillings} \div 3 = 4 \text{ shillings}$,
so also $12 \text{ fifth-parts} \div 3 = 4 \text{ fifth-parts}$;
that is, $\frac{12}{5} \div 3 = \frac{4}{5}$.

Again,

$$\frac{12}{11} \div 2 = 6 \text{ twenty-eighth parts} \div 2 \\ = 3 \text{ twenty-eighth parts};$$

that is,

$$\frac{12}{11} \div 2 = \frac{3}{28}.$$

If we divide a quantity into 14 equal parts, each part is $\frac{1}{14}$ of the whole quantity. And if we divide each of these *fourteenth-parts* again into 2 equal parts, we get 28 equal parts, each of which is $\frac{1}{28}$ of the whole quantity; therefore, if each *fourteenth-part* in $\frac{3}{14}$ of the whole quantity be divided into 2 equal parts, we get 6 *twenty-eighth parts* of the whole; and if we divide this by 2, we get 3 *twenty-eighth parts*.

Hence, to divide a fraction by a whole number, we multiply the denominator by the number, or (when possible) divide the numerator by the number.

Example. Divide $2\frac{1}{6}$ by 7.

$$2\frac{1}{6} \div 7 = \frac{13}{6} \div 7 = \frac{13}{6 \times 7} = \frac{13}{42}.$$

Factors common to numerator and denominator should be removed first.

EXERCISE 55.

Divide

- | | | |
|------------------------------------|-------------------------------------|--------------------------------------|
| (1) $\frac{2}{3}$ by 4, 5, 7, 10. | (2) $\frac{2}{3}$ by 3, 5, 7, 9. | (3) $1\frac{1}{2}$ by 2, 3, 4, 6. |
| (4) $1\frac{1}{8}$ by 2, 4, 8, 16. | (5) $1\frac{1}{2}$ by 3, 5, 30, 45. | (6) $\frac{3}{10}$ by 6, 12, 24, 36. |
| (7) $2\frac{1}{2}$ by 2, 6, 14. | (8) $3\frac{3}{4}$ by 4, 9, 11. | (9) $16\frac{2}{3}$ by 5, 7, 15. |
| (10) $3\frac{2}{3}$ by 17, 34, 85. | (11) $4\frac{2}{3}$ by 19, 38, 95. | (12) $3\frac{1}{2}$ by 5, 11, 46. |

Multiplication by a Fraction.

159. To find $\frac{2}{3}$ of a quantity, the quantity must be divided into 3 equal parts, and 2 of these parts be taken. Hence, to find $\frac{2}{3}$ of $\frac{5}{7}$, we first divide $\frac{5}{7}$ by 3, and then multiply the result by 2.

Thus
$$\frac{2}{3} = 2 \times \frac{1}{3},$$

$$\therefore \frac{2}{3} \text{ of } \frac{5}{7} = 2 \times \frac{1}{3} \text{ of } \frac{5}{7} = 2 \times \frac{5}{3 \times 7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21};$$

and similarly
$$\frac{3}{5} \text{ of } \frac{4}{11} = \frac{3 \times 4}{5 \times 11} = \frac{12}{55}.$$

Thus, to multiply one fraction by another, we multiply the numerators together to obtain the numerator of the product, and the denominators to obtain the denominator of the product.

A fraction of a fraction is called a **compound fraction**.

160. If there are any factors common to either numerator and either denominator, they should be removed before multiplication.

Thus
$$\frac{8}{15} \times \frac{5}{12} = \frac{4 \times 2}{3 \times 5} \times \frac{5}{4 \times 3} = \frac{2 \times 1}{3 \times 3} = \frac{2}{9}.$$

161. It follows from Art. 159, that such an expression as

$$\frac{1}{2} \text{ of } \frac{3}{5} \text{ of } \frac{7}{11} = \frac{1}{2} \text{ of } \frac{3 \times 7}{5 \times 11}, \text{ and also that the latter} = \frac{1 \times 3 \times 7}{2 \times 5 \times 11}.$$

Thus the product of any number of fractions is a fraction which has for its numerator the product of the original numerators, and for its denominator the product of the original denominators.

If there should be any *mixed numbers* among the fractions to be multiplied, we substitute for them their equivalent improper fractions.

Example. Simplify $3\frac{1}{2} \times 5\frac{2}{3} \times 3\frac{2}{5} \times \frac{3}{17} \times 2$.

$$\text{The product} = \frac{2}{16} \times \frac{5}{3} \times \frac{32}{9} \times \frac{3}{22} \times \frac{2}{1} = \frac{192}{11} = 17\frac{5}{11}.$$

EXERCISE 56.

Multiply

- | | |
|---|---|
| (1) $\frac{1}{2}$ by $\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{10}$. | (2) $\frac{2}{3}$ by $\frac{3}{4}$, $\frac{3}{5}$, $\frac{3}{10}$. |
| (3) $\frac{1}{3}$ by $\frac{2}{3}$, $\frac{2}{5}$, $\frac{2}{15}$. | (4) $5\frac{2}{3}$ by $\frac{1}{11}$, $\frac{1}{10}$, $\frac{1}{11}$. |
| (5) $9\frac{7}{8}$ by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$. | (6) $3\frac{2}{3}$ by $2\frac{1}{2}$, $5\frac{1}{2}$, $7\frac{2}{3}$. |
| (7) $7\frac{1}{2}$ by $1\frac{1}{2}$, $4\frac{1}{2}$, $3\frac{1}{2}$. | (8) $5\frac{2}{3}$ by $3\frac{2}{3}$, $2\frac{1}{10}$, $5\frac{1}{2}$. |
| (9) $8\frac{1}{2}$ by $6\frac{1}{11}$, $3\frac{1}{2}$, $2\frac{1}{2}$. | (10) $1\frac{2}{3}$ by $2\frac{1}{10}$, $17\frac{1}{2}$, $6\frac{1}{2}$. |

Find the value of

- | | | |
|---|---|---|
| (11) $3\frac{1}{2} \times 5\frac{7}{8}$. | (12) $7\frac{7}{8} \times 1\frac{1}{2}$. | (13) $8\frac{1}{10} \times 7\frac{1}{11}$. |
| (14) $3\frac{2}{3} \times 7\frac{1}{2}$. | (15) $4\frac{1}{7} \times 1\frac{1}{2}$. | (16) $8\frac{1}{2} \times 7\frac{1}{2}$. |
| (17) $6\frac{1}{2} \times 2\frac{2}{3}$. | (18) $8\frac{1}{2} \times 1\frac{1}{2}$. | (19) $10\frac{1}{2} \times 3\frac{1}{2}$. |
| (20) $10\frac{1}{2} \times 4\frac{1}{2}$. | (21) $\frac{1}{11} \times 2\frac{1}{2}$. | (22) $3\frac{1}{2} \times 2\frac{1}{2}$. |
| (23) $5\frac{1}{2} \times 3\frac{1}{2}$. | (24) $10\frac{1}{2} \times 2\frac{2}{3}$. | (25) $7\frac{1}{2} \times 13\frac{1}{2}$. |
| (26) $3\frac{1}{2} \times 6\frac{1}{2}$. | (27) $6\frac{1}{2} \times 2\frac{1}{2}$. | (28) $13\frac{1}{2} \times 14\frac{1}{2}$. |
| (29) $2\frac{1}{2} \times 3\frac{1}{2}$. | (30) $1\frac{2}{3} \times 2\frac{2}{3}$. | (31) $6\frac{1}{2} \times 4\frac{1}{2}$. |
| (32) $4\frac{1}{2} \times 3\frac{1}{2}$. | (33) $6\frac{1}{2} \times 2\frac{1}{2}$. | (34) $8\frac{1}{2} \times 9\frac{1}{2}$. |
| (35) $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$. | (36) $\frac{2}{3}$ of $\frac{2}{3}$ of $9\frac{2}{3}$. | |
| (37) $2\frac{1}{2} \times \frac{1}{2} \times 3\frac{1}{2}$. | (38) $\frac{2}{3} \times 14 \times \frac{2}{3} \times 3\frac{1}{2}$. | |
| (39) $1\frac{1}{2} \times 1\frac{1}{2} \times 1\frac{1}{2} \times 1\frac{1}{2} \times 1\frac{1}{2}$. | (40) $2\frac{1}{2} \times 3\frac{1}{2} \times 1\frac{1}{2} \times \frac{2}{3}$ of 10. | |
| (41) $\frac{2}{3}$ of $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$ of 18. | (42) $\frac{2}{3}$ of $8\frac{1}{2} \times \frac{2}{3} \times 9\frac{2}{3}$. | |
| (43) $5\frac{7}{8}$ of $\frac{1}{2} \times 2\frac{1}{2}$ of $3\frac{1}{2}$. | (44) $3\frac{1}{2}$ of $3\frac{1}{2} \times \frac{1}{2} \times 1\frac{1}{2}$. | |
| (45) $5\frac{1}{2} \times 5\frac{2}{3} \times 2\frac{1}{2} \times 1\frac{1}{2}$. | (46) $\frac{1}{2} \times 2\frac{1}{2} \times 7\frac{1}{2} \times 3\frac{1}{2}$. | |
| (47) $10\frac{1}{2} \times 1\frac{1}{2} \times \frac{2}{3} \times 2\frac{1}{2}$. | (48) $4\frac{1}{2} \times 7\frac{1}{2} \times \frac{1}{2} \times 2\frac{1}{2}$. | |
| (49) $4\frac{1}{2} \times 1\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{2}$. | (50) $1\frac{1}{2} \times 1\frac{1}{2} \times 2\frac{1}{2} \times 1\frac{1}{2}$. | |
| (51) $2\frac{1}{2} \times 6\frac{1}{2} \times 5\frac{1}{2} \times 3\frac{1}{2}$. | (52) $7\frac{1}{2} \times 8\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{2}$. | |
| (53) $3\frac{1}{2} \times 2\frac{1}{2} \times 1\frac{1}{2} \times 1\frac{1}{2}$. | (54) $6\frac{1}{2} \times \frac{1}{2} \times 3\frac{1}{2} \times \frac{1}{2}$. | |
| (55) $3\frac{1}{2} \times 1\frac{1}{2} \times 1\frac{1}{2} \times \frac{1}{2}$. | (56) $\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{2} \times 1\frac{1}{2}$. | |
| (57) $4\frac{1}{2} \times 3\frac{1}{2} \times 2\frac{1}{2} \times 1\frac{1}{2}$. | (58) $1\frac{1}{2} \times 2\frac{1}{2} \times 1\frac{1}{2} \times 2\frac{1}{2}$. | |
| (59) $4\frac{1}{2} \times \frac{1}{2} \times 2\frac{1}{2} \times 1\frac{1}{2}$. | (60) $2\frac{1}{2} \times 1\frac{1}{2} \times 2\frac{1}{2} \times 1\frac{1}{2}$. | |

Division by a Fraction.

162. It is essential to the common definition of division that the divisor should be a whole number, but if we extend the definition and consider *dividend*, *divisor*, and *quotient*, merely as quantities, such that

$$\text{Dividend} = \text{Divisor} \times \text{Quotient},$$

we may allow the divisor to be a fraction or a mixed number, and we may consider the quotient as merely answering the question—*“By what must the divisor be multiplied in order that the result may be the dividend?”*

Thus the quotient of 7 divided by 8 is $\frac{7}{8}$; that of $3\frac{1}{2}$ divided by $\frac{1}{2}$ is 7; that of 7 by $3\frac{1}{2}$ is 2 ; and that of 28 by 3 is $9\frac{1}{3}$.

We may thus consider a fraction to represent such a quantity, that *when it is multiplied by the denominator the result is the numerator*.

163. Again, to divide $\frac{9}{16}$ by $\frac{5}{7}$, we have to find a quantity such that if we multiply it by $\frac{5}{7}$, the product will be $\frac{9}{16}$;

$$\begin{aligned}\therefore \frac{5}{7} \times \text{the quantity} &= \frac{9}{16}; \\ \therefore 7 \times \frac{5}{7} \times \text{the quantity} &= 7 \times \frac{9}{16}; \\ \therefore 5 \times \text{the quantity} &= 7 \times \frac{9}{16}; \\ \therefore \text{the quantity} &= \frac{7}{5} \times \frac{9}{16} = \frac{63}{80};\end{aligned}$$

Hence, *to divide one fraction by another, we invert the divisor and multiply.*

Example 1. Divide $\frac{4}{9}$ by $1\frac{1}{12}$.

$$\frac{4}{9} \div \frac{16}{12} = \frac{4}{9} \times \frac{3}{4} = \frac{7}{12}.$$

We cancel factors common to numerators and denominators *after* the divisor has been inverted.

Example 2. Divide $3\frac{5}{12}$ by $4\frac{1}{12}$.

$$3\frac{5}{12} \div 4\frac{1}{12} = \frac{41}{12} \times \frac{12}{28} = \frac{7}{9}.$$

Mixed numbers must be first expressed as improper fractions.

EXERCISE 57.

Divide

(1) $\frac{9}{16}$ by $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{7}{8}$.

(3) $\frac{8}{15}$ by $\frac{9}{10}$, $\frac{8}{9}$, $\frac{1}{15}$, $\frac{9}{15}$.

(5) 50 by $\frac{1}{15}$, $\frac{5}{12}$, $\frac{1}{17}$, $1\frac{1}{11}$.

(7) $\frac{3}{8}$ by $5\frac{1}{2}$, $2\frac{1}{8}$, $4\frac{1}{8}$, $6\frac{1}{8}$.

(2) $1\frac{1}{2}$ by $1\frac{2}{3}$, $\frac{2}{14}$, $\frac{7}{8}$, $\frac{5}{8}$.

(4) $3\frac{1}{6}$ by $1\frac{1}{6}$, $\frac{1}{15}$, $\frac{1}{14}$, $\frac{9}{15}$.

(6) 55 by $1\frac{1}{2}$, $1\frac{5}{12}$, $2\frac{5}{12}$, $\frac{3}{8}$.

(8) $4\frac{3}{4}$ by $5\frac{1}{2}$, $3\frac{1}{8}$, $9\frac{1}{4}$, $4\frac{1}{4}$.

Perform the operation indicated:—

(9) $31\frac{1}{7} \div 6\frac{4}{7}$. (10) $11\frac{1}{8} \div 2\frac{5}{8}$. (11) $8\frac{1}{10} \div 2\frac{1}{10}$. (12) $18\frac{1}{11} \div 4\frac{5}{11}$.

(13) $31\frac{1}{8} \div 2\frac{3}{8}$. (14) $1\frac{2}{11} \div 5\frac{7}{11}$. (15) $\frac{3}{11} \div 6\frac{2}{11}$. (16) $5\frac{7}{11} \div 1\frac{3}{11}$.

(17) $33\frac{1}{11} \div 24\frac{7}{11}$. (18) $10\frac{2}{11} \div 3\frac{1}{11}$. (19) $13\frac{1}{11} \div 18\frac{1}{11}$. (20) $24\frac{1}{11} \div 18\frac{1}{11}$.

- (21) $10\frac{1}{2} \div 3\frac{1}{2}$. (22) $15\frac{1}{2} \div 3\frac{1}{2}$. (23) $7\frac{1}{2} \div 5\frac{1}{2}$. (24) $1\frac{1}{2} \div 2\frac{1}{2}$.
 (25) $5\frac{1}{2} \div 1\frac{1}{2}$. (26) $2\frac{1}{2} \div 3\frac{1}{2}$. (27) $2\frac{1}{2} \div 15\frac{1}{2}$. (28) $3\frac{1}{2} \div 6\frac{1}{2}$.
 (29) $9\frac{1}{2} \div 3\frac{1}{2}$. (30) $10\frac{1}{2} \div 6\frac{1}{2}$. (31) $4\frac{1}{2} \div 2\frac{1}{2}$. (32) $17\frac{1}{2} \div 2\frac{1}{2}$.
 (33) $3\frac{1}{2} \div 2\frac{1}{2}$. (34) $2\frac{1}{2} \div 3\frac{1}{2}$. (35) $6\frac{1}{2} \div 4\frac{1}{2}$. (36) $\frac{1}{2} \div 5\frac{1}{2}$.
 (37) $2\frac{1}{2} \div 1\frac{1}{2}$. (38) $44\frac{1}{2} \div 6\frac{1}{2}$. (39) $3\frac{1}{2} \div 2\frac{1}{2}$. (40) $4\frac{1}{2} \div 2\frac{1}{2}$.
 (41) $\frac{1}{2} \div 1\frac{1}{2}$. (42) $1\frac{1}{2} \div 1\frac{1}{2}$. (43) $1\frac{1}{2} \div 4\frac{1}{2}$. (44) $2\frac{1}{2} \div 6\frac{1}{2}$.
 (45) $1\frac{1}{2} \div 3\frac{1}{2}$. (46) $1\frac{1}{2} \div 15\frac{1}{2}$. (47) $17\frac{1}{2} \div 1\frac{1}{2}$. (48) $8\frac{1}{2} \div 3\frac{1}{2}$.

Simplification of Expressions involving Fractions.

164. The forms (), { }, [], are called **brackets**, and are used to *bind together* two or more quantities into one.

Thus $8 - (3 + 2)$ means that the sum of 3 and 2 is to be taken from 8, the result being 3. If we were to omit the brackets, the meaning would be quite different; for $8 - 3 + 2$ would mean that from 8 we must take 3, and then add 2, which would give 7 as the result.

Again, $24 \div (3 + 5)$, means that 24 is to be divided by the sum of 3 and 5, the result being 3. If we were to omit the brackets, and write it as $24 \div 3 + 5$, the expression would mean that 24 is to be divided by 3, and 5 to be then added to the result, which would make 13.

Similarly, $(8 - 2)(3 + 2)$, means that the difference between 8 and 2 is to be multiplied by the sum of 3 and 2.

From these illustrations we learn that *numbers enclosed in brackets must be reduced to their simplest form before any of the operations indicated by signs outside the bracket can be performed.*

Example. Simplify $(3\frac{1}{2} + 2\frac{1}{2}) \div (1\frac{1}{2} + 5\frac{1}{2})$.

$$\begin{aligned} (3\frac{1}{2} + 2\frac{1}{2}) \div (1\frac{1}{2} + 5\frac{1}{2}) &= 5\frac{1}{2} \div 7\frac{1}{2} \\ &= \frac{2\frac{1}{2}}{1\frac{1}{2}} \times \frac{1\frac{1}{2}}{1\frac{1}{2}} \\ &= 1\frac{1}{2} \div 1\frac{1}{2}. \end{aligned}$$

EXERCISE 58.

Simplify the expressions:—

- (1) $15\frac{1}{2} - (7\frac{1}{2} + 6\frac{1}{2})$. (2) $15\frac{1}{2} - (7\frac{1}{2} - 6\frac{1}{2})$.
 (3) $4\frac{1}{2} \div (5\frac{1}{2} + 1\frac{1}{2})$. (4) $1\frac{1}{2} \div (\frac{1}{2} - \frac{1}{2})$.
 (5) $(15\frac{1}{2} - 3) \div 7\frac{1}{2}$. (6) $(10\frac{1}{2} + 3) \times 4\frac{1}{2}$.
 (7) $(15\frac{1}{2} + 3)(7\frac{1}{2} - 6\frac{1}{2})$. (8) $(\frac{1}{2} + \frac{1}{2} + \frac{1}{2})(17\frac{1}{2} - 11\frac{1}{2})$.
 (9) $(8\frac{1}{2} - 1\frac{1}{2}) \div (3\frac{1}{2} + 7\frac{1}{2})$. (10) $(9\frac{1}{2} + 3\frac{1}{2}) \div (4\frac{1}{2} - 1\frac{1}{2})$.

165. It must be noticed carefully that

1st. Operations of multiplication are to be performed in order from left to right.

2nd. Quantities connected by the sign "of" must be taken as forming a single quantity, just as if they were in brackets.

NOTE.—The difference in meaning between

$$3\frac{1}{2} \div 2\frac{1}{2} \times 7\frac{1}{4} \text{ and } 3\frac{1}{2} \div 2\frac{1}{2} \text{ of } 7\frac{1}{4}$$

is that, in the former, the sign \div extends only to the next quantity $2\frac{1}{2}$, and no farther; but, in the latter, $2\frac{1}{2}$ of $7\frac{1}{4}$ is a single quantity.

EXERCISE 59.

Simplify the expressions:—

- | | |
|--|---|
| (1) $2\frac{1}{4} \times 1\frac{1}{2} \div 1\frac{1}{8}$ of $2\frac{3}{8}$. | (2) $2\frac{1}{4}$ of $1\frac{1}{2} \div 1\frac{1}{8} \times 2\frac{3}{8}$. |
| (3) $2\frac{1}{4}$ of $1\frac{1}{2} \div 1\frac{1}{8}$ of $2\frac{3}{8}$. | (4) $\frac{7}{8}$ of $\frac{1}{2} \div \frac{5}{8}$ of $\frac{3}{4}$. |
| (5) $2\frac{1}{4}$ of $\frac{7}{8} \div \frac{1}{2} \times \frac{6}{11}$. | (6) $\frac{2}{3}$ of $4\frac{1}{2} \div \frac{4}{5}$ of $3\frac{1}{4}$. |
| (7) $2\frac{1}{4}$ of $\frac{7}{8} \div \frac{1}{2}$ of $\frac{6}{11}$. | (8) $\frac{5}{7}$ of $1\frac{6}{8} \div 2\frac{5}{7}$ of $\frac{4}{5}$. |
| (9) $6\frac{1}{7}$ of $10\frac{1}{2} \div 2\frac{1}{10}$ of $7\frac{1}{8}$. | (10) $\frac{4}{5}$ of $5\frac{1}{2} \div 1\frac{1}{7}$ of $1\frac{3}{4}$. |
| (11) $3\frac{2}{3}$ of $4\frac{3}{7} \div 1\frac{2}{3}$ of $3\frac{4}{7}$. | (12) $\frac{2}{3}$ of $\frac{5}{7}$ of $3\frac{8}{9} \div \frac{1}{7}$ of $\frac{2}{3}$ of $1\frac{3}{8}$. |

Example 1. Simplify $\frac{1}{8}$ of $2\frac{5}{7} - 10\frac{5}{8} \div 1\frac{1}{2} + \frac{3}{7}$ of $3\frac{3}{4}$ of $3\frac{4}{5}$.

$$\begin{aligned} \text{The expression} &= \frac{1}{8} \times \frac{19}{7} - \frac{85}{8} \times \frac{1}{2} + \frac{3}{7} \times \frac{15}{4} \times \frac{19}{5} \\ &= \frac{19}{56} - \frac{1625}{112} + \frac{171}{56} \\ &= \frac{19}{56} - 5\frac{13}{8} + 6\frac{3}{8} \\ &= 1 + \frac{228 - 206 + 45}{112} \\ &= 1\frac{7}{112} \\ &= 1\frac{1}{16}. \end{aligned}$$

Example 2. Simplify $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} \div (\frac{3}{4} - \frac{1}{8})$.

$$\begin{aligned} \text{The expression} &= \frac{1}{2} + \frac{1}{3} + \frac{1}{6} \div \frac{5}{8} \\ &= \frac{1}{2} + \frac{1}{3} + \frac{1}{6} \times \frac{8}{5} \\ &= \frac{1}{2} + \frac{1}{3} + \frac{2}{3} \\ &= \frac{15 + 10 + 12}{30} \\ &= \frac{37}{30} \\ &= 1\frac{7}{30}. \end{aligned}$$

EXERCISE 60.

Find the value of

- | | |
|---|---|
| (1) $15\frac{1}{4} + \frac{2}{3}$ of $7\frac{3}{4}$. | (2) $\frac{3}{7} \div 1\frac{4}{11}$ of $1\frac{1}{21} - \frac{1}{5}$. |
| (3) $\frac{5}{13}$ of $5\frac{4}{7} - \frac{2}{3}$ of $\frac{5}{4}$ of $2\frac{1}{3}$. | (4) $3\frac{1}{5} + 4\frac{1}{3} - 5\frac{1}{4}$ of $\frac{9}{7}$. |
| (5) $8\frac{2}{3} - 3\frac{5}{8}$ of $1\frac{7}{8} + 2\frac{3}{8}$ of $2\frac{3}{8}$. | (6) $3\frac{1}{2} - 4\frac{1}{3}$ of $\frac{7}{8} + 4\frac{1}{3}$ of $\frac{3}{7}$. |
| (7) $3\frac{7}{8} \div \frac{9}{10} - 1\frac{3}{8}$. | (8) $15\frac{1}{4} + \frac{2}{3} \div 7\frac{3}{4}$. |
| (9) $15\frac{1}{4} - \frac{2}{3} \div 7\frac{3}{4} - 6\frac{3}{8}$. | (10) $5\frac{2}{3} + 3\frac{1}{3} \div 1\frac{1}{3} - 4 \div 1\frac{9}{11}$. |
| (11) $\frac{1}{11}$ of $21\frac{6}{7} - \frac{1}{11}$ of $10\frac{5}{9} + \frac{1}{11} \div \frac{3}{11}$. | (12) $\frac{7}{11}$ of $5\frac{1}{2} \times 1\frac{4}{7}$ of $\frac{4}{11} - \frac{2}{3} \div 1\frac{1}{4}$. |
| (13) $(15\frac{1}{4} + \frac{2}{3})$ of $7\frac{3}{4} - 6\frac{3}{8}$. | (14) $15\frac{1}{4} + \frac{2}{3}$ of $(7\frac{3}{4} - 6\frac{3}{8})$. |
| (15) $15\frac{1}{4} + \frac{2}{3} \div (7\frac{3}{4} - 6\frac{3}{8})$. | (16) $15\frac{1}{4} - \frac{2}{3} \div (7\frac{3}{4} - 6\frac{3}{8})$. |

- (17) $(3\frac{1}{2} + 7\frac{1}{2})$ of $\frac{2}{3} + 9\frac{2}{3}$. (18) $\frac{1}{2}$ of $\frac{1}{2} \div (\frac{7}{8} + \frac{1}{2}$ of 20).
- (19) $\frac{2}{3}$ of $4\frac{1}{2}$ of $3\frac{2}{7} - (5\frac{1}{2} - 2\frac{3}{4})$. (20) $\frac{1}{2} + 1\frac{1}{2} + \frac{2}{3} \times (\frac{4}{5} - \frac{2}{5})$.
- (21) $\frac{1}{4}$ of $(3\frac{1}{2} + 4\frac{1}{2}) \div \frac{2}{3}$ of $(4\frac{2}{5} - 3\frac{2}{5})$. (22) $1 - (\frac{2}{3}$ of $\frac{2}{3} + \frac{2}{3} + \frac{2}{3}$ of $\frac{7}{10})$.
- (23) $2\frac{2}{3} - 1\frac{1}{2} \times (2\frac{1}{2} - \frac{1}{2})$. (24) $(2\frac{2}{3} + 1\frac{1}{2}) \div 2\frac{1}{2} - \frac{1}{2}$.
- (25) $2\frac{2}{3} \times 2\frac{2}{3} \div (2\frac{2}{3} - 2\frac{1}{2})$. (26) $(3\frac{2}{3} + 2\frac{4}{7})$ of $(1\frac{1}{5} + 1 - \frac{1}{2})$ of $\frac{1}{2}$.
- (27) $\frac{1}{10}$ of $(\frac{1}{2} + \frac{2}{3}) + \frac{1}{10}$ of $(\frac{1}{2} - \frac{1}{5})$. (28) $(\frac{1}{2} + \frac{2}{3} - \frac{1}{2}) \times 7 \times (\frac{2}{3} + \frac{1}{5}) \div 4\frac{1}{2}$.
- (29) $(3\frac{1}{2} \times 5\frac{1}{2}) \div (3\frac{1}{2}$ of $3\frac{3}{4}) + \frac{1}{2}$.
- (30) $(17\frac{1}{2} \div 3\frac{2}{5}) - (1\frac{1}{2}$ of $\frac{4}{5}) + 5\frac{2}{3} \quad 3\frac{1}{2}$.

Fractions of Concrete Quantities.

166. To find the value of a fraction of a given quantity.

To do this we divide the quantity by the denominator of the fraction, and multiply the result by the numerator; or we may first multiply the quantity by the numerator, and then divide the result by the denominator.

Example 1. Find the value of $\frac{2}{11}$ of £78. 16s. 2½d.

$$\begin{aligned} \frac{2}{11} \text{ of } £78. 16s. 2\frac{1}{2}d. &= \frac{£78. 16s. 2\frac{1}{2}d.}{11} \times 2 & \text{Or thus} & \begin{array}{r} £ \quad s. \quad d. \\ 78 \quad 16 \quad 2\frac{1}{2} \\ 3 \\ \hline 11 \overline{) 236 \quad 8 \quad 7\frac{1}{2}} \\ \underline{£21 \quad 9 \quad 10\frac{1}{2}} \end{array} \\ &= £7. 3s. 3\frac{1}{2}d. \times 2 \\ &= \underline{£21. 9s. 10\frac{1}{2}d.} \end{aligned}$$

Example 2. Find the value of $5\frac{1}{2}$ of £18. 13s. 9d.

$$\begin{aligned} \text{The value} &= £18. 13s. 9d. \times 5 + £18. 13s. 9d. \times \frac{1}{2} \\ &= £93. 8s. 9d. + £12. 9s. 2d. \\ &= \underline{£105. 17s. 11d.} \end{aligned}$$

Here it is best to multiply separately by 5 and $\frac{1}{2}$, and to add the products.

EXERCISE 61.

Find the value of

- (1) $\frac{1}{4}$ of £16. 2s. 4d. (2) $\frac{1}{2}$ of £7. 18s. 2d.
- (3) $5\frac{1}{2}$ of £10. 9s. 6½d. (4) $8\frac{1}{2}$ of £3. 12s. 6½d.
- (5) £2. 7s. 6d. $\div 1\frac{1}{2}$. (6) £35. 14s. 8½d. $\div 1\frac{1}{2}$.
- (7) £600. 12s. 6d. $\div 2\frac{1}{2}$. (8) £219. 9s. 7d. $\div 12\frac{1}{2}$.
- (9) £17. 2s. 6d. $\times \frac{1}{10}$. (10) 16s. 2½d. $\div 5\frac{1}{2}$.
- (11) £9. 10s. 9d. $\times 5\frac{1}{2}$. (12) £617. 13s. 6½d. $\times 1\frac{1}{2}$.
- (13) 1 ton 1 qr. 20 lb. $\times \frac{1}{10}$. (14) 2 qrs. 10 lb. 7 oz. 9 drs. $\times \frac{1}{2}$.
- (15) 17 tons 12 cwt. 56 lb. $\times \frac{3}{10}$. (16) 7 cwt. 7 st. 7 lb. $\times 6\frac{1}{2}$.
- (17) 3 tons 5 cwt. 1 qr. $\times 5\frac{1}{2}$. (18) 1 ton 77 lb. $\div 4\frac{1}{2}$.
- (19) 1 ton 13 cwt. 3 qrs. $\div 1\frac{1}{2}$. (20) 6 tons 9 cwt. 16 lb. $\div 6\frac{1}{2}$.
- (21) 33 yds. 0 ft. 8 in. $\div 13\frac{1}{2}$. (22) 11 qrs. 5 bush. 3 pks. $\div 3\frac{1}{2}$.
- (23) 5 days 4 hrs. 12 min. $\times 6\frac{1}{2}$. (24) 3 weeks 19 hrs. $\div 3\frac{1}{2}$.

Example 3. Find the value of $1\frac{1}{2}$ of £1. 10s. $4\frac{1}{2}d.$ + $2\frac{1}{2}$ of £2. 8s. 4d. + $\frac{3}{4}$ of half-a-guinea.

$$\begin{aligned}\text{Value} &= \text{£1. 10s. } 4\frac{1}{2}d. + 3s. 4\frac{1}{2}d. \times 7 \\ &\quad + \text{£4. 16s. 8d.} + 9s. 8d. \times 4 \\ &\quad + \quad \quad 4s. 6d. \\ &= \text{£6. 11s. } 6\frac{1}{2}d. + \text{£1. 3s. } 7\frac{1}{2}d. + \text{£1. 18s. 8d.} \\ &= \underline{\underline{\text{£9. 13s. 10d.}}}\end{aligned}$$

EXERCISE 62.

Find the value of

- (1) $\frac{3}{8}s.$ + $\frac{2}{7}$ guin. + $\frac{1}{12}$ h.-cr. + $\text{£}\frac{1}{12}$. (2) $\frac{5}{7}$ guin. + $\text{£}\frac{3}{8}$ + $\frac{7}{10}$ cr. + $\frac{1}{4}s.$
- (3) $\text{£}\frac{1}{16}$ + $\frac{5}{8}s.$ + $\frac{5}{8}$ guin. + $\frac{3}{8}$ of £1. 6s. 8d.
- (4) $\frac{1}{12}$ of £5 + $\frac{1}{12}$ of 15s. + $\frac{3}{4}$ of £1. 6s. 8d.
- (5) $\frac{2}{3}$ of £33. 5s. 10d. - $\frac{1}{11}$ of £31. 2s. 5d.
- (6) $\frac{1}{3}$ of £18. 2s. 1d. - $\frac{1}{3}$ of £2. 0s. $4\frac{1}{2}d.$
- (7) $\frac{1}{11}$ of £2. 8s. $9\frac{3}{4}d.$ + $\frac{1}{8}$ of £3. 19s. 4d. - $\frac{3}{4}$ of 17s. $9\frac{1}{2}d.$
- (8) $\frac{7}{10}$ of £3 $\frac{3}{8}$ + $6\frac{3}{8}$ of £3. 0s. 9d. - $4\frac{1}{3}$ of £3. 2s.
- (9) $\text{£}\frac{2}{3}$ + $\frac{1}{3}$ guin. + $5\frac{2}{3}$ cr. - $2\frac{3}{4}$ fl. - 2s. $7\frac{3}{4}d.$
- (10) $\frac{7}{8}$ of £1. 10s. $4\frac{1}{2}d.$ + $\frac{4}{5}$ of £2. 8s. 4d. + $\frac{3}{7}$ of 10s. 6d.
- (11) $\frac{1}{8}$ of $4\frac{2}{3}$ of 3 lb. 8 oz. + $2\frac{4}{5}$ of $11\frac{1}{2}$ of 3 oz. 2 drs.
- (12) $3\frac{1}{2}$ of 6 cwt. 1 qr. 1 lb. 9 oz. + $5\frac{1}{2}$ of 3 cwt. 1 qr. 10 lb. 1 oz.
- (13) $3\frac{3}{8}$ of 11 cwt. 3 lb. - $1\frac{5}{8}$ of 1 ton 10 lb.
- (14) $\frac{8}{9}$ of 1 ton 6 cwt. 2 qrs. 2 lb. - $\frac{2}{3}$ of 1 cwt. 3 qrs. 24 lb.
- (15) $\frac{1}{3}$ of 1 ton - $\frac{1}{16}$ of 5 cwt. + $\frac{7}{10}$ of 1 qr. - $1\frac{7}{10}$ of 1 stone.
- (16) $\frac{4}{11}$ of 5 fur. 7 ch. 11 yds. + $\frac{1}{2}$ of $\frac{4}{5}$ of $1\frac{5}{8}$ yds.
- (17) $\frac{1}{2}$ of an acre + $\frac{1}{8}$ of a rood + $\frac{3}{8}$ of a perch.
- (18) $3\frac{5}{8}$ ac. + $3\frac{3}{8}$ ro. + $7\frac{3}{8}$ per. + $23\frac{1}{4}$ sq. yds.
- (19) $1\frac{1}{2}$ of $4\frac{3}{4}$ of $1\frac{3}{8}$ of $\frac{1}{11}$ of 3 ac. 1 ro. 30 per. - $\frac{3}{8}$ of $1\frac{1}{2}$ of 3 ro. 15 per.
- (20) $1\frac{1}{2}$ of a min. - $\frac{3}{8}$ of an hour + $6\frac{7}{10}$ days - $\frac{1}{4}$ of a week.

167. To find what fraction one quantity is of another of the same kind.

Example 1. What fraction of £5 is 4s. 6d.?

We take sixpence for the common unit; then since

$$4s. 6d. = 9 \text{ sixpences,}$$

$$\text{and } \text{£}5 = 200 \dots\dots\dots,$$

$$\therefore 1 \text{ sixpence} = \frac{1}{200} \text{ of } \text{£}5,$$

$$\text{and } 9 \text{ sixpences} = \frac{9}{200} \text{ of } \text{£}5;$$

$$\therefore 4s. 6d. = \frac{9}{200} \text{ of } \text{£}5;$$

$$\therefore \text{fraction required} = \frac{9}{200}.$$

Example 2. Reduce 2 qrs. 8 lb. to the fraction of 2 cwt.

$$\text{The fraction} = \frac{2 \text{ qrs. } 8 \text{ lb.}}{2 \text{ cwt.}} = \frac{64}{224} = \frac{2 \times 8 \times 2}{7 \times 32} = \frac{2}{7}.$$

Example 3. What fraction of £2. 2s. 7½d. is ⅔ of £2. 2s. 9d.?

⅔ of £2. 2s. 9d. = ⅔ of 1026 halfpence,
and £2. 2s. 7½d. = 1023 halfpence.

$$\text{The fraction required} = \frac{\frac{2}{3} \times 1026}{1023} = \frac{2}{3} \times \frac{1026}{1023} = \frac{2}{11}.$$

NOTE.—To find what fraction one quantity is of another is the same as to find how many times the former contains the latter.

EXERCISE 63.

What fraction of

- (1) 19s. 5¾d. is 11s. 8½d.?
- (2) £1. 5s. 7½d. is 15s. 4½d.?
- (3) £20. 13s. 3¾d. is £13. 15s. 6½d.?
- (4) £38. 16s. 10½d. is £15. 10s. 9d.?
- (5) £48. 19s. 10½d. is £20. 14s. 6¾d.?
- (6) 1 cwt. 1 qr. 3 lb. is 1 qr. 5 lb.?
- (7) 2 cwt. 3 qrs. 11 lb. is 1 qr. 5 lb.?
- (8) 5 cwt. 3 qrs. 6 lb. is 3 qrs. 20 lb.?
- (9) 2½ furlongs is 2 poles 3 yds. 2 ft.?
- (10) 3 quarters is 5 bush. 1 gall. 2 qts. 1 pt.?
- (11) 6 acres is 3 roods 27 per.?
- (12) 365½ days is 319 days 14 hrs. 15 min.?
- (13) 3 cubic yards is 3 cub. feet 648 cub. inches?
- (14) 27s. is ⅔ of 1s. 10½d.?
- (15) £49. 10s. is 2¾ of £10. 6s. 3d.?
- (16) 2⅝ of £4. 5s. is 2½ of £5. 11s.?
- (17) ⅔ of ⅔ of £1. 8s. 4d. is ⅔ of ⅔ of 19s. 6d.?
- (18) ⅔ of ⅔ of 3 guineas is ⅔ of ⅔ of 16s. 3d.?
- (19) 1 ton is 2¾ of 1½ of ⅔ of 3 cwt.?
- (20) 2 cwt. 1 qr. 12 lb. is 3½ of 1 cwt. 3 qrs. 2 lb.?
- (21) £33 is £1. 4s. 0¾d.?
- (22) £15. 4s. 10½d. is 2s. 7½d.?
- (23) £2. 9s. 1d. is 4s. 9d.?
- (24) £6. 15s. 10¾d. is £4. 6s. 5¾d.?
- (25) £3. 18s. 9d. is £2. 3s. 9d.?
- (26) 19 cwt. is 3 qrs. 11 lb.?
- (27) 3 cwt. 3 qrs. 9 lb. is 2⅛ cwt.?
- (28) 5 tons is 6½ oz.?
- (29) 2 oz. 15 dwt. 3 grs. is 1 oz. 11 dwt. 3 grs. (troy)?
- (30) 3 qrs. 2 bush. 1 pk. is 3 bush. 1 pk. 1 gall.?
- (31) 2 miles is 11 fur.?
- (32) 1 mile is 1540 yds. 2 ft. 9 in.?

- (33) 1 sq. mile is $2\frac{1}{2}$ roods? (34) 27 acres is 3 ac. 1 ro. 22 per.?
 (35) £41. 4s. $1\frac{1}{2}d.$ is $\frac{1}{8}$ of £34. 14s.?
 (36) $\frac{2}{3}$ of $\frac{1}{2}$ of £1. 12s. 6d. is $\frac{1}{4}$ of $\frac{2}{3}$ of £4. 10s.?
 (37) 1 gallon is $\frac{1}{4}$ of $\frac{2}{3}$ of 3 qts. 1 pt.?
 (38) 3 cwt. 16 lb. is $\frac{3}{11}$ of 2 qrs. 7 lb.?
 (39) 50 yds. 10 in. is $1\frac{1}{2}$ of $3\frac{1}{2}$ of 5 yds. 1 ft.?
 (40) 2 ac. 3 ro. 10 per. is $\frac{2}{3}$ of $1\frac{1}{2}$ of 3 ro. 15 per.?

Complex Fractions.

168. A fraction which has a fractional numerator or a fractional denominator, or both, is called a **complex** fraction.

Complex fractions are simplified by making use of the modified definition of a fraction given in Arts. 134—135.

Example 1. Simplify $\frac{\frac{1}{2} + \frac{3}{4}}{\frac{1}{2} + \frac{3}{8}}$.

$$\begin{aligned}\text{The fraction} &= \frac{\frac{1}{2} + \frac{3}{4}}{\frac{1}{2} + \frac{3}{8}} \\ &= \frac{\frac{2}{2} + \frac{3}{4}}{\frac{4}{4} + \frac{3}{8}} \\ &= \frac{\frac{2}{2} + \frac{3}{4}}{\frac{8}{8} + \frac{3}{8}} \\ &= \frac{\frac{2}{2} + \frac{3}{4}}{\frac{11}{8}} \\ &= 1\frac{5}{11}.\end{aligned}$$

Otherwise, as in Ex. 2, we may multiply numerator and denominator by 24, and we thus find at once that the fraction

$$= \frac{8+18}{12+9} = \frac{26}{21} = 1\frac{5}{21}.$$

Example 2. Simplify $\frac{3\frac{1}{2} - 2\frac{1}{4}}{3\frac{1}{2} + \frac{5}{8}}$.

A fraction is not altered in value if its numerator and denominator are both multiplied by the same number.

Hence, if we here multiply numerator and denominator by 12 (the L.C.M. of denominators 2, 3, 4, and 6), both the numerator and the denominator become integral and the fraction is no longer complex.

$$\begin{aligned}\text{Thus the fraction} &= \frac{42-28}{39+10} \\ &= \frac{14}{49} \\ &= \frac{2}{7}.\end{aligned}$$

Example 3. Simplify

$$\frac{1\frac{1}{8} \div 1\frac{1}{2}}{\frac{2}{3} \text{ of } \frac{5}{9} \div 10\frac{1}{2}} \times \frac{1\frac{1}{2} \text{ of } 4\frac{1}{8}}{6\frac{1}{2} \text{ of } 5\frac{1}{2}}.$$

The expression

$$\begin{aligned}&= \frac{\frac{9}{8} \times \frac{5}{9}}{\frac{2}{3} \times \frac{5}{9} \times \frac{21}{2}} \times \frac{\frac{3}{2} \times \frac{37}{8}}{1\frac{1}{2} \times \frac{16}{8}} \\ &= \frac{9}{8} \times \frac{5}{9} \times \frac{8}{2} \times \frac{8}{8} \times \frac{31}{8} \times \frac{8}{2} \times \frac{37}{8} \times \frac{8}{8} \times \frac{8}{8} \\ &= \frac{272}{64} \\ &= 4\frac{23}{8}.\end{aligned}$$

Example 4. Simplify $\frac{1\frac{1}{2}}{1 + \frac{1}{1 + \frac{1}{2}}}$.

We begin by multiplying numerator and denominator of $\frac{1}{1 + \frac{1}{2}}$ by 4. Thus the expression

$$= \frac{1\frac{1}{2}}{1 + \frac{1}{2}} = \frac{15}{10+8} = \frac{15}{18} = \frac{5}{6}.$$

A fraction of this kind is sometimes called a **continued fraction**.

EXERCISE 64.

Simplify the following expressions:—

- (1) $\frac{\frac{7}{8}}{\frac{3}{11}}$. (2) $\frac{7\frac{1}{2}}{1\frac{5}{7}}$. (3) $\frac{28\frac{1}{2}}{7\frac{1}{8}}$. (4) $\frac{19\frac{1}{7}}{32\frac{4}{7}}$. (5) $\frac{\frac{2}{3}}{\frac{5}{6} \times \frac{7}{8}}$.
- (6) $\frac{\frac{5}{7} \times 1\frac{6}{13}}{2\frac{5}{7} \times \frac{1}{13}}$. (7) $\frac{4\frac{2}{3} \text{ of } 5\frac{1}{7} \text{ of } 8}{28\frac{1}{2} \times \frac{8}{7}}$. (8) $\frac{\frac{1}{5} \text{ of } 1\frac{1}{4} \text{ of } 4\frac{1}{2}}{\frac{5}{8} \times 1\frac{1}{8} \times 3\frac{1}{2}}$. (9) $\frac{1\frac{1}{3} \times 7\frac{1}{2}}{3\frac{7}{8} \div 6\frac{1}{2}}$.
- (10) $\frac{3\frac{2}{3} \div 1\frac{2}{3}}{6\frac{2}{3} \div 5\frac{1}{3}}$. (11) $\frac{\frac{2}{3} \times \frac{5}{7} \div \frac{1}{18}}{\frac{4}{5} \times 6 \div 1\frac{7}{12}}$. (12) $\frac{\frac{7}{8} \times 3\frac{2}{3} \div \frac{2}{25}}{\frac{1}{3} \times 8\frac{3}{4} \div 7\frac{1}{2}}$. (13) $\frac{2\frac{1}{2} + 3\frac{1}{4}}{1\frac{1}{3} - \frac{2}{8}}$.
- (14) $\frac{6\frac{3}{4} - 1\frac{5}{14}}{2\frac{1}{8} + 1\frac{2}{7}}$. (15) $\frac{\frac{3}{5} \times \frac{4}{13}}{\frac{5}{8} + \frac{4}{13}}$. (16) $\frac{4\frac{4}{15} - 2\frac{1}{3}}{6\frac{1}{2} \times 2\frac{1}{5}}$. (17) $\frac{2\frac{2}{7} \text{ of } 2\frac{5}{8}}{2\frac{2}{8} - 2\frac{2}{7}}$.
- (18) $\frac{\frac{1}{2} \text{ of } 1\frac{2}{3}}{3\frac{3}{8} + 1\frac{1}{18}}$. (19) $\frac{\frac{2}{3} \text{ of } 1\frac{2}{3} \text{ of } 2\frac{2}{3}}{\frac{7}{9} + 1\frac{5}{12} - \frac{2}{4}}$. (20) $\frac{3\frac{1}{2} - 8\frac{2}{3} + 5\frac{1}{8}}{4\frac{1}{18} \text{ of } 2\frac{6}{13}}$.
- (21) $\frac{2\frac{1}{3} - \frac{7}{5} + \frac{2}{7}}{1 + \frac{6}{7} - \frac{5}{8}}$. (22) $\frac{\frac{2}{7} - \frac{3}{8} + \frac{1}{5}}{\frac{2}{7} + \frac{3}{8} - \frac{1}{5}}$. (23) $\frac{4\frac{1}{5} - \frac{2}{3} \text{ of } \frac{5}{8}}{1\frac{2}{3} + 2\frac{3}{7} - \frac{2}{8}}$.
- (24) $\frac{15\frac{3}{4} - 1\frac{4}{3} \times 1\frac{5}{8}}{\frac{1}{5} \times 23\frac{1}{3} + \frac{2}{3}\frac{1}{8}}$. (25) $\frac{2\frac{1}{4} - \frac{2}{3} \text{ of } 1\frac{5}{8}}{\frac{1}{5} \text{ of } 3\frac{1}{3} + 1\frac{2}{3}}$. (26) $\frac{\frac{1}{2}\frac{3}{4} \times \frac{1}{6} - \frac{1}{2}\frac{1}{8} \times \frac{1}{9}}{\frac{1}{2}\frac{1}{1} \times 1\frac{1}{2} - 1\frac{6}{7} \times \frac{1}{3}\frac{1}{8}}$.
- (27) $\frac{3\frac{1}{5} + 4\frac{1}{3} - 5\frac{1}{2} \text{ of } \frac{6}{7}}{3\frac{1}{2} - 4\frac{1}{3} \text{ of } \frac{7}{8} + 4\frac{1}{3} \text{ of } \frac{2}{7}}$. (28) $\frac{\frac{2}{4} + \frac{9}{11} - \frac{7}{8}}{(\frac{5}{8} + \frac{6}{8}) \div (3 - \frac{1}{15})}$.
- (29) $\frac{(2\frac{1}{2} - 1\frac{1}{8}) \div (\frac{2}{4} + 1\frac{1}{2})}{\frac{2}{4} - (\frac{7}{8} - \frac{2}{4}) + 1\frac{2}{10}}$. (30) $\frac{(\frac{2}{4} + \frac{4}{5}) \text{ of } \frac{5}{8} + \frac{7}{8}}{\frac{2}{4} + \frac{4}{5} \text{ of } (\frac{5}{8} + \frac{7}{8})}$.
- (31) $\frac{21\frac{3}{4} \text{ of } \frac{4\frac{2}{3}}{2}}{3\frac{5}{12}}$. (32) $\frac{5\frac{1}{4}}{4\frac{1}{8}} \times \frac{1\frac{4}{11}}{2\frac{5}{11}} \div \frac{5\frac{2}{3}}{6\frac{5}{7}}$. (33) $\frac{1\frac{7}{8} \text{ of } \frac{2\frac{7}{8}}{11} \div \frac{4\frac{4}{7} \text{ of } \frac{2\frac{1}{10}}{13\frac{5}{4}}}{\frac{1}{12} \text{ of } 9\frac{9}{11}}$.
- (34) $\frac{7\frac{1}{5} \text{ of } 5\frac{1}{7} \div \frac{5\frac{1}{3} \times 3\frac{1}{5}}{5\frac{1}{3} + 3\frac{1}{5}}}{7\frac{1}{5} - 5\frac{1}{7}}$. (35) $\frac{7}{8} + \frac{7\frac{3}{4} - 6\frac{1}{2}}{2\frac{1}{3} \times 9}$. (36) $\frac{1}{1\frac{1}{2}} + \frac{1}{1\frac{1}{3}} + \frac{1}{1\frac{1}{4}}$.
- (37) $\frac{1}{3\frac{1}{8}} - \frac{2\frac{1}{4}}{9} + \frac{3\frac{5}{8}}{2} + \frac{4}{4\frac{2}{7}}$. (38) $\frac{\frac{1}{3} + \frac{2}{4} \text{ of } \frac{2}{3}}{\frac{7}{3} + 1\frac{1}{2}} \times \frac{4 + \frac{1}{3}}{\frac{1}{4} + 1\frac{5}{12}}$.
- (39) $\frac{1\frac{5}{17} - \frac{5}{7} \div \frac{9}{11} - 1\frac{2}{13}}{1\frac{2}{3} + 1\frac{2}{5}} \div \frac{9}{13} + \frac{2}{3}$. (40) $\frac{1}{2 + \frac{1}{1 + \frac{2}{3}}}$. (41) $\frac{5}{6 + \frac{7}{8 + 1\frac{10}{11}}}$.
- (42) $\frac{1}{2 + \frac{1}{3 + \frac{1}{5}}}$. (43) $\frac{1}{3 + \frac{1}{7 + 1\frac{1}{8}}}$. (44) $\frac{1}{10 + \frac{1}{2 + \frac{1}{30}}}$. (45) $\frac{2}{3 + \frac{4}{5 - \frac{1}{7}}}$.

169. The following are examples of problems involving vulgar fractions.

Example 1. A commercial traveller rode by rail $\frac{1}{3}$ of a journey, drove $\frac{2}{5}$ of it, and walked the rest; what part of it did he walk?

Fraction of whole journey which he did *not* walk = $\frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{8}{15} = 1\frac{1}{3}$;
 \therefore fraction of whole journey which he *did* walk = $1 - 1\frac{1}{3} = \frac{2}{3}$.

Example 2. A boy spends $\frac{2}{3}$ of his money, then $\frac{2}{3}$ of the remainder, and then has 3s. 6d.; what had he at first?

$$\begin{aligned}\text{Amount spent} &= \frac{2}{3} \text{ of orig. money} + \frac{2}{3} \text{ of the remainder} \\ &= \frac{2}{3} \text{ of orig. money} + \frac{2}{3} \text{ of } \frac{1}{3} \text{ of orig. money} \\ &= \left(\frac{2}{3} + \frac{2}{9}\right) \text{ of orig. money} \\ &= \frac{8}{9} \text{ of original money;} \end{aligned}$$

$$\therefore \frac{1}{9} \text{ of original money} = 3\text{s. } 6\text{d.};$$

$$\therefore \text{original money} = \underline{\underline{\pounds 1. 11\text{s. } 6\text{d.}}}$$

Example 3. A working alone can do a piece of work in $2\frac{1}{2}$ days, B in $3\frac{3}{4}$ days, and C in 5 days; in what time could they do it if they all worked together?

A can do the whole in $2\frac{1}{2}$ days,

B $3\frac{3}{4}$

C 5

\therefore A can do $\frac{2}{5}$ of it in 1 day,

B $\frac{4}{15}$ 1 ... ,

C $\frac{1}{5}$ 1 ... ;

\therefore A, B and C together can do $\frac{2}{5} + \frac{4}{15} + \frac{1}{5}$ in one day,

\therefore $\frac{6+4+3}{15}$

\therefore $\frac{13}{15}$

\therefore the whole in $1\frac{5}{13}$ days;

\therefore time required = $1\frac{5}{13}$ days.

Example 4. A vessel has 3 pipes connected with it, 2 to supply and 1 to draw off. The first alone can fill the vessel in 4 hours, the second in 3 hours, and the third can empty it in $1\frac{1}{2}$ hours. If all the pipes be opened when the cistern is full, how soon will the cistern be empty?

1st pipe fills $\frac{1}{4}$ of cistern in 1 hour,

2nd $\frac{1}{3}$

3rd pipe empties $\frac{2}{3}$

\therefore when all the pipes are open,

$\left(\frac{2}{3} - \frac{1}{4} - \frac{1}{3}\right)$ of cistern will be emptied in 1 hour,

i.e. $\left(\frac{1}{3} - \frac{1}{4}\right)$

i.e. $\frac{1}{12}$

\therefore the whole will be emptied in 12 hours.

MISCELLANEOUS EXAMPLES.

Fractions.

EXERCISE 65.

(1) In a battle $\frac{1}{4}$ of the soldiers are killed, and $\frac{2}{3}$ wounded; what fraction of the army will now be unable to fight?

(2) A man paid $\frac{2}{3}$ of an account in notes, $\frac{1}{4}$ in gold, and the rest in silver; what fraction of the account was paid in silver?

(3) A man travelled $\frac{5}{8}$ of his journey by coach, $\frac{7}{10}$ by rail, and walked the remaining distance; what fraction of his journey did he walk?

(4) One quarter of an estate is left to the eldest daughter, $\frac{1}{3}$ to the second daughter, and $\frac{2}{3}$ of the remainder to a son; how much was over?

(5) If I pay away $\frac{2}{3}$ of my money, and then $\frac{1}{4}$ of what remains, how much of the whole is left?

(6) A person at the first shop spends $\frac{1}{3}$ of his money, at the second $\frac{2}{3}$ of the remainder, at the third $\frac{1}{4}$ of the rest; what fraction of his original money has he left?

(7) A boy after giving away $\frac{1}{4}$ of his pocket-money to one companion, and $\frac{1}{3}$ of the remainder to another, has 9d. left; how much had he at first?

(8) A boy spent $\frac{2}{3}$ of his money, and then found that $\frac{1}{3}$ of the remainder was half-a-crown; what had he at first?

(9) A gives away $\frac{2}{3}$ of a guinea. He gives $\frac{1}{2}$ of this to B, $\frac{1}{3}$ of it to C, and the remainder to D; how much does each get?

(10) One pipe fills $\frac{1}{3}$ of a cistern in a minute, another fills $\frac{1}{4}$, and another $\frac{1}{6}$ in the same time; how much of the cistern will be filled in a minute if all three pipes be running?

(11) A can do a piece of work in 6 days, B in 8 days, and C in 9 days; in how many days would all three working together finish it?

(12) A alone can do a certain work in $2\frac{1}{2}$ hours, B in $3\frac{3}{4}$, C in 4, and D in $7\frac{1}{2}$; in what time could it be done if they all worked together?

(13) If a cistern can be filled by one pipe in 3 hours, by a second in 5 hours, and by a third in 7 hours, in what time will it be filled by all three in action together?

(14) A cistern can be filled by 2 pipes, A and B, in 20 and 24 minutes respectively, and can be emptied by a pipe C in 30 minutes; if all be turned on when the cistern is empty, in what time will it be filled?

(15) Pipes A and B can fill a cistern in 3 minutes and 5 minutes respectively, and C can empty it in $7\frac{1}{2}$ minutes; in what time will the cistern be filled when A, B and C are all turned on?

(16) A bag contains 365 oranges and 425 lemons, and I wish to arrange them in boxes, putting an equal number in each box; what is the greatest number I can put in a box?

(17) A farmer wishes to put 51 bushels of wheat, 221 bushels of barley, and 391 bushels of oats into the largest bags of equal size that will exactly hold each kind of corn; how many bushels must each bag hold?

(18) The children of a school can be arranged in classes of 32, 48, or 40; how many children are there at least in the school?

(19) Seven men, beginning together, fire at a target at intervals of 2, 3, 5, 7, 10, 12 and 14 minutes respectively; after how many hours will they again fire together?

(20) A copy-book contains 24 pages, and a boy has written 16 of them; what fraction of the whole has he yet to write?

(21) A $4\frac{1}{2}$ -gallon barrel of ale loses $\frac{3}{4}$ of a gallon by leakage; how many gallons will it now contain?

(22) If in $4\frac{1}{2}$ hours a train travels $105\frac{4}{5}$ miles, what is its rate per hour (in miles)?

(23) A brig started from Cardiff for Portsmouth with 608 tons of coal. During a storm $\frac{5}{9}$ of the cargo was thrown overboard; how many tons of coal did she deliver?

(24) A cask contains $46\frac{2}{3}$ gallons of beer; if $\frac{3}{4}$ of a gallon leak out each week, in how many weeks will the barrel be empty?

(25) From a pole 18 ft. long, as many pieces as possible are cut off, each $3\frac{1}{3}$ ft. in length; how much of the pole will be left?

DECIMALS.

170. If we write down 555, we know that this means five hundred and fifty-five, or in other words that the 5 to the right denotes 5 *units*; that the next digit represents 5 *tens*; and that the left-hand digit represents 5 *hundreds*. In other words, the **place-value of a figure increases 10 times for each place as we move from right to left, and diminishes one-tenth for each place as we go from left to right.** The last figure always represents *units*.

We will now extend this notation to the right of the units' place. If we put a figure 5 to the right of the units' place, this 5 may be considered to represent a *tenth* of a unit, and a figure 5 placed to the right of this to represent a *hundredth* of a unit, and a third 5 a *thousandth* of a unit, and so on. In other words, the first figure to the right of the units' place may be considered to be *divided* by 10, the second by 100, the third by 1000, and so on.

171. It is evident that the units' place must be distinguished in some way. This is done by placing a dot immediately after it. If, for example, we write down 555.555 we know that the figure to the *left* of the dot denotes 5 units; that the first figure to the *right* of the dot represents $\frac{5}{10}$, that the next represents $\frac{5}{100}$, and the next $\frac{5}{1000}$.

It will be seen that any number of noughts may be placed at, or

omitted from, the *end* of a decimal without at all changing its value. For example, $\cdot 035200$ and $\cdot 03520$ have exactly the same value as $\cdot 0352$.

Similarly, $\cdot 005$ means 5 thousandths; $50\cdot 05$ means 50 units and 5 hundredths.

172. An expression of the form $\cdot 555$ is called a **Decimal**, and the point is called a **decimal point**.

173. We read a decimal by naming the figures in order. Thus, 15 is "*decimal, one five*," not "*decimal, fifteen*."

EXERCISE 66.

Express the following in decimal notation :—

- | | |
|--------------------------|--------------------------|
| (1) 4 tenths. | (2) 7 tenths. |
| (3) 14 hundredths. | (4) 75 hundredths. |
| (5) 3 hundredths. | (6) 795 thousandths. |
| (7) 52 thousandths. | (8) 8 thousandths. |
| (9) 583 ten-thousandths. | (10) 29 ten-thousandths. |
| (11) 5 ten thousandths. | (12) 95 tenths. |
| (13) 17 tenths. | (14) 306 tenths. |
| (15) 853 hundredths. | (16) 2005 thousandths. |

To multiply a Decimal by 10, 100, 1000, etc.

174. It follows from what has been said about place-values increasing as we move from the decimal point to the left, and decreasing as we move to the right, that **to multiply by 10, 100, 1000, etc., we move the decimal point 1, 2, 3, etc., places respectively to the right.**

Examples.

$$\begin{aligned} 8\cdot 765 \times 10 &= 87\cdot 65. \\ 8\cdot 765 \times 100 &= 876\cdot 5. \\ 8\cdot 765 \times 1000 &= 8765. \end{aligned}$$

EXERCISE 67.

Multiply each of the following by 10 :—

- | | | | |
|--------------------|-------------------|------------|-------------------|
| (1) 32·425. | (2) $\cdot 025$. | (3) 58·02. | (4) 5·002. |
| (5) $\cdot 0002$. | (6) $\cdot 45$. | (7) 4·5. | (8) $\cdot 045$. |

Multiply each of the following by 100 :—

- | | | | |
|------------|----------------------|----------------------|---------------------|
| (9) 4 245. | (10) $\cdot 06275$. | (11) $\cdot 00025$. | (12) $\cdot 0525$. |
| (13) 10·1. | (14) $\cdot 001$. | (15) $\cdot 11$. | (16) 10·01. |

Multiply each of the following by 1000.

- | | | | |
|--------------------|---------------------|---------------|----------------------|
| (17) 37 2751. | (18) 5·2675. | (19) 52·4278. | (20) $\cdot 04725$. |
| (21) $\cdot 001$. | (22) $\cdot 0075$. | (23) 15·625. | (24) $\cdot 00005$. |

To divide a Decimal by 10, 100, 1000, etc.

175. Similarly, to divide by 10, 100, 1000, etc., we move the decimal point 1, 2, 3, etc. places respectively to the left. If it happens that there are no figures to the left of the decimal point, we imagine 0's to be there.

Examples.

$$54.321 \div 10 = 5.4321.$$

$$54.321 \div 100 = .54321.$$

$$54.321 \div 1000 = .054321.$$

EXERCISE 68.

Divide each of the following by 10:—

(1) $16.375.$

(2) $.65.$

(3) $.352.$

(4) $13.5.$

(5) $125.025.$

(6) $.01.$

(7) $.11.$

(8) $.011.$

Divide each of the following by 100:—

(9) $47.75.$

(10) $324.005.$

(11) $365.2.$

(12) $.0465.$

(13) $.4.$

(14) $.05.$

(15) $.006.$

(16) $.0005.$

Divide each of the following by 1000:—

(17) $1002.65.$

(18) $365.275.$

(19) $25.56.$

(20) $5.825.$

(21) $.5.$

(22) $.025.$

(23) $.0035.$

(24) $.04.$

To express a Decimal by an equivalent Vulgar Fraction.

176. Since $.76 = \frac{7}{10} + \frac{6}{100} = \frac{76}{100}$, and $.025 = \frac{0}{10} + \frac{2}{100} + \frac{5}{1000} = \frac{25}{1000}$; we see that the vulgar fraction equivalent to a decimal has for its denominator, unity, followed by as many noughts as there are figures after the decimal point.

Thus $.762 = \frac{762}{1000}$; $.0762 = \frac{762}{10000}$; and $2.05 = 2\frac{5}{100}$.

The vulgar fractions must of course be subsequently reduced to their lowest terms.

EXERCISE 69.

Express the following decimals as vulgar fractions, and reduce the fractions to their lowest terms:—

(1) $.2.$

(2) $.4.$

(3) $.14.$

(4) $.12.$

(5) $.45.$

(6) $.575.$

(7) $.125.$

(8) $.025.$

(9) $.015.$

(10) $.002.$

(11) $.005.$

(12) $.0125.$

(13) $.0052.$

(14) $4.016.$

(15) $5.0225.$

(16) $.15625.$

(17) $.59375.$

(18) $.078125.$

(19) $.071875.$

(20) $6.4375.$

(21) $7.0475.$

(22) $5.0125.$

(23) $2.0725.$

(24) $4.2745.$

(25) $13.01125.$

(26) $21.00625.$

(27) $15.750125.$

(28) $29.03205.$

(29) $82.20085.$

(30) $90.00025.$

Addition of Decimals.

177. In addition, we place the decimals so that their decimal points are exactly one under another; we then add as if the decimals were whole numbers, and place the decimal point in the result under the other decimal points.

Example 1.

$$\begin{array}{r}
 115.3804 \\
 -1100 \\
 23.0000 \\
 1.5000 \\
 5.2460 \\
 \hline
 145.2364
 \end{array}$$

Example 2.

$$\begin{array}{r}
 .08 \\
 26.3 \\
 9 \\
 .803 \\
 .01 \\
 \hline
 36.193
 \end{array}$$

We may, for convenience as in Ex 1, write noughts at the end of some of the decimals so as to have the same number of places in all; the noughts cannot affect the result

EXERCISE 70.

Add together

- (1) 23.3045, 136.489, 14.0364, 8.37.
- (2) 361.1264, 8.351, 10.0004, 136.28.
- (3) 10.164, .23243, .00323, .0001.
- (4) .0002, .001, .0001, .01.
- (5) 10. .01, 9.00101, 1 101.
- (6) 15.15, 1.515, .1515, .01515.
- (7) 2.0002, 2 002, 1.001, .02.
- (8) 349, 31.36, 365.005, 2.07, .76.
- (9) .05, .0051, .0001, 365.00009, .000001
- (10) 459.1, 64.35, 17.00103, 50.05051, .001.
- (11) 47.8543, 8.05647, .094886, and 437.506.
- (12) 438.506, 3.06321, 84.0508, and .090831.
- (13) 39.00643, .84506, 210.057, and .095386.
- (14) 85 0609, 4.68403, .00689, and 529 873.
- (15) 69 053, .5075, .81789, and 23.261587.
- (16) .003256, 11.98, 432 876, and 1532.
- (17) 29.885, .70985, 540.043, and .0065978.
- (18) 113.723, .0548, 1063, and 82.94167.
- (19) .074, 341.8962, 13.279, and 5370.
- (20) 218.832, .00458, 6321, and 28.74196.
- (21) .084, 413.2968, 14.972, and 3680.
- (22) 13.0129, 105.6201, .46054, and .009367.
- (23) 170.0644, .96437, 3.8405, and 25.00673.
- (24) 5.6043, .06547, 673.48, and 37.005.
- (25) 43.726, 9.00321, .2017, 2.3, 1001.001, and .00002.

Subtraction of Decimals.

178. To subtract one decimal from another we place the one below the other so that the decimal points may be together; we then subtract as if the decimals were whole numbers, and place the decimal point in the result under the other decimal points.

Example 1. Subtract 5.0019 from 12.001.

$$\begin{array}{r} 12.0010 \\ 5.0019 \\ \hline 6.9991 \end{array}$$

We have placed a nought at the end of the first decimal to make the number of figures the same in both lines.

Example 2. Subtract 1.053 from 3.13.

$$\begin{array}{r} 3.13 \\ 1.053 \\ \hline 2.077 \end{array}$$

Here we *imagine* a nought placed at the end of the minuend. It need not be written down.

EXERCISE 71.

Subtract

- | | | |
|--------------------------------|-------------------------------|-------------------------|
| (1) 4.75 from 6.825. | (2) .931 from 1.002. | (3) .001 from 1. |
| (4) 3.01 from 4. | (5) 1.98 from 8.471. | (6) 1.875 from 11.2. |
| (7) .4761 from 3. | (8) 19.003 from 20.1. | (9) .127 from 86.4. |
| (10) .0864 from .183. | (11) .076 from 6.2. | (12) 1.00005 from 2.14. |
| (13) 57.84936 from 63.299148. | (14) 46.648307 from 57.4802. | |
| (15) 219.546813 from 320.2795. | (16) 59.98063 from 72.480527. | |
| (17) 45.9328 from 87.625. | (18) 78.95392 from 91.023. | |
| (19) 48.43798 from 59.082. | (20) 73.841265 from 101.0963. | |
| (21) 84.941562 from 108.00641. | (22) 119.226709 from 180.015. | |
| (23) 6.4271 from 11.08573. | (24) 92.64382 from 203.07604. | |

Simplify

- (25) $.012 + 4.21 - .0012 + .00045 - 1.720004.$
 (26) $.096 - 4.007 - .00769 + 960 - 875.9.$
 (27) $10.007 - .006 + .0307 - 2.3 - .006.$
 (28) $3.0008 + 5.037 - 2.3 - 1.01 + 1.001.$
 (29) $.0005 - 2.705 + 30 - 5.01275 + .005.$
 (30) $6.752 - .005 - 2.75 + 5.2756 - .0005.$

Multiplication of Decimals.

179. (a) To multiply by a whole number.

We proceed exactly as in ordinary multiplication; that is, we carry forward multiples of 10, and set down remainders. The decimal point is so placed in the result that it may have as many figures after it there as it has in the multiplicand.

Example 1. Multiply 6.834 by 17.

$$\begin{array}{r} 6.834 \\ 17 \\ \hline 47838 \\ 6834 \\ \hline 116.178 \end{array}$$

Example 2. Multiply .00025 by 26.

We multiply 25 by 26, and prefix noughts to the product until there are five figures altogether, five being the number of figures after the point in the multiplicand.

We omit the final nought at the right of the product, and write it .0065.

$$\begin{array}{r} .00025 \\ 26 \\ \hline 150 \\ 50 \\ \hline .00650 \end{array}$$

EXERCISE 72.

Multiply

- | | |
|-------------------------------------|---------------------------------------|
| (1) .5 by 2, 3, 5, 9, 10. | (2) .125 by 2, 3, 4, 8, 5, 25, 125. |
| (3) .15 by 3, 4, 5, 6, 8, 9, 11. | (4) .05 by 8, 14, 18, 21, 25. |
| (5) 3.24 by 4, 7, 56, 48, 60. | (6) 3.1678 by 3, 45, 63, 112, 108. |
| (7) .0015 by 20, 35, 48, 150, 1100. | (8) 3.005 by 30, 45, 57, 205, 1250. |
| (9) 51.05 by 45, 54, 63, 360, 1020. | (10) 51.005 by 50, 65, 71, 450, 1520. |

180. (b) To multiply by a decimal.

We multiply as with whole numbers, and in the product we count off, beginning at the right, as many figures as there are decimal places in the two original decimals together, and we insert a decimal point to the left of these.

If the number of figures in the product be less than the number of decimal places in the two original decimals together, we prefix noughts until the number is made up, and we put the decimal point in front of all.

Example 1. Multiply 82.6 by 8.4.

There is one decimal place in each factor; therefore there will be *two* decimal places in the product, and the result is 693.84.

$$\begin{array}{r} 82.6 \\ 8.4 \\ \hline 3304 \\ 6608 \\ \hline 693.84 \end{array}$$

Example 2. Multiply .0125 by .00025.

The product of 125 and 25 is 3125; and since there are 4 decimal places in one factor and 5 in the other, there will be 9 in the required product. The result is .000003125.

$$\begin{array}{r} .0125 \\ .00025 \\ \hline 625 \\ 250 \\ \hline .000003125 \end{array}$$

Example 3. Multiply 625·2 by ·0125.

The result is 7·815. We strike out the 00 at the end, after counting 5 places and inserting the decimal point.

$$\begin{array}{r} 625\cdot2 \\ \cdot0125 \\ \hline 31260 \\ 12504 \\ 6252 \\ \hline 7\cdot81500 \end{array}$$

EXERCISE 73.

Multiply

- (1) 19·75 by 7·2.

(2) 13·13 by 11·17.

(3) 692 by ·305.
- (4) 1672 by ·014.

(5) 679·85 by ·00056.

(6) 12·375 by ·00125.
- (7) 78·125 by 40·96.

(8) ·001501 by 6·5.

(9) 654·1 by ·00167.
- (10) ·0076 by ·017.

(11) ·3785 by ·0003.

(12) ·0703 by ·00055.
- (13) 300·75 by 2·908.

(14) 396·8 by ·01609.

(15) 52·0468 by ·4715.
- (16) ·08435 by ·12042.

(17) ·57063 by ·025078.

(18) ·98236 by 370·95.
- (19) 29·5062 by ·05401.

(20) 54·9575 by 31·058.
- (21) 52·08631 by ·38056.

(22) 534·701 by ·80547.
- (23) 3·0259 by 24·029.

(24) 436·108 by ·75046.
- (25) 4·0952 by 48·038.

(26) 17·6531 by ·30095.
- (27) 1·438 by ·0362.

(28) ·0001234 by ·05123.
- (29) 4·572001 by ·000372.

(30) 4·30216 by 30·00209.

Simplify each of the following expressions :—

- (31) $52\cdot63 \times \cdot0004 \times \cdot525.$

(32) $\cdot0013 \times \cdot013 \times \cdot169.$
- (33) $\cdot1 \times \cdot1 \times \cdot1 \times \cdot1 \times \cdot1 \times \cdot1.$

(34) $\cdot01 \times \cdot001 \times \cdot1 \times 1\cdot1 \times 10\cdot1.$
- (35) $\cdot1 \times 10\cdot2 \times \cdot3 \times \cdot5 \times \cdot2.$

(36) $\cdot5 \times \cdot5 \times 1\cdot5 \times 5\cdot5 \times \cdot05.$

Division of Decimals.

181. (a) To divide by a whole number.

Divide the decimal as though it were a whole number, observing that the units' figure in the quotient is that which we get by using the units' figure in the dividend.

Example 1. Divide 6376·1275 by 5.

$$\begin{array}{r} 5 \) \ 6376\cdot1275 \\ \underline{1275\cdot2255} \end{array}$$

Example 2. Divide 56·787 by 12.

$$\begin{array}{r} 12 \) \ 56\cdot787 \\ \underline{4\cdot73225} \end{array}$$

When we come to the end of the figures given in the dividend, we can affix noughts, and continue the operation until it terminates or until we have obtained as many places as we require.

Example 3. Divide 1.0076 by 11.

$$\begin{array}{r} 11 \overline{) 1.0076} \\ \underline{0.0916} \end{array}$$

The first figure in the quotient, 9, must be put in the second place to the right of the decimal point, immediately below the last of the figures, 1.00, used in obtaining it.

Example 4 Divide 00176 by 44.

We must divide by factors when possible.

$$\begin{array}{r} 4 \overline{) .00176} \\ 11 \overline{) .00044} \\ \underline{.00004} \end{array}$$

Example 5. Divide .00213435 by 85.

The integral part of the dividend is 0; hence the integral part of the quotient is 0. Then comes the decimal point in the quotient. Then follow four noughts in the quotient, produced by the digits 0, 0, 2, 1 of the dividend, for we have to bring down five digits before we get a number 213, which is greater than 85. The fifth digit of the quotient is 2. The operation then goes on as in ordinary long division

$$\begin{array}{r} 85 \overline{) .00213435} \quad (.00002511) \\ \underline{170} \\ 434 \\ \underline{425} \\ 93 \\ 85 \\ \underline{85} \\ 85 \end{array}$$

EXERCISE 74.

Divide

- | | | |
|--------------------------------------|------------------------------------|------------------------|
| (1) 255 by 3, 5, 15, 17, 150, 170 | (2) 255 by 3, 5, 34, 60, 102, 255. | |
| (3) 8 0288 by 3, 7, 9, 63, 126, 189. | (4) 12 45 by 5, 83, 166, 332, 415. | |
| (5) 773 682 by 6, 13, 78, 109, 1014. | (6) 0289 by 17, 34, 680. | |
| (7) 53 018 by 4, 190, 392. | (8) 932 463 by 287, 861, 574. | |
| (9) 783 5 by 25, 125, 625, 6250. | (10) .00750116 by 677, 1354, 27 8. | |
| (11) 24 9216 by 128. | (12) 884 576 by 320. | (13) 22 7088 by 3800. |
| (14) 8 775 by 2340. | (15) 6 031094 by 7382. | (16) 8 56406 by 22537. |
| (17) .08008 by 2002. | (18) 001224 by 180. | |
| (19) 45 1732 by 45173200. | (20) .000256 by 1600. | |

182. (b) To divide by a decimal.

The quotient obtained by dividing one number by another is the same as the quotient obtained by dividing any multiple of the first number by the same multiple of the second.

When the divisor is a decimal we may therefore multiply both divisor and dividend by such a power of 10 as will make the divisor a whole number.

But multiplication by 10 is equivalent to moving the decimal point one place to the right. Hence the operation just stated is equivalent to moving the decimal point in both divisor and dividend as many places to the right as there are decimal places in the divisor. The divisor then becomes a whole number, and we proceed as in Art. 181.

The rule stated briefly is : **Make the divisor a whole number.**

Example 1. Divide .0011355 by .003.

$$\begin{array}{r} 3 \overline{) 1.1355} \\ \underline{.3785} \end{array}$$

In each number, we move the decimal point three places to the right. We thus get 3 for our new divisor, and 1.1355 for our dividend. Then proceeding as before, the required quotient is .3785.

Example 2. Divide 15.46875 by 12.375.

$$\begin{array}{r} 12375 \overline{) 15468.75} \quad (\underline{1.25} \\ \underline{12375} \\ 30937 \\ \underline{24750} \\ 61875 \\ \underline{61875} \end{array}$$

This is equivalent to dividing 15468.75 by 12375.

Example 3. Divide 12.5 by .000025.

This is equivalent to dividing 12500000 by 25.

$$\begin{array}{r} 25 \overline{) 12500000} \\ \underline{500000} \end{array}$$

EXERCISE 75.

Divide

- | | |
|--------------------------|---------------------------|
| (1) 46.4856 by .4. | (2) .0096 by 1.2. |
| (3) 4.00004 by .0011. | (4) 815.804 by .00011. |
| (5) .0189 by 2.5. | (6) 12.305 by .25. |
| (7) 4.8 by .0016. | (8) 19819.8 by .066. |
| (9) .3087 by .0049. | (10) .504 by .00042. |
| (11) .08748 by .0108. | (12) 25.5 by 1.7. |
| (13) 1028.5 by .0017. | (14) 28.3696 by 1.49. |
| (15) 384.006 by .123. | (16) .011977 by 2.03. |
| (17) 1126.08 by .00276. | (18) .00112608 by .017. |
| (19) .2107206 by 42.06. | (20) .000133 by 8.75. |
| (21) 1320.223 by .02806. | (22) 46.901343 by 15.423. |
| (23) 14.9352 by .00294. | (24) .37914426 by 37.062. |
| (25) 6.031094 by .7382. | (26) .00918 by .017. |
| (27) 3498.464 by 8.024. | (28) .00160778 by .019. |

- | | |
|---------------------------------|---------------------------------|
| (29) 8.32187 by $.4357$. | (30) 49.0248 by 1.857 . |
| (31) $.20705$ by $.0101$. | (32) 19.323135 by $.0603$. |
| (33) $.0830304$ by 86.4 . | (34) 46.1131776 by $.0576$. |
| (35) 210.6144185 by 27.35 . | (36) 155806.56 by $.006552$. |
| (37) 3072 by $.8192$. | (38) 2400 by $.0192$. |
| (39) 64 by $.00512$. | (40) 13877136 by $.6552$. |

In the following cases find the quotient as far as the 3rd figure after the decimal point:—

- | | | |
|------------------------------|------------------------------|-----------------------------|
| (41) $23.746 \div .0059$. | (42) $.185 : 34.06$. | (43) $.482 \quad 37.164$. |
| (44) $281.5 \div 13.789$. | (45) $21.387 \div .005192$. | (46) $46.75 \div .0038$. |
| (47) $.101 \div 80.351$. | (48) $.694 : 17.96$. | (49) $8870.6 \div 329.01$. |
| (50) $.983784 \div .623$. | (51) $112.03 \div 98.52$. | (52) $12.008 \div 54.1$. |
| (53) $1.758498 \div .0943$. | (54) $16.3704 : 72.6$. | (55) $15.125 \div 17.5$. |
| (56) $.78 \div 361.059$. | (57) $347.15 \div 10.5$. | (58) $74.3253 \div 1.33$. |
| (59) $92.3784 \div .623$. | (60) $46.75 : .0038$. | |

To express a Vulgar Fraction in the form of a Decimal.

183. If we divide the numerator by the denominator, the quotient is the decimal required.

Example 1. Express $\frac{3}{4}$ in a decimal form.

The numerator may be written $3.000\dots$, and if we divide it by 4 the quotient is .75.

$$\begin{array}{r} 4 \overline{) 3.00} \\ \underline{.75} \end{array}$$

Example 2. Express $7\frac{3}{8}$ in a decimal form.

The whole number remains unaltered. The required decimal is .09375.

$$\begin{array}{r} 4 \overline{) 3.00000} \\ 8 \overline{) .75000} \\ \underline{.09375} \end{array}$$

EXERCISE 76.

Express each of the following vulgar fractions as a decimal:—

- | | | | | |
|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| (1) $\frac{1}{20}$. | (2) $\frac{7}{8}$. | (3) $\frac{8}{25}$. | (4) $\frac{1}{2}$. | (5) $\frac{1}{20}$. |
| (6) $\frac{1}{16}$. | (7) $\frac{1}{8}$. | (8) $10\frac{1}{8}$. | (9) $5\frac{1}{8}$. | (10) $8\frac{1}{8}$. |
| (11) $\frac{1}{10}$. | (12) $\frac{1}{16}$. | (13) $\frac{1}{8}$. | (14) $\frac{1}{16}$. | (15) $\frac{1}{16}$. |

In the following cases find the quotient as far as the 6th figure after the decimal point:

- | | | | | |
|------------------------|------------------------|------------------------|------------------------|-----------------------|
| (16) $8\frac{1}{4}$. | (17) $9\frac{1}{8}$. | (18) $24\frac{1}{7}$. | (19) $4\frac{1}{2}$. | (20) $2\frac{1}{7}$. |
| (21) $8\frac{1}{4}$. | (22) $10\frac{1}{4}$. | (23) $7\frac{1}{2}$. | (24) $3\frac{1}{2}$. | (25) $9\frac{1}{8}$. |
| (26) $12\frac{1}{8}$. | (27) $14\frac{1}{4}$. | (28) $16\frac{1}{4}$. | (29) $18\frac{1}{4}$. | (30) $7\frac{1}{4}$. |

Complex Fractions involving Decimals.

184. The following examples should be noticed.

EXERCISE 77.

Simplify, without converting the decimals into vulgar fractions:—

- | | | |
|--|--|---|
| (1) $\frac{.0038425 - .00183}{.035}$ | (2) $\frac{.0013133}{2.3 \times 57.1}$ | (3) $\frac{851 \times .00016}{.032}$ |
| (4) $\frac{.00281 \times .0625}{1.405}$ | (5) $\frac{.23 - .023}{.0023 \div 23000}$ | (6) $\frac{.203 \times .0003 \times 16}{.008 \times .0029}$ |
| (7) $\frac{(3.45 - 2.014) \times 17.002}{.0008}$ | (8) $\frac{(1.005 + .201) \times (1.005 - .201)}{1.005 \times .201}$ | |
| (9) $\frac{2.004}{.167} \times \frac{3.375}{4}$ | (10) $\frac{22.4}{.25} + \frac{250}{.8} + \frac{1.2}{.0075}$ | |

Recurring Decimals.

185. We will now consider cases of division in which the quotient does not terminate. For example,

$$\frac{2}{3} = .6666\ldots, \text{ and } \frac{1}{7} = .142857142857\ldots,$$

in neither of which will the decimal ever end.

Whenever this happens, it will however be found that a figure, or a set of figures, is repeated continually. In $\frac{2}{3}$ the figure 6 is repeated, and in $\frac{1}{7}$ the figures 1, 4, 2, 8, 5, 7.

186. A decimal in which a digit, or a set of digits, is repeated continually is called a **Recurring**, or a **Circulating**, decimal.

187. The digit, or set of digits, which is repeated is called the **recurring period**.

The period is marked by dots placed over its first and last digits. Thus the decimal equivalent to $\frac{2}{3}$ is written $.6\dot{6}$, and that equivalent to $\frac{1}{7}$ is written $.1\dot{4}285\dot{7}$.

188. Such a decimal as $.14285\dot{7}$, in which all the digits recur, is called a **pure** recurring decimal; and one such as $.9\dot{5}4$, in which any do not recur, is called a **mixed** recurring decimal.

189. The prime factors of 10 are 2 and 5; hence, if the denominator of a vulgar fraction in its lowest terms be 2 or 5, or be the product of any number of 2's and 5's, the equivalent decimal will terminate. *In all other cases it will recur*; because, if the divisor be 7, for example, there can be only 6 *different* remainders, namely the numbers 1 to 6; a remainder must therefore soon recur, and then consequently the quotient digit will also recur.

We can tell beforehand that $\frac{3}{14}$, for example, will yield a recurring decimal, and $\frac{19}{125}$ one that terminates.

EXERCISE 78.

Express each of the following vulgar fractions as a recurring decimal:—

- | | | | | |
|------------------------|------------------------|------------------------|------------------------|------------------------|
| (1) $\frac{1}{2}$. | (2) $\frac{37}{100}$. | (3) $\frac{5}{7}$. | (4) $\frac{1}{15}$. | (5) $\frac{1}{11}$. |
| (6) $\frac{7}{8}$. | (7) $\frac{1}{3}$. | (8) $\frac{3}{8}$. | (9) $\frac{1}{4}$. | (10) $\frac{1}{11}$. |
| (11) $\frac{1}{3}$. | (12) $\frac{3}{12}$. | (13) $\frac{4}{11}$. | (14) $\frac{5}{10}$. | (15) $\frac{6}{15}$. |
| (16) $\frac{10}{11}$. | (17) $\frac{12}{11}$. | (18) $\frac{14}{17}$. | (19) $\frac{15}{11}$. | (20) $\frac{16}{15}$. |
| (21) $\frac{21}{11}$. | (22) $\frac{32}{7}$. | (23) $\frac{43}{15}$. | (24) $\frac{54}{11}$. | (25) $\frac{60}{15}$. |

To find a Vulgar Fraction equivalent to a Recurring Decimal.

190. A recurring decimal is equivalent to a vulgar fraction which has for its numerator the difference between the number formed by all the digits to the end of the first period, and the number formed by the digits which do not recur (if any): and for its denominator the number formed by as many nines as there are recurring digits, followed by as many noughts as there are non-recurring digits.

Example. Express $\cdot 12\dot{3}$ as a vulgar fraction.

$$\cdot 12\dot{3} = \frac{123}{100} - \frac{3}{10} = \frac{9}{10}.$$

The correctness of this method may be shown thus:—

$$\begin{aligned} \cdot 12\dot{3} &= \cdot 1232323 \dots \\ \therefore 10 \times \cdot 12\dot{3} &= 1 \cdot 232323 \dots \\ \therefore 1000 \times \cdot 12\dot{3} &= 123 \cdot 232323 \dots \\ \therefore 990 \times \cdot 12\dot{3} &= 123 - 1 \text{ (by subtraction),} \\ \therefore \cdot 12\dot{3} &= \frac{123}{990} - \frac{1}{990} = \frac{9}{10}. \end{aligned}$$

NOTE.—It follows from the rule that $\cdot \dot{9} = \frac{9}{9} = 1$, a result which should not be passed over without notice.

We know that $\cdot \dot{9}$ denotes $\cdot 99999\dots$. Now

$$\begin{aligned} \cdot \dot{9} &= \frac{9}{10}, \text{ and differs from unity by } \frac{1}{10}, \\ \cdot 99 &= \frac{99}{100}, \dots \dots \dots \frac{1}{100}, \\ \cdot 999 &= \frac{999}{1000}, \dots \dots \dots \frac{1}{1000}, \\ \cdot 9999 &= \frac{9999}{10000}, \dots \dots \dots \frac{1}{10000}; \end{aligned}$$

hence, for each additional nine the difference between the decimal and unity becomes one-tenth of what it was before. As we increase the number of nines this difference rapidly diminishes. And if we take an indefinitely large number of nines, the difference becomes so small as to be inappreciable; that is, the decimal becomes equivalent to unity itself.

Similarly, $\cdot 0\dot{3} = 1$, $\cdot 06\dot{9} = \cdot 07$, $\cdot 013\dot{9} = \cdot 014$; and generally, whenever $\dot{9}$ occurs at the end of a decimal, it should be omitted, and the preceding digit increased by unity.

EXERCISE 79.

Determine the vulgar fractions equivalent to the following recurring decimals, and reduce them to their lowest terms:—

- | | | | | |
|--|---|--|--|--------------------------------------|
| (1) $\cdot\dot{3}$. | (2) $\cdot\dot{4}\dot{2}$. | (3) $\cdot\dot{1}0\dot{8}$. | (4) $\cdot\dot{6}2\dot{1}$. | (5) $\cdot\dot{1}88\dot{1}$. |
| (6) $\cdot\dot{7}\dot{3}$. | (7) $\cdot\dot{0}\dot{5}\dot{6}$. | (8) $\cdot\dot{1}\dot{5}\dot{6}$. | (9) $\cdot\dot{0}0\dot{3}\dot{3}$. | (10) $\dot{1}5\cdot\dot{8}\dot{6}$. |
| (11) $\cdot\dot{0}11\dot{3}\dot{6}$. | (12) $\cdot\dot{0}0\dot{3}\dot{7}$. | (13) $\cdot\dot{0}00\dot{4}\dot{1}\dot{6}$. | (14) $3\cdot\dot{0}\dot{8}571\dot{4}\dot{2}$. | |
| (15) $5\cdot\dot{2}\dot{2}8571\dot{4}$. | (16) $10\cdot\dot{1}931\dot{8}$. | (17) $3\cdot\dot{0}38541\dot{6}$. | (18) $6\cdot\dot{1}004273\dot{5}$. | |
| (19) $12\cdot\dot{7}09\dot{3}$. | (20) $\cdot\dot{4}018\dot{5}$. | (21) $\cdot\dot{3}748\dot{1}$. | (22) $5\cdot\dot{1}6188\dot{1}$. | |
| (23) $\cdot\dot{4}3\dot{6}5\dot{7}$. | (24) $8\cdot\dot{0}07\dot{6}4\dot{8}$. | (25) $1\cdot\dot{2}0134\dot{3}\dot{8}$. | | |

Express as non-recurring decimals:—

- | | | | | |
|---------------------------------|-------------------------------------|-------------------------------|---------------------------------------|-----------------------------------|
| (26) $\cdot\dot{0}0\dot{9}$. | (27) $\cdot\dot{0}00\dot{9}$. | (28) $\cdot\dot{1}\dot{9}$. | (29) $\cdot\dot{3}1\dot{9}$. | (30) $\cdot\dot{0}62\dot{9}$. |
| (31) $\cdot\dot{0}244\dot{9}$. | (32) $\cdot\dot{6}\dot{9}\dot{9}$. | (33) $9\cdot\dot{9}\dot{9}$. | (34) $99\cdot\dot{9}\dot{9}\dot{9}$. | (35) $100\cdot\dot{8}99\dot{9}$. |

To add or subtract Recurring Decimals.

191. The method will be seen from the following examples.

Example 1. Add together $5\cdot\dot{7}6\dot{2}$, $\cdot\dot{5}4\dot{9}$, and $1\cdot\dot{2}0\dot{4}$.

We first write the decimals so that the periods may begin at the same, i.e. at the 3rd, decimal place in all. They thus become

$$5\cdot\dot{7}6\dot{2}\dot{6}, \cdot\dot{5}4\dot{9}\dot{5}\dot{4}, 1\cdot\dot{2}0\dot{4}.$$

If we now place the numbers one under another in the usual way, and carry on each decimal to the 8th place, it will be seen that the 9th column after the decimal points will be the same as the 3rd, the 10th the same as the 4th, etc.; and again, the 15th the same as the 3rd and 9th, the 16th the same as the 4th and 10th, and so on: hence the *columns* will recur in periods of six. It follows therefore that the digits at the foot of these columns will recur; that is, the decimal part of the sum will also be recurring, and its period will consist of six digits.

It will moreover be seen that the number of columns that recur, and therefore the number of digits in the recurring part of the result, is the L.C.M. of the numbers of digits in the periods of the given decimals.

Next we notice that the sum of the 8th column must be increased by some number carried to it from the sum of the 9th. In order therefore that the 8th place in the result may be correct, we carry on each decimal to *two* places more; but the 9th and 10th places may be ignored when the addition has been performed.

$$\begin{array}{r}
 5\cdot\dot{7}6\dot{2}6\dot{2}6\dot{2}6\ldots \\
 \cdot\dot{5}4\dot{9}54\dot{9}54\ldots \\
 1\cdot\dot{2}0444444\ldots \\
 \hline
 7\cdot\dot{5}1\dot{6}6202\dot{5}
 \end{array}$$

Adding the decimals, we find that the sum is $7\cdot\dot{5}1\dot{6}6202\dot{5}$.

NOTE.—The two chief properties to be remembered are

(1) The sum of any number of recurring decimals is also a recurring decimal, and

(2) The number of digits in its period is generally the L.C.M. of the numbers of digits in the given periods.

Example 2. Subtract $2\dot{4}\dot{6}21\dot{6}$ from $9\cdot\dot{2}1\dot{4}$.

We replace the 2nd decimal by $9\cdot\dot{2}1\dot{4}\dot{2}$, so that its period may begin at the same place as the other.

There are 4 digits in one period and 3 in the other; hence there will be 12 in the recurring part of the result.

The result is $6\cdot\dot{7}\dot{5}2047997592\dot{5}$.

$$\begin{array}{r} 9\dot{2}1\dot{4}214214214\dot{2}14 \\ 2\dot{4}\dot{6}2166216621\dot{6}62 \\ \hline 6\cdot\dot{7}\dot{5}2047997592\dot{5} \end{array}$$

EXERCISE 80.

Find the value of

- | | | |
|---|---|--|
| (1) $\cdot\dot{3} + \cdot\dot{6} + \cdot\dot{1}\dot{2}$. | (2) $\dot{6}4\dot{8} + 2\dot{6}2\dot{1} + \cdot\dot{1}0\dot{8}$ | |
| (3) $\cdot00\dot{5} + \dot{1}\dot{8} + \cdot\dot{4}\dot{2}$. | (4) $\cdot\dot{7}64\dot{1} + 1\cdot\dot{1}88\dot{1} + 6\cdot\dot{7}67\dot{1}$. | |
| (5) $2\cdot\dot{1}88\dot{1} + \cdot0\dot{8} + 3\cdot\dot{1}\dot{6}$. | (6) $7\cdot500\dot{1} + \cdot800\dot{2} + \cdot7\dot{8}9 + 85\cdot\dot{1}\dot{2}$. | |
| (7) $\cdot0\dot{3}\dot{6} + 5\cdot015 + \cdot0\dot{5}\dot{9}$. | (8) $\dot{6}4\dot{8} + \cdot0\dot{2} + 1\ 006$ | |
| (9) $43\cdot\dot{1}\dot{5} - 9\cdot\dot{1}\dot{4}$. | (10) $20\cdot\dot{3}\dot{1} - 17\cdot\dot{2}\dot{5}$. | (11) $6\cdot\dot{4} - 5\cdot\dot{5}$. |
| (12) $174\cdot\dot{1} - 143\cdot0\dot{2}\dot{9}$. | (13) $18\cdot8\dot{7}\dot{6} - 13\cdot2\dot{5}\dot{1}$. | (14) $32\ \dot{1}8\dot{1} - 25\cdot20\dot{8}\dot{6}$. |
| (15) $46\cdot3\dot{5}\dot{6} - 4\ 5\dot{2} + 4\cdot039\dot{7} - 3\ 215$. | (16) $15\ \dot{3}4\dot{3} - 5\cdot24\dot{3}\dot{3} + 5\cdot24\dot{3}$. | |
| (17) $9\ 87\dot{5} - 291\dot{6} - \cdot41\dot{6} - 1\cdot6$. | (18) $1 - \cdot4 - \cdot04 - \cdot0\dot{4} - \cdot00\dot{4}$. | |
| (19) $7 - 2\cdot5 - 3\cdot\dot{3} - \cdot\dot{6} - 075$. | | |
| (20) $1\cdot313\dot{2} + \cdot100\dot{1} - \cdot54\dot{3} + \cdot031\dot{4} - \cdot34\dot{5} + \cdot244\dot{1}$. | | |

To multiply or divide a Recurring Decimal by a whole number or by a terminating decimal.

192. The method will be seen from the following examples.

Example 1. Multiply $3\cdot1\dot{5}3\dot{2}$ by 7.

We multiply as in ordinary multiplication, but we carry the decimal part of the multiplicand two places beyond the end of the period, in order that, as in addition, we may know what number to carry. We thus ensure the correctness of the last digit retained in the product.

$$\begin{array}{r} 3\cdot1\dot{5}3\dot{2}\ 53 \\ 71 \\ \hline 22\cdot07\dot{2}\dot{7}1 \end{array}$$

Example 2. Multiply $1\ 8\dot{4}\dot{7}$ by $6\cdot3185$.

We multiply by the digits in succession as in Ex. 1, placing the successive products as in ordinary multiplication. Thus we get (a).

We now add these lines together, and to do this correctly we extend each line by repeating the digits of its period. The whole operation will then appear as in (b).

The number of decimal places as far as the end of the first period in the result must be $3 + 4$, i.e. 7.

We therefore put the point to the left of the 7th figure from the end, and the required product is $11\cdot673260\dot{1}$.

(a)	$1\ 8\dot{4}\dot{7}\ 47$	(b)	$1\cdot8\dot{4}\dot{7}\ 47$
	$6\ 3185$		$6\ 3185$
	$\hline 9237$		$\hline 9237\ 97$
	$1477\dot{9}$		$147797\ 97$
	$184\dot{7}$		$184747\ 47$
	$554\dot{2}$		$5542424\ 24$
	$110\dot{6}\dot{4}$		$110848484\ 84$
			$\hline 11\cdot673260\dot{1}$

55692.9

7-2

Example 3. Divide $15\cdot7\dot{6}20\dot{3}$ by 5.

We proceed as in ordinary division; but instead of affixing 0's, as in Art. 181, Example 2, we continue to bring down the digits of the period in rotation.

$$\begin{array}{r} 5 \overline{) 15\cdot7\dot{6}20\dot{3}620\dot{3}620\dot{3}\dots} \\ \underline{3\cdot1524072407240\dots} \end{array}$$

The quotient is $3\cdot15\dot{2}40\dot{7}$.

Example 4. Divide $73\cdot\dot{2}$ by 37.

After finding the fourth figure of the quotient we see that the remainder is the same as the remainder after the first figure of the quotient; hence the figures 9, 7, 8, must recur.

The quotient is $1\cdot97\dot{8}$.

$$\begin{array}{r} 37 \overline{) 73\cdot222\dots} \quad (1\cdot978\dots \\ \underline{37} \\ 362 \\ \underline{333} \\ 292 \\ \underline{259} \\ 332 \\ \underline{296} \\ 36 \end{array}$$

EXERCISE 81.

Find the value of

- (1) $1\cdot08\dot{2} \times 365.$

(4) $\cdot008\dot{3}7\dot{6} \times 762.$

(7) $\cdot\dot{3} \div 7.$

(10) $38\cdot0\dot{6}9\dot{4} \div \cdot025.$

(13) $\cdot\dot{6}3\dot{9} \times \cdot259.$

(16) $30\cdot\dot{3} \times 3\cdot265.$

(19) $15\cdot\dot{6} \div 1\cdot5.$

(22) $20\cdot13\dot{9}7\dot{2} \div 42\cdot1.$

(2) $7\cdot36\dot{7}\dot{4} \times 560.$

(5) $\cdot\dot{8}\dot{1} \times \cdot81.$

(8) $37\cdot\dot{0}8\dot{7} \div 5.$

(11) $2\cdot4\dot{7}\dot{2} \div 3\cdot4.$

(14) $3\cdot45\dot{6} \times \cdot425.$

(17) $32\cdot6\dot{7}\dot{3} \times 2\cdot834.$

(20) $3\cdot3\dot{2} \div \cdot004.$

(23) $\cdot011423\dot{9} \div 18\cdot36.$

(3) $\cdot00\dot{7}0\dot{4} \times 834.$

(6) $3\cdot\dot{0}9\dot{7} \times \cdot00061.$

(9) $\cdot332\dot{5} \div 25.$

(12) $\cdot200\dot{1} \div 1001.$

(15) $\cdot00\dot{1}\dot{2} \times 4\cdot56.$

(18) $\cdot03\dot{1}3\dot{2} \times 7\cdot095.$

(21) $16\cdot3\dot{2} \div \cdot004.$

(24) $4\cdot\dot{7}1428\dot{5} \div 2\cdot75.$

To multiply or divide one Recurring Decimal by another.

193. When the divisor is a recurring decimal, or when both multiplier and multiplicand are recurring decimals, we reduce both to equivalent vulgar fractions.

Example 1. Multiply $\cdot\dot{3}$ by $1\cdot\dot{0}\dot{6}$.

$$\begin{aligned} \cdot\dot{3} \times 1\cdot\dot{0}\dot{6} &= \frac{3}{9} \times 1\frac{6}{9} \\ &= \frac{1}{3} \times \frac{2}{3} \\ &= \frac{2}{9} \\ &= \underline{\underline{\cdot3\dot{5}}}. \end{aligned}$$

Example 2. Divide $\cdot\dot{3}$ by $1\cdot\dot{0}\dot{6}$.

$$\begin{aligned} \cdot\dot{3} \div 1\cdot\dot{0}\dot{6} &= \frac{3}{9} \div 1\frac{6}{9} \\ &= \frac{1}{3} \div \frac{2}{3} \\ &= \frac{1}{2} \\ &= \frac{2}{4} \\ &= \underline{\underline{\cdot3\dot{1}428\dot{5}\dot{7}}}. \end{aligned}$$

EXERCISE 82.

Find the value of

- | | | |
|-----------------------------------|----------------------------------|-----------------------------------|
| (1) $8\cdot3 \times \cdot519$. | (2) $\cdot054 \div 0\dot{2}$. | (3) $\cdot018 \times \cdot03$. |
| (4) $\cdot089 \times 0\dot{2}3$. | (5) $\cdot3 \times \cdot031$. | (6) $\cdot0009 \div 0\dot{9}09$. |
| (7) $1\cdot83 \times 3\cdot63$. | (8) $6\cdot8351 \times \cdot7$. | (9) $5\cdot43 \div 44\cdot45$. |
| (10) $1\ 753 \div 7\cdot51$. | (11) $9\ 04 \times 2\ 27$. | (12) $6\ 30 \div 14\cdot18$. |

Decimals of Concrete Quantities.

194. To find what decimal one quantity (*A*) is of another (*B*) of the same kind.

(a) When *B* is a simple quantity we proceed as in Examples 1 and 2.

Example 1. Express £1. 17s. 3½d. as a decimal of £5.

$$\begin{array}{r|l} 12 & 3\cdot75 \text{ pence} \\ 20 & 17\cdot3125 \text{ shillings} \\ 5 & 1\cdot865625 \text{ pounds} \\ & \cdot373125 \text{ of } \pounds 5. \end{array}$$

The decimal = $\cdot373125$.

Example 2. Reduce 3 pks. 1 gall. 2 qts. 1 pt. to the decimal of 8 bushels.

$$\begin{array}{r|l} 2 & 1 \text{ pt.} \\ 4 & 2\ 5 \text{ qts.} \\ 2 & 1\cdot625 \text{ gall.} \\ 4 & 3\ 8125 \text{ pks.} \\ 8 & 953125 \text{ bush.} \\ & \cdot119140625 \text{ of } 8 \text{ bush.} \end{array}$$

The decimal = $\cdot119140625$.

In Ex. 1, we first express 3½d. as the decimal 3·75d. We then divide by 12 to reduce to shillings, and we get 3125s. We add the 17s from the given amount, and we get 17·3125s.; this, reduced to pounds, becomes 1·865625. We then add the £1 from the given amount, and divide by 5.

In Ex. 2, the process is similar.

EXERCISE 83.

In the following examples reduce the first of the two given quantities to the decimal of the second:

- | | | |
|---|------------------------------------|--------------------------|
| (1) 18s. 9d.; £1. | (2) 19s. 5½d.; £1. | (3) £3. 4s. 7½d.; £1. |
| (4) £1. 11s. 1½d.; £2. | (5) 12s. 8½d.; £10. | (6) £5 17s. 7½d.; £100 |
| (7) 6s. 3d.; £50. | (8) 12s. 6¾d.; 15s. | (9) 7s. 10½d.; 1 guinea. |
| (10) 1 lb. 5 oz.; 1 cwt. | (11) 3 qrs. 14 lb.; 1 ton. | |
| (12) 2 cwt. 3 qrs. 3 lb. 8 oz.; 1 ton. | | |
| (13) 4 cwt. 3 qrs. 8 lb. 12 oz.; 1 ton. | (14) 3 cwt. 2 qrs. 7 lb.; 19 tons. | |
| (15) 33 yds.; 1 mile. | (16) 1 fur. 4 po.; 1 mule. | |
| (17) 7 ac. 3 ro. 28 per.; 20 acres. | (18) 3 gall. 2 qts.; 1 bushel. | |
| (19) 5 yds. 1 ft. 6 in.; 1 mile. | (20) 1 gall. 1 qt. 1 pt.; 1 peck. | |

(b) When B is a compound quantity we proceed as in Examples 3 and 4.

Example 3. Reduce 5s. to the decimal of 13s. 4d.

The vulgar fraction = $\frac{5}{13\frac{4}{3}} = \frac{15}{40} = \frac{3}{8}$; \therefore the decimal = .375.

Example 4. Reduce 5 ac. 3 ro. 15 p. to the decimal of 1 ac. 2 ro. 32 p.

The vulgar fraction = $\frac{5 \text{ ac. } 3 \text{ ro. } 15 \text{ p.}}{1 \text{ ac. } 2 \text{ ro. } 32 \text{ p.}} = \frac{935}{272} = 3\frac{113}{272} = 3\frac{7}{16}$;

\therefore the decimal = 3.4375.

EXERCISE 84.

In the following examples reduce the first of the two given quantities to the decimal of the second:—

- | | |
|---|--|
| (1) 3s. 3 $\frac{3}{4}$ d.; 4s. 5d. | (2) 6s. 6 $\frac{3}{4}$ d.; 10s. 6d. |
| (3) £1. 19s. 4 $\frac{1}{2}$ d.; £3. 3s. | (4) 16s. 1 $\frac{1}{2}$ d.; £5. 7s. 6d. |
| (5) £1. 8s. 2 $\frac{3}{4}$ d.; £2. 5s. 2d. | (6) £4. 12s. 6d.; £115. 12s. 6d. |
| (7) £4; £10. 8s. 4d. | (8) 2 cwt. 3 qrs. 7 lb.; 1 cwt. 1 qr. |
| (9) 2 lb. 6 oz.; 4 cwt. 27 lb. | |
| (10) 2 cwt. 1 qr. 12 lb.; 1 ton 15 cwt. 2 qrs. 24 lb. | |
| (11) 5 cwt. 1 qr. 1 lb. 8 oz.; 1 cwt. 2 qrs. 12 lb. | |
| (12) 11 ac. 3 ro. 10 per.; 1 ac. 3 ro. | |
| (13) 125 ac. 34 per.; 115 ac. 3 ro. 30 per. | |
| (14) 21 hrs. 54 min.; 2 days 10 hrs. 24 min. | |
| (15) 1 mile 550 yds.; 1 mile 240 yds. | |
| (16) 4s. 8 $\frac{3}{4}$ d.; 7s. 6d. | (17) 16s. 10d.; £3. 15s. 9d. |
| (18) £1. 16s. 11d.; £33. 4s. 6d. | (19) £3. 10s. 7d.; 11 guineas. |
| (20) 4 cwt. 2 qrs. 12 lb.; 6 cwt. 3 qrs. 18 lb. | |

195. To find the value of a given decimal of a quantity.

Example 1. Find the value of .36875 of a sovereign.

$$\begin{array}{r} .36875 \text{ £} \\ \quad 20 \\ \hline 7.37500 \text{ s.} \\ \quad 12 \\ \hline 4.500 \text{ d.} \end{array}$$

The required value = 7s. 4 $\frac{1}{2}$ d.

Example 2. Find the value of .07890625 of a ton.

$$\begin{array}{r} .07890625 \text{ ton} \\ \quad 20 \\ \hline 1.57812500 \text{ cwt.} \\ \quad 4 \\ \hline 2.312500 \text{ qrs.} \\ \quad 28 \\ \hline 25000 \\ \quad 6250 \\ \hline 8.75 \text{ lb.} \end{array}$$

Value = 1 cwt. 2 qrs. 8 lb. 12 oz.

Example 3. Find the value of 1.234375 of £2 6s. 8d.

$$1.234375 \text{ of } £2. 6s. 8d. = 1.234375 \text{ of } 560d.$$

$$= (1.234375 \times 560) \text{ pence}$$

$$= 691.875 \times 70 \text{ pence}$$

$$= 691.25 \text{ pence}$$

$$= 57s. 7\frac{1}{2}d.$$

$$= £2. 17s. 7\frac{1}{2}d.$$

Example 4. Find the value of .8125 of 2 tons 4 cwt.

$$.8125 \text{ of } 2 \text{ tons } 4 \text{ cwt.} = .8125 \text{ of } 44 \text{ cwt.}$$

$$= 3.25 \times 11 \text{ cwt.}$$

$$= 35.75 \text{ cwt.}$$

$$= 1 \text{ ton } 15 \text{ cwt } 3 \text{ qrs.}$$

EXERCISE 85 a.

Find the value of

- | | |
|---|--|
| (1) .025 of £1 in pence. | (2) .375 of 13s. 4d. in pence. |
| (3) .065 of 16s. 8d. in pence. | (4) .6 of £1. 17s. 6d. in half-crowns. |
| (5) .003125 of a ton in pounds. | (6) .028125 of a ton in pounds. |
| (7) .046875 of a lb. (avoir.) in drams. | (8) .325 of a mile in yards. |
| (9) .6875 of a lunar month in hours. | (10) .125 of 3 tons 4 cwt. in cwt. |

EXERCISE 85 b.

Find the value of

- | | | |
|-----------------------------------|-------------------------------------|----------------------------|
| (1) £ 0.625. | (2) £.9375. | (3) £3.6125. |
| (4) £26 975. | (5) £50.928125. | (6) 2 8125 of a cwt. |
| (7) 18.225 of a ton. | (8) 8421875 of a ton. | |
| (9) .203125 of a qr. (corn). | (10) .890625 of a bushel. | |
| (11) .1425 of a year (365 days). | (12) .7875 of a m.le. | |
| (13) .5875 of an acre. | (14) .384375 of £5. | (15) 9.68125 of £5. |
| (16) .053375 of £100. | (17) 3.03 of 10s. 5d. | (18) .0625 of £24. 7s. 4d. |
| (19) 271875 of 5 acres | (20) .856 of 2 cwt. 26 lb. | |
| (21) .625 of a year (365½ days). | (22) 3.4375 of 1 ac. 2 ro. 32 per. | |
| (23) 3 725 of 25 miles 4 fur. | (24) .390625 of 6 ac. 1 ro. 24 per. | |
| (25) .252 of 3 qrs. 7 bush. 1 pk. | (26) .36 of 12s. 1½d. | |
| (27) .572 of £1. 7s. 6d. | (28) .083 of £12. 19s. 6d. | |
| (29) .327 of £7. 10s. | (30) 7.83 of 5½ guineas. | |

198. The method of Simple Practice will be understood from the following examples.

Example 1. Find the value of 3125 things at £2. 10s. each.

We find the value of 3125 things at £2. 10s. each by adding their value at 10s. each to their value at £2 each. The work appears thus:—

£	s.	d.	
3125	0	0	= value at £1 each.
		2	
10s. = $\frac{1}{2}$ of £1	6250	0	0 = £2
	1562	10	0 = 10s.....
	£7812	10	0 = value at £2. 10s. each.

Example 2. Find the value of 1243 things at £4. 17s. 6d. each.

The value at £1 each is clearly £1243. The value at £4 each is found by multiplying the value at £1 by 4.

Again, 10s. is an aliquot part, namely $\frac{1}{2}$, of £1; hence the value at 10s. each is found by dividing the value at £1 by 2.

Similarly, we find the value at 5s. each by dividing the value at 10s. by 2, and the value at 2s. 6d. each by dividing the value at 5s. by 2. The sum of these results will be the value at £4. 17s. 6d. each.

£	s.	d.	
1243	0	0	= value at £1 each.
		4	
10s. = $\frac{1}{2}$ of £1.	4972	0	0 = £4
5s. = $\frac{1}{2}$ of 10s.	621	10	0 = 10s.
2s. 6d. = $\frac{1}{2}$ of 5s.	310	15	0 = 5s.
	155	7	6 = 2s. 6d....
	£6059	12	6 = value at £4. 17s. 6d. each.

It is often easier to find the value at a "round" sum *higher* than the given price, and then to subtract from the amount thus found the value at the difference between this "round" sum and the given price. Thus:—

£	s.	d.	
1243	0	0	= value at £1 each.
		5	
2s. 6d. = $\frac{1}{2}$ of £1	6215	0	0 = £5 each.
	155	7	6 = 2s. 6d. each.
By subtraction,	£6059	12	6 = value at £4. 17s. 6d. each.

EXERCISE 88 a.

Find, by Practice, the value of the following numbers of things; the given price being the price of *one*:—

- | | | |
|---------------------|---------------------|--------------------|
| (1) 175 at 10s. | (2) 387 at 5s. | (3) 269 at 4s. |
| (4) 795 at 2s. | (5) 968 at 1s. | (6) 419 at 2s. 6d. |
| (7) 136 at 6s. 8d. | (8) 227 at 3s. 4d. | (9) 526 at 1s. 8d. |
| (10) 697 at 1s. 4d. | (11) 853 at 1s. 3d. | (12) 369 at 10d. |

- | | | |
|----------------------|-----------------------------------|-------------------------------|
| (13) 277 at 8d. | (14) 497 at 6d. | (15) 531 at 3d. |
| (16) 419 at 4d. | (17) 759 at 2d. | (18) 827 at $1\frac{1}{2}$ d. |
| (19) 698 at 1d. | (20) 867 at $\frac{1}{2}$ d. | (21) 829 at 15s. |
| (22) 367 at 12s. 6d. | (23) 453 at 12s. | (24) 280 at 13s. 4d. |
| (25) 526 at 11s. 8d. | (26) 637 at 11s. | (27) 819 at 6s. |
| (28) 743 at 7s. 6d. | (29) 397 at 4s. 6d. | (30) 574 at 4s. 8d. |
| (31) 954 at 3s. 9d. | (32) 613 at 2s. 9d. | (33) 725 at 2s. 8d. |
| (34) 863 at 2s. 3d. | (35) 838 at 2s. 4d. | (36) 459 at 1s. 6d. |
| (37) 831 at 1s. 2d. | (38) 745 at 1s. $1\frac{1}{2}$ d. | (39) 823 at 1s. 1d. |
| (40) 459 at 10s. 6d. | (41) 593 at 14s. | (42) 676 at 16s. 8d. |
| (43) 791 at 9s. | (44) 897 at 7s. | (45) 625 at 3s. 6d. |

EXERCISE 88 b.

Find, by Practice, the value of the following numbers of things; the given price being the price of *one* :—

- | | | |
|--|---|-------------------------------------|
| (1) 6347 at £1. 10s. | (2) 7389 at £2. 5s. | (3) 3874 at £3. 4s. |
| (4) 6286 at £5. 2s. 6d. | (5) 7398 at £7. 6s. 8d. | (6) 9274 at £8. 2s. |
| (7) 4293 at £9. 1s. 8d. | (8) 9649 at £5. 3s. 4d. | (9) 4962 at £6. 1s. |
| (10) 2736 at £2. 15s. | (11) 3869 at £3. 12s. | (12) 4887 at £4. 13s. 4d. |
| (13) 6948 at £5. 12s. 6d. | (14) 7329 at £6. 11s. 8d. | |
| (15) 8693 at £9. 11s. | (16) 7294 at £8. 6s. | |
| (17) 6385 at £6. 7s. 6d. | (18) 4721 at £5. 4s. 6d. | |
| (19) 1978 at £3. 9s. 2d. | (20) 9634 at £12. 8s. 5d. | |
| (21) 5362 at £1. 5s. $3\frac{1}{2}$ d. | (22) 7257 at £5. 16s. $9\frac{3}{4}$ d. ✓ | |
| (23) 3829 at £2. 14s. $8\frac{1}{2}$ d. | (24) 8210 at £1. 18s. $6\frac{3}{4}$ d. | |
| (25) 3254 at £4. 17s. $2\frac{1}{2}$ d. | (26) 7263 at £2. 12s. $4\frac{1}{2}$ d. | |
| (27) 7432 at £5. 18s. $10\frac{1}{2}$ d. | (28) 6724 at £6. 14s. $5\frac{1}{2}$ d. | |
| (29) 8974 at £8. 17s. $11\frac{1}{2}$ d. | (30) 6327 at £10. 19s. $2\frac{1}{2}$ d. | |
| (31) 8369 at 14s. | (32) 6348 at 17s. 6d. | |
| (33) 4527 at 16s. | (34) 8210 at 18s. 6d. | |
| (35) 5670 at 11s. $9\frac{3}{4}$ d. | (36) 1842 at 6s. $9\frac{1}{2}$ d. | (37) 6480 at 4s. $3\frac{1}{2}$ d. |
| (38) 1296 at 16s. $10\frac{1}{2}$ d. | (39) 4750 at 9s. $10\frac{3}{4}$ d. | (40) 4069 at 13s. $4\frac{1}{2}$ d. |

Example 3 Find the value of 3245 $\frac{3}{4}$ things at 13s. 9d. each.

Since £9-15s., the value of 3245 $\frac{3}{4}$ things at £1 each is £3245. 15s.
The most convenient aliquot parts are 10s., 2s. 6d., and 1s. 3d.

	£	s.	d.	
	3245	15	0	— value at £1 each.
10s. — $\frac{1}{4}$ of £1	1622	17	6	= 10s.
2s. 6d. — $\frac{1}{4}$ of 10s.	405	14	$4\frac{1}{2}$	= 2s. 6d. ...
1s. 3d. — $\frac{1}{2}$ of 2s. 6d.	202	17	$2\frac{1}{2}$	= 1s. 3d. ...
	£2231	9	$0\frac{3}{4}$	= value at 13s. 9d. each.

EXERCISE 89.

Find, by Practice, the value of the following numbers of things; the given price being the price of *one* :—

- | | |
|----------------------------|-----------------------------|
| (1) 369½ at 1s. 8d. | (2) 428½ at 2s. 6d. |
| (3) 537¾ at 3s. 9d. | (4) 247½ at 4s. |
| (5) 369¾ at 5s. 3d. | (6) 415¾ at 3s. 6½d. |
| (7) 425¾ at 3s. 4d. | (8) 636¾ at 4s. 6d. |
| (9) 345⅞ at 5s. 8d. | (10) 524½ at 2s. 10½d. |
| (11) 7950½ at £1. 7s. 4d. | (12) 2037½ at £1. 19s. 4½d. |
| (13) 2178¾ at £3. 18s. 4d. | (14) 1762¾ at £1. 1s. 10½d. |
| (15) 1178¾ at £4. 16s. 8d. | (16) 8243⅞ at £2. 12s. 4d. |
| (17) 3548½ at £5. 7s. 1½d. | (18) 1816¾ at £6. 15s. 3¾d. |
| (19) 2734¾ at £7. 13s. 3d. | (20) 1346⅞ at £8. 9s. 6d. |

Example 4. Find the value of 32475 things at £1. 13s. 6d. per dozen.

	£	s.	d.	
12	32475	0	0	= value at £1 each.
	2706	5	0	= £1 per dozen.
	1353	2	6	= 10s.
	451	0	10	= 3s. 4d.
	22	11	0½	= 2d.
	£4532	19	4½	= value at £1. 13s. 6d. per dozen.

EXERCISE 90.

Find, by Practice, the value of the following numbers of things :—

- | | |
|--|---------------------------------------|
| (1) 87642 at 16s. 3d. per dozen. | (2) 416472 at £1. 2s. 9d. per dozen. |
| (3) 49021 at 17s. 6d. per score. | (4) 57625 at £1. 11s. 7d. per score. |
| (5) 32562 at £3. 9s. per gross. | (6) 489816 at £4. 14s. 6d. per gross. |
| (7) 24000 at 5s. 9d. per hundred. | |
| (8) 26475 at £8. 12s. 11d. per hundred. | |
| (9) 63000 at 3s. 9d. per thousand. | |
| (10) 939500 at £1. 5s. 6d. per thousand. | |
| (11) 83476 at 17s. 4½d. for 8. | (12) 43872 at £3. 18s. for 6. |
| (13) 20973 at £11. 15s. 4½d. for 6. | (14) 6358½ at £7. 17s. 10½d. for 9. |
| (15) 479635 at £3. 14s. 2d. for 10. | |

COMPOUND PRACTICE.

199. In Compound Practice we find the value of a compound quantity, when the value of a unit of *one* denomination is given.

Example. Find the value of 11 tons 6 cwt. 2 qrs. 14 lb. at £3. 13s. 4d. per ton.

The value of 11 tons is obtained by multiplying £3. 13s. 4d. by 11.

To the product we add the value of 6 cwt. 2 qrs. 14 lb., obtained by taking aliquot parts. Thus

	£	s.	d.	
	3	13	4	= value of 1 ton
			11	
4 cwt. = $\frac{1}{5}$ of 1 ton	40	6	8	= 11 tons
2 cwt. = $\frac{1}{5}$ of 4 cwt.	0	14	8	= 4 cwt.
2 qrs. = $\frac{1}{4}$ of 2 cwt.	0	7	4	= 2 cwt.
14 lb. = $\frac{1}{4}$ of 2 qrs.	0	1	10	= 2 qrs.
	0	0	5 $\frac{1}{2}$	= 14 lb.
	£41	10	11 $\frac{1}{2}$	= value of 11 tons 6 cwt. 2 qrs. 14 lb.

EXERCISE 91 a.

Find, by Practice, the value of

- | | |
|--|---|
| (1) 15 tons at 1s. 7 $\frac{1}{2}$ d. per cwt. | (2) $\frac{1}{2}$ cwt. at 2 $\frac{3}{4}$ d. per lb. |
| (3) $\frac{3}{4}$ cwt. at 4 $\frac{1}{2}$ d. per lb. | (4) 10 cwt. at 2 $\frac{1}{2}$ d. per lb. |
| (5) 1 $\frac{1}{2}$ cwt. at 11 $\frac{3}{4}$ d. per lb. | (6) 2 $\frac{1}{2}$ cwt. at 4 $\frac{3}{4}$ d. per lb. |
| (7) 4 $\frac{3}{4}$ cwt. at 8 $\frac{1}{2}$ d. per lb. | (8) 1 qr. 23 lb. at 3 $\frac{3}{4}$ d. per lb. |
| (9) 5 $\frac{1}{2}$ lb. at 2 $\frac{1}{2}$ d. per oz. | (10) 1 $\frac{1}{2}$ cwt. at 3 $\frac{1}{2}$ d. per oz. |
| (11) 3 $\frac{3}{4}$ qrs. at 3s. 11 $\frac{1}{2}$ d. per bush. | (12) 7 gall. at 4s. 6d. per qt. |
| (13) 3 $\frac{1}{2}$ gall. at 5 $\frac{1}{2}$ d. per pint. | (14) 5 gall. 3 qts. at 2 $\frac{1}{2}$ d. per qt. |
| (15) 55 yds. at 2s. per ft. | (16) 130 yds. at 8d. per ft. |
| (17) 4 cwt. at 17s. 6d. per ton. | (18) 14 lb. at £2. 4s. 4d. per cwt. |
| (19) 10 lb. 8 oz. at 10 $\frac{1}{2}$ d. per lb. | (20) 6 tons 10 cwt. at £1. 2s. per ton. |
| (21) 8 lb. 10 oz. at 4d. per lb. | (22) 9 oz. at 3s. 4d. per lb. |
| (23) 21 lb. at £2. 2s. per cwt. | (24) 15 cwt. at 12s. 6d. per ton. |
| (25) 4 lb. 2 oz. at 1s. 2d. per lb. | (26) 7 lb. 4 oz. at 11d. per lb. |
| (27) 14 lb. 12 oz. at 9d. per lb. | (28) 3 gall. 2 qts. at 1s. 9d. per gall. |
| (29) 10 gall. 3 qts. at 1s. 2d. per gall. | |
| (30) 36 qrs. 4 bush. at £1. 10s. 9d. per qr. | |
| (31) 16 qrs. 6 bush. at £1. 18s. 9d. per qr. | |
| (32) 9 ft. 6 in. at 2s. 3d. per ft. | (33) 12 yds. 1 ft. at £1. 16s. 9d. per yd. |
| (34) 3 sq. ft. at 5s. 9d. per sq. yd. | (35) 9 ac. 1 ro. at £2. 6s. per ac. |
| (36) 5 ac. 3 ro. at £3. 3s. 4d. per ac. | |

EXERCISE 91 b.

Find, by Practice, the value of

- (1) 12 tons 15 cwt. at £1. 5s. 6d. per ton.
- (2) 14 cwt. 3 qrs. at 18s. 4d. per cwt.
- (3) 8 lb. 12 oz. at 3s. 9d. per lb.
- (4) 15 lb. 6 oz. 12 drs. at 2s. 8d. per lb.
- (5) 2 cwt. 3 qrs. 7 lb. at £4. 15s. 8d. per cwt.

- (6) 13 cwt. 3 qrs. 4 lb. at £2. 18s. 4d. per cwt.
- (7) 3 tons 5 cwt. 2 qrs. 21 lb. at £12 per ton.
- (8) 10 tons 18 cwt. 56 lb. at £4. 17s. 6d. per ton.
- (9) 6 tons 13 cwt. 49 lb. at £7. 6s. 8d. per ton.
- (10) 1 ton 2 cwt. 2 qrs. 20 lb. at £9. 6s. 8d. per cwt.
- (11) 2 tons 10 cwt. 15 lb. 12 oz. at £2. 6s. 8d. per cwt.
- (12) 12 tons 8 cwt. 76 lb. at 7s. per cwt.
- (13) 7 cwt. 73 lb. 8½ oz. at 5s. 4d. per lb.
- (14) 3 cwt. 3 qrs. 14 lb. at 16s. 8d. per ton.
- (15) 3 cwt. 1 qr. 10½ lb. at £8 per ton.
- (16) 7 tons 13 cwt. 3 qrs. 26½ lb. at £9. 6s. 8d. per ton.
- (17) 8 oz. 12 dwt. 18 grs. at £3. 18s. 4d. per oz. (troy).
- (18) 8 oz. 6 dwt. 16 grs. at 5s. 6d. per oz. (troy).
- (19) 27 yds. 2 ft. 7 in. at 15s. 9d. per yard.
- (20) 3 miles 5 fur. 119 yds. at £5. 10s. per mile.
- (21) 19 miles 7 fur. 176 yds. at £12. 13s. 4d. per mile.
- (22) 80 miles 5 fur. 5 chains at £1. 3s. 4d. per mile.
- (23) 3 miles 8 chains 5½ yds. at £8. 13s. 4d. per mile.
- (24) 13 ac. 1 ro. 16 per. at £2. 3s. 4d. per acre.
- (25) 1 ro. 24 per. at £70. 11s. 8d. per acre.
- (26) 55 ac. 3 ro. 10 per. at 13s. 4d. per rood.
- (27) 25 sq. yds. 7 sq. ft. 126 sq. in. at £1. 10s. per sq. yard.
- (28) 18 cu. yds. 9 cu. ft. 486 cu. in. at £1. 16s. per cu. yard.
- (29) 3 cu. yds. 1 cu. ft. 216 cu. in. at 5s. 6d. per cu. foot.
- (30) 3 pks. 1 gall. 3 qts. at 2s. 6d. per peck.
- (31) 14 bush. 1 pk. 1 gall. 1 qt. at 9s. 4d. per bushel.
- (32) 12 qrs. 3 bush. 1 pk. at £1. 14s. 8d. per quarter.
- (33) 3 gall. 2 qts. 1 pt. at 18s. 6d. per gallon.
- (34) 12 barrels 27 gall. at £1. 10s. 8d. per barrel.
- (35) 13 weeks 2 days 6 hours at £2. 3s. 9d. per week.
(8 hrs.=1 day; 5 days=1 week.)
- (36) 6 months 3 weeks 4 days 7 hours at £1. 13s. per week.
(8 hours=1 day; 6 days=1 week; 4 weeks=1 month.)

MISCELLANEOUS EXAMPLES.

Practice (Simple and Compound).

EXERCISE 92.

- (1) Find the cost of 575 lb. of beef at 9s. 10d. per stone of 8 lb.
- (2) Find the value of 47365 cwt. of coal at 13s. 9½d. for 10 cwt.

- (3) Find the cost of a ton of tea at $2s. 5d.$ per lb
- (4) Find the value of 273 days' work at $\pounds 4. 11s. 7\frac{1}{2}d.$ per week of 6 days
- (5) Find the value of 68922 articles, sold at $7s. 3d.$ per packet, each containing 14 articles.
- (6) Find the cost of 28 parcels of cloth, each containing 116 yards, at $9s. 8d.$ per yard.
- (7) Find the value of 3 cwt. 2 qrs. 14 lb. of tea at $2s. 1\frac{1}{2}d.$ per lb.
- (8) What amount of wages will be paid in 318 days to 31 men, if each man receives $2s. 9d.$ per day?
- (9) What must I pay for 85700 bricks at $35s.$ per thousand?
- (10) When coal costs $11s. 6d.$ a ton, what will be the annual cost of fuel for an engine whose daily consumption is $2\frac{1}{2}$ tons?
- (11) A grocer buys half a ton of sugar for $\pounds 28. 10s.$, and sells it at $7\frac{1}{2}d.$ per lb.; what does he gain by the sale?
- (12) Find the cost of four trucks of potatoes, each containing 14 tons 5 cwt. 2 qrs. 24 lb., at $7s.$ per cwt.
- (13) Find the value of six farms, each containing 237 ac. 1 ro. 30 per., at $\pounds 43. 15s.$ per acre.
- (14) If I buy 3 tons 6 cwt. of tea at $\pounds 10. 14s. 8d.$ per cwt., and sell it at $2s. 3d.$ per lb., what do I gain by the transaction?
- (15) If a plot containing 3 ac. 3 ro. 25 per. of land is sold at $\pounds 120$ per acre, what is the value of the plot?
- (16) What is the cost of making a railway 17 miles 3 fur. 53 yds. long at $\pounds 3476$ per mile?
- (17) A grocer bought 4 cwt. 3 qrs. 14 lb. of sugar at $\pounds 1. 16s. 8d.$ per cwt., and retailed it at $4\frac{1}{2}d.$ per lb.; what did he gain or lose by the outlay?
- (18) Find a servant's wages for 7 months 3 weeks 6 days at $\pounds 1. 6s. 8d.$ per month, reckoning 7 days to a week and 4 weeks to a month
- (19) What is the value of a piece of gold plate weighing 10 oz. 8 dwt. 12 grs. at $\pounds 4. 2s. 6d.$ per oz.?
- (20) Find the value of six silver table-spoons and a dozen tea-spoons at $3s. 6d.$ per oz., if a table-spoon and a tea-spoon weigh 3 oz. 10 dwt. and 15 dwt. respectively.

INVOICES.

200. An **Invoice** is a written list of the articles bought, with a statement of the cost of them attached, sent by a seller to a buyer. Each separate entry in the list is called an **item**.

The following is the usual form of an Invoice:—

PORTSMOUTH, 22nd March, 1898.

MRS. MARSHALL

Bought of PARKER BROTHERS,
Drapers.

	£	s.	d.
5 yds. Muslin @ 1s. 2½d. per yard	0	6	0½
39 yds. Merino @ 4s. 3¼d. per yard	8	6	6½
3 doz. Buttons @ 4¾d. per dozen	0	1	2½
16½ yds. Sheeting @ 1s. 3½d. per yard	1	1	3½
	<u>£9 . 15 . 1½</u>		

EXERCISE 93.

Make out the following invoices, with names and dates:—

(1) 6 lb. tea at 2s. 6d. per lb.; 9 lb. rice at 3d. per lb.; 8 lb. currants at 4d. per lb.; 8 lb. coffee at 1s. 8d. per lb.; 10 lb. sugar at 2½d. per lb.

(2) 14 lb. beef at 9½d. per lb.; 10 lb. pork at 7½d. per lb.; 12 lb. mutton at 8½d. per lb.; 8 lb. veal at 9d. per lb.; 9 lb. lamb at 11d. per lb.

(3) 8 loaves at 2¾d. each; 11 lb. ginger-nuts at 8d. per lb.; 30 cheese cakes at ¾d. each; 13 bags flour at 3d. per bag; 7 loaves at 5d. each.

(4) 64 yds. sheeting at 1s. 4½d. per yd.; 20 yds. calico at 5¾d. per yd.; 15 yds. flannel at 1s. 6d. per yd.; 21 yds. velvet at 2s. 4¾d. per yd.; 12 yds. linen at 1s. 8¾d. per yd.

(5) 12 pairs blankets at 12s. 6¾d. per pair; 13 pairs worsted stockings at 1s. 9d. per pair; 36 pairs gloves at 3s. 11½d. per pair; 17 pairs muslin curtains at 9s. 11d. per pair; 9 pairs thread stockings at 8¾d. per pair.

(6) 17 locks at 1s. 1½d. each; 45 yds. paper at 7½d. per yd.; 30 pots paint at 3s. 10d. each; 130 yds. wire fencing at 2s. per yd.; 60 knives at 2s. 3d. each.

(7) 14 lamps at 8s. 6½d. each; 25 gallons oil at 1s. 1½d. per gallon; 400 boxes matches at ¼d. each; 34 pairs hinges at 3½d. per pair; 12 coal vases at 18s. 9d. each.

(8) Loin of veal, 18 lb., at 6½d. per lb.; sitchbone of beef, 12 lb., at 7½d. per lb.; sirloin of beef, 28 lb., at 8d. per lb.; leg of mutton, 10 lb., at 8½d. per lb.; saddle of mutton (New Zealand), 14 lb., at 5d. per lb.

(9) 6 dozen knives at 11s. 6d. per dozen; 4 sets of carvers at 29s. 6d. per set; 3 dozen plated forks at 1s. 11d. each; 2 dozen spoons at 6½d. each; 5 sets of fire-irons at 10s. 9d. per set.

(10) 5 dozen copy-books at 2d. each; 18 boxes steel pens at 1s. 5d. per box; 4 dozen slates at 5½d. each; 3 gross lead pencils at 1s. 9d. per dozen; 15 dozen account books at 15s. per dozen.

(11) Ham, 29 lb., at $8\frac{1}{2}d.$ per lb.; sirloin of beef, $18\frac{1}{2}$ lb., at $11d.$ per lb.; fillet of veal, 16 lb., at $7\frac{1}{2}d.$ per lb.; leg of mutton, 10 lb., at $10\frac{1}{2}d.$ per lb.; fore quarter of mutton, $14\frac{1}{2}$ lb., at $8\frac{1}{2}d.$ per lb.

(12) $16\frac{1}{2}$ yds. flannel at $1s. 2d.$ per yd.; $13\frac{1}{2}$ yds. cambric at $10s. 6d.$ per yd.; $15\frac{1}{2}$ yds. lace at $8s. 9d.$ per yd.; 12 pairs gloves at $3s. 4d.$ per pair; 50 yds. calico at $5\frac{1}{2}d.$ per yd.

(13) $10\frac{1}{2}$ lb. gingerbread at $6\frac{1}{2}d.$ per lb.; $19\frac{1}{2}$ lb. biscuits at $5d.$ per lb.; $5\frac{1}{2}$ lb. cake at $1s. 6d.$ per lb.; 56 lb. wheaten bread at $1\frac{3}{4}d.$ per lb.; 100 tarts at $9d.$ per dozen.

(14) $12\frac{1}{2}$ lb. coffee at $1s. 8d.$ per lb.; $18\frac{1}{2}$ lb. sugar at $5\frac{1}{2}d.$ per lb.; $7\frac{1}{2}$ lb. tea at $2s. 3d.$ per lb.; $29\frac{1}{2}$ lb. cheese at $7\frac{1}{2}d.$ per lb.; $11\frac{1}{2}$ lb. butter at $1s. 7d.$ per lb.

(15) 26 lemons at $\frac{3}{4}d.$ each; 90 pears at $1s. 4d.$ per doz.; 1000 oranges at $9d.$ per doz.; 36 apples, 3 to the lb., at $5\frac{1}{2}d.$ per lb.; 3 score lb. potatoes at 5 lb. for $3\frac{1}{2}d.$

(16) $10\frac{1}{2}$ yds. floor-cloth at $2s. 7\frac{1}{2}d.$ per yd.; 30 stair rods at $12s. 6d.$ per doz.; 84 lb. feathers at $1s. 2\frac{1}{2}d.$ per lb.; 9 yds. drugget at $1s. 11\frac{1}{2}d.$ per yd.; 15 yds. damask at $4s. 9d.$ per yd.

(17) 19 lb. butter at $1s. 1\frac{1}{2}d.$ per lb.; 300 eggs at $9d.$ per doz.; $1\frac{1}{2}$ cwt. cheese at $7d.$ per lb.; $\frac{1}{2}$ cwt. jam at $7\frac{1}{2}d.$ per lb.; 14 gall. milk at $2d.$ per pint.

(18) 153 reels thread at $11d.$ per doz.; 936 reels cotton at $15s.$ per gross, $17\frac{1}{2}$ yds. muslin at $6\frac{1}{2}d.$ per yd., 96 yds. braid at $3d.$ per piece of 12 yds.; $13\frac{1}{2}$ yds. velvet at $7s. 6d.$ per yd.

(19) 40 yds. tape at $6\frac{1}{2}d.$ per doz. yds., 375 yds. calico at $4s. 5d.$ per doz. yds.; 37 yds. chintz at $9\frac{1}{2}d.$ per yd.; 164 yds. fringe at $1s. 3\frac{1}{2}d.$ per doz. yds.; 2 dozen blankets at $16s.$ per pair.

(20) 100 Herrings at 2 for $1\frac{1}{2}d.$, 56 bunches smoked haddock at $28s.$ per doz. bunches; 9 lobsters at $4s. 3d.$ each; 176 lb. cod at $2\frac{1}{2}d.$ per lb., 15 lb. salmon at $1s. 10d.$ per lb.

(21) 8 lb. moist sugar at $\pounds 2. 2s.$ per cwt.; $\frac{3}{4}$ cwt. soap at $3s. 3d.$ per stone (14 lb.); $5\frac{1}{2}$ dozen lb. candles at $8\frac{1}{2}d.$ per lb.; 10 cwt. rice at $2\frac{1}{2}d.$ per lb.; $23\frac{1}{2}$ lb. tea at $2s. 6d.$ per lb.

(22) 500 nails at $3s. 4d.$ per thousand; $2\frac{1}{2}$ cwt. lead pipe at $4\frac{1}{2}d.$ per lb.; 130 yards wire fencing at $8d.$ per ft.; $7\frac{1}{2}$ gross screws at $3\frac{1}{2}d.$ per doz.; 51 gunlets at $5s. 9d.$ per doz.

(23) 6 lb. 12 oz. cake at $3s. 6d.$ per lb.; $5\frac{1}{2}$ doz. loaves at $6\frac{1}{2}d.$ each; 25 pots marmalade at $3s. 6d.$ per doz.; 282 biscuits at $7\frac{1}{2}d.$ per doz.; $3\frac{1}{2}$ cwt. flour at $2s. 8d.$ per stone (14 lb.).

(24) 30 boxes matches at $4\frac{1}{2}d.$ per score; 7 gall. 2 qts. oil at $1s. 9d.$ per gall; $2\frac{1}{2}$ cwt. putty at $4\frac{1}{2}d.$ per lb.; $5\frac{1}{2}$ doz. lb. candles at $8\frac{1}{2}d.$ per lb.; 9 gross wick at $2\frac{1}{2}d.$ per dozen.

(25) 10 lb. 8 oz. mutton at $11d.$ per lb.; $1\frac{1}{2}$ cwt. beef at $11\frac{1}{2}d.$ per lb.; 100 lb. pork at $\pounds 2. 16s.$ per cwt.; $8\frac{1}{2}$ lb. lamb at $1s. 1d.$ per lb., $7\frac{1}{2}$ lb. steak at $11\frac{1}{2}d.$ per lb.

(26) $\frac{1}{2}$ doz. pairs socks at 2s. 6d. per pair; 52 collars at 6s. 3d. per doz.; 3 doz. pairs gloves at 2s. 3d. per pair; $3\frac{1}{4}$ doz. handkerchiefs at 4d. each; 102 scarfs at 7s. 9d. per doz.

(27) 23 bush. coke at $4\frac{1}{2}$ d. per bush.; 19 gall. tar at $1\frac{3}{4}$ d. per gall.; 17 gas burners at 2s. $7\frac{1}{2}$ d. each; 3500 cubic feet gas at 3s. 8d. per 1000 ft.; hire of meter, 7 months, at 7s. 6d. per year.

(28) 31 lb. rice at $3\frac{1}{4}$ d. per lb.; $3\frac{1}{2}$ lb. tobacco at 4d. per oz.; 5 stones salt at 4s. 8d. per cwt.; $3\frac{1}{2}$ gall. vinegar at $5\frac{1}{2}$ d. per pint; 3 cwt. soap at $2\frac{1}{4}$ d. per lb.

(29) $11\frac{1}{2}$ yds. flannel at 1s. 5d. per yd.; 29 yds. edging at $2\frac{1}{4}$ d. per yd.; 37 yds. ribbon at 1s. $0\frac{1}{2}$ d. per yd.; $4\frac{1}{2}$ dozen yds. dress material at 4s. 11d. per yd.; 5 lb. wool at 3d. per oz.

(30) 10 gall. 3 qts. oil at 1s. 2d. per gall.; 10000 boxes matches at 3d. per doz.; 64 cakes blacking at $11\frac{1}{4}$ d. per doz.; $5\frac{1}{2}$ dozen lb. candles at $8\frac{1}{2}$ d. per lb.; $\frac{3}{4}$ cwt. soap at 3s. 3d. per stone (14 lb.).

SIMPLE PROPORTION.

201. In the solution of simple problems in which the idea of proportion is involved we will generally employ *a mode of expression*, which is called the **Unitary Method**. It will be seen from the following examples what the method is.

Example 1. If 5 yards of cloth cost 15s., what will be the cost of 13 yards?

$$\begin{aligned} \text{The cost of } 5 \text{ yards} &= 15s.; \\ \therefore \dots\dots\dots 1 \text{ yard} &= 3s.; \\ \therefore \dots\dots\dots 13 \text{ yards} &= 39s. \\ &= \underline{\underline{\pounds 1. 19s.}} \end{aligned}$$

Example 2. If 49 lb. of tea cost £6. 10s. 8d., what will be the cost of 25 lb.?

$$\begin{aligned} \text{The cost of } 49 \text{ lb.} &= \pounds 6. 10s. 8d.; \\ \therefore \dots\dots\dots 7 \text{ lb.} &= 18s. 8d.; \\ \therefore \dots\dots\dots 1 \text{ lb.} &= 2s. 8d.; \\ \therefore \dots\dots\dots 5 \text{ lb.} &= 13s. 4d.; \\ \therefore \dots\dots\dots 25 \text{ lb.} &= \underline{\underline{\pounds 3. 6s. 8d.}} \end{aligned}$$

EXERCISE 94.

- (1) If 10 eggs cost 1s. 3d., what is the price of a dozen?
- (2) If 16 oranges cost 1s., what is the price of a box containing 100?
- (3) If 13 tons of coal cost £19. 10s., what will be the cost of 7 tons?
- (4) If 50 packets of envelopes cost 12s. 6d., what will 121 packets cost?
- (5) If 6 yards of silk cost £1. 7s. 9d., what will 47 yards cost?
- (6) If 145 sheep cost £169. 3s. 4d., what is the price of a score at the same rate?

- (7) Find the cost of 37 yards of silk, when 25 yards cost £4. 7s. 6d.
 (8) If 37 yards of satin cost £8. 11s. 1½d., what will 152 yards cost?
 (9) If 100 copies of a book cost £27. 10s., what is the price of 29 copies of it?
 (10) If 85 men earn £153, how much will 64 men earn in the same time, and at the same rate of wages?

Example 3. If 2 cwt. 24 lb. of cheese cost £8. 5s. 4d., find the cost of 3 cwt. 10 lb.

By reduction, 2 cwt. 24 lb. = 248 lb.; 3 cwt. 10 lb. = 346 lb.;
 £8. 5s. 4d. = 1984d.

The cost of 248 lb. = 1984d.,

$$\therefore \dots\dots\dots 1 \text{ lb.} = \frac{1984d.}{248} = 8d.;$$

$$\therefore \dots\dots\dots 346 \text{ lb.} = 8d. \times 346 = 2768d. = 230s. 8d. \\ = \underline{\underline{£11. 10s. 8d.}}$$

- (11) If 4 cwt. of cheese cost £5. 16s. 8d., what is the cost of 7 tons 5 cwt.?
 (12) What is the cost of 3 cwt. 42 lb. of butter, if 33 lb. cost £1. 18s. 6d.?
 (13) What is the cost of 17 lb. of sweets, if 2 lb. 2 oz. cost 4s. 3d.?
 (14) If 2½ cwt. of salt cost £2. 18s. 4d., what will 42 lb. cost?
 (15) If 5½ cwt. of beef cost £24. 10s., what is the value of 97 lb.?
 (16) If 1 cwt. of tea cost £19. 12s., find the value of 4 cwt. 1 qr. 7 lb.
 (17) If 6 cwt. 1 qr. 10 lb. of sugar cost £11. 16s. 8d., what is the value of 2 qrs. 15 lb.?
 (18) If 4 cwt. 2 qrs. 7 lb. of coffee cost £36. 3s. 11d., what is the price of 3 qrs. 14 lb.?
 (19) If 1 cwt. 1 qr. 21 lb. of copper cost £4. 0s. 6d., what is the price per ton?
 (20) If 15 yards of netting cost £1. 13s. 9d., how much will 20 yards 1 foot cost?
 (21) Find the value of 3 bushels of corn, if 17 qrs. 1 bush. cost £32. 5s. 0½d.
 (22) If 1 bush. 2 pks. of wheat cost 7s. 3d., what must be given for 5 qrs. 3 bush.?

Example 4. If 6 articles cost £3. 16s. 6d., how many can be bought for £8. 5s. 9d.?

The cost of 6 articles = £3. 16s. 6d.;

$\therefore \dots\dots\dots 1 \text{ article} = 12s. 9d.;$

$$\therefore \text{number of articles worth £8. 5s. 9d.} = \frac{£8. 5s. 9d.}{12s. 9d.} \\ = \underline{\underline{13.}}$$

(23) If a dozen bottles of ale cost 6s. 9d., how many bottles could be bought for £8. 12s. 1½d.?

(24) How many lb. of coffee could I buy for £3. 3s., if I pay £1. 2s. 9d. for 13 lb.?

(25) If 87 yards of cloth cost £6. 3s. 3d., how many can be bought for £4. 5s.?

(26) If the wages of 14 men for a month be £143. 10s., how many men must work for the same time to receive £174. 5s.?

(27) A person in 87 days spends £38. 19s. 4½d.; in how many days will he spend £163. 9s. 9½d. at the same rate?

(28) If corn is sold at £1. 11s. 6d. for 7 bushels, how many quarters can be bought for £143. 11s.?

Example 5. How many lb. of sugar at 5d. per lb. are worth 36 lb. of coffee at 1s. 8d. per lb.?

The cost of 36 lb. of coffee = 20d. × 36 = 720d.,

and 1 lb. of sugar = 5d.;

$$\therefore \text{number of lb. of sugar worth 720d.} = \frac{720}{5} \\ = \underline{144.}$$

(29) How many books worth 4s. 6d. each are worth as much as 84 books at 7s. 6d. each?

(30) How many yards of cloth worth 19s. 3d. per yard must be given in exchange for 3 qrs. 21 lb. of cheese worth 11d. per lb.?

(31) How many yards of cloth at 1s. 10½d. per yard would be worth as much as 345 pairs of boots at 18s. per pair?

(32) A merchant exchanged 600 yards of silk, worth 11s. 3d. a yard, for 480 yards of velvet; what was the price to him of the velvet per yard?

(33) If 270 boxes of cigars, each box being worth 14s. 7d., are equal in value to 378 gallons of wine, what is the wine worth per gallon?

(34) A brewer receives 12 doz. of brandy in exchange for 5 barrels of ale, worth 78s. a barrel; what does the brandy cost him per bottle?

Example 6. If 19 men can complete a work in 437 days, in how many days will 23 men be able to do it?

The time required by 19 men = 437 days;

∴ 1 man = 19 × 437 days;

∴ 23 men = $\frac{19 \times 437}{23}$ days

= 361 days.

(35) If 25 men can do a piece of work in 24 days, in how many days will 15 men do it?

(36) If 18 men can do a piece of work in 10 days, in how many days will 45 men do the same work?

(37) How many men in 19 days would do a piece of work which 171 could do in 12 days?

(38) If 98 men can do a piece of work in 24 days, how many men could do it in 21 days?

(39) If 8 men would take 75 days to finish a piece of work, how many men would be needed to finish it in 40 days?

(40) If a gang of workmen can do a piece of work in 8 days, working 9 hours a day, how many hours a day must they work to do it in 6 days?

(41) If 63 oxen can be grazed in a field for 16 days, for how many days may 84 oxen be grazed in the same field?

(42) If 40 horses eat a certain quantity of hay in 9 days, how many days will the same amount of hay last 15 horses?

Example 7. If I borrow £112 for 12 months, for how long ought I to lend £42 in return?

The time for which £112 can be lent = 12 months;

∴ £1 = 12×112 months;

∴ £42 = $\frac{12 \times 112}{42}$ months

= 32 months

= 2 years 8 months.

(43) If I borrow £1800 for 4 months, for how many months ought I to lend £450 in return?

(44) I lent a friend £75 for 20 days; for how many days should he lend me £125 in return?

(45) A person lent me £150 for 8 months; what sum ought I to lend him for 12 months in return?

(46) I lent a friend £420 for 30 days; what sum ought he to lend me for 35 days in return?

(47) If £180 gain £10 in 8 months, what sum will gain the same amount in 3 months?

(48) If £375 gain £20 in 8 months, what sum will gain an equal sum in 12 months?

202. The following are examples of problems introducing fractions.

Example 1. If $\frac{3}{5}$ of a house is worth £210, what is the value of the whole house?

The value of $\frac{3}{5}$ of the house = £210;

∴ $\frac{1}{5}$ = £70;

∴ the whole house = £70 \times 5
= £350.

Example 2. If $\frac{3}{4}$ of a yard of silk cost 4s., what will $\frac{3}{2}$ of a yard cost?

The cost of $\frac{3}{4}$ yd. of silk = 4s.;
 \therefore $\frac{1}{4}$ yd. = 2s.;
 \therefore 1 yd. = 6s.;
 \therefore $\frac{3}{2}$ yd. = 6s. $\times \frac{3}{2}$
= 4s. 6d.

EXERCISE 95.

- (1) If $\frac{3}{4}$ lb. of tea costs 1s. 3d., what is the price per lb.?
- (2) What will 1 lb. of butter cost, if $\frac{5}{8}$ lb. cost 10d.?
- (3) If $\frac{5}{7}$ oz. of gold be worth £1 $\frac{1}{2}$, what is the value of an ounce?
- (4) If 3 $\frac{1}{2}$ tons of coal cost £4. 10s., what is the cost of a ton?
- (5) What is the value of a sack of flour, if 2 $\frac{1}{2}$ sacks cost £3. 6s.?
- (6) If $\frac{3}{8}$ of a property be worth £882, what is the value of $\frac{1}{4}$ of it?
- (7) If $\frac{1}{2}$ yard of silk cost 5s. 9 $\frac{1}{2}$ d., what is the cost of $\frac{3}{4}$ of a yard?
- (8) If 2 $\frac{1}{2}$ yards of silk cost £1. 15s., what will $\frac{1}{4}$ yard cost?
- (9) If 13 $\frac{5}{8}$ lb. of tea cost £1. 2s. 8 $\frac{1}{2}$ d., what is the cost of 2 $\frac{1}{4}$ lb.?
- (10) If 3 $\frac{1}{2}$ lb. of tea cost 8 $\frac{1}{2}$ s., what will 97 $\frac{3}{8}$ lb. cost?
- (11) A owns $\frac{3}{4}$ of a ship; if at the end of a year the total profits are £4523. 14s. 8d., what is A's share?
- (12) If $\frac{1}{2}$ of a share cost £440, what will 16 $\frac{1}{2}$ shares cost?
- (13) If 3 $\frac{3}{4}$ shares are worth £27. 10s., what are 4 $\frac{5}{8}$ shares worth?
- (14) If $\frac{1}{17}$ of a legacy amount to £3473. 17s. 9d., what is the value of $\frac{1}{17}$?
- (15) If 3 $\frac{3}{4}$ lb. of tea can be bought with $\frac{1}{8}$ of a sovereign, how many pounds can be bought with £11 $\frac{1}{4}$?
- (16) If 17 $\frac{7}{8}$ yds. of muslin cost £1. 6s. 9 $\frac{3}{4}$ d., how many yards can be purchased for £3. 2s. 3d.?
- (17) If $\frac{3}{4}$ of an acre of land is let for £2. 5s. a year, how many acres should be let for £23. 2s.?
- (18) If an engine consume 52 $\frac{4}{10}$ tons of coal in 29 $\frac{7}{10}$ days, how many days would 91 $\frac{5}{8}$ tons last?
- (19) If 2.25 yds. of cloth cost 3.75s., what will be the cost of 13.625 yds.?
- (20) If .48 of an estate be worth £721.25, what is the value of .3 of the estate?
- (21) If 75.75 acres can be rented for £15. 3s., what will be the rent of 168.375 acres of similar land?
- (22) If .4 of a rood of land cost £.85, what will be the cost of .04 of an acre?
- (23) If 2.625 tons of salt cost £36.75, find the cost of 1.5 lb.
- (24) If 3.5625 lb. of tea cost £.675, what should be paid for a chest of similar tea weighing 2 qrs. 20 lb.?

COMPOUND PROPORTION.

203. In the preceding examples we have found the change in *one* quantity corresponding to the change in *one* other. Sometimes however *three* quantities may be involved, two of which change in a given way, and we have to find the consequent change in the third.

In the first line of each of the following examples we re-state the *data*, so that the quantity whose change is to be determined may come last. This re-arrangement should always be made.

Example 1. If 16 horses can plough 1280 acres in 8 days, how many acres can 12 horses plough in 5 days?

Here we have to find the change in the number of acres which corresponds to a change in the number of horses from 16 to 12, and in the time from 8 days to 5 days.

In 8 days 16 horses can plough 1280 acres,			
\therefore	1 day 16	160	
\therefore	5 days 16	800	
\therefore	5 4	200	
\therefore	5 12	600	
\therefore the number of acres = <u>600</u> .			

In the second and third lines we have changed the time from 8 days to 5 days, without changing the number of horses; and in the last two lines we have changed the number of horses from 16 to 12.

Example 2. If 15 pumps working 8 hours a day can raise 1260 tons of water in 7 days, how many pumps working 12 hours a day will be required to raise 7560 tons of water in 14 days?

Here we have to change the number of tons of water from 1260 to 7560, and the number of hours from 7×8 to 12×14 , and to find the corresponding change in the number of pumps required.

1260 tons can be raised in 56 hours by 15 pumps,			
\therefore 7560	4×14	90	
\therefore 7560	12×14	80	
\therefore the number of pumps required = <u>30</u> .			

EXERCISE 96.

- (1) If 120 men can build a house 60 ft. high in 15 days, how many men will it take to build one 55 ft. high in 10 days?
- (2) If 9 men reap a field of 8 acres in 12 hours, how many men will reap a field of 28 acres in 18 hours?
- (3) If 32 horses eat 9 bushels of corn in 21 days, for how many days will 60 bushels feed 7 horses?

(4) If 3 fires consume 65 cwt. of coal in 26 days, in how many days will 12 fires consume 4 tons?

(5) If 6 men can do a piece of work in 30 days of 9 hours each, how many men will it take to do 10 times the amount, if they work 25 days of 8 hours each?

(6) If 27 men mow a field of 90 acres in 7 days, working 8 hours a day, how many men will be required to mow 200 acres in 16 days if they work 10 hours a day?

(7) If 3 men can reap 8 acres in 5 days, working 8 hours a day, in how many days can 8 men, working 12 hours a day, reap 192 acres?

(8) If the wages of 4 men for 12 days be £6, what would be the wages of 6 men for 10 days?

(9) If a family of 7 persons can live on £140 for 28 weeks, how long can a family of 9 persons live on £135?

(10) The wages of 5 men for 6 weeks being £14. 5s., how many weeks will 4 men work for £19?

(11) What is a quarter's rent of 350 acres of land, if £11. 5s. 9d. per annum be given for 9 acres?

(12) If 25 horses can be kept 13 days for £32. 10s., how many can be kept 6 weeks for 20 guineas?

(13) If the wages of 29 men for 54 days amount to £80. 9s. 6d., how many men must work 12 days to receive £407?

(14) A person completes a journey of 1056 miles in 12 days, travelling 11 hours a day; in how many days would he travel 480 miles, going for 6 hours a day at the same rate?

(15) If 200 men can make an embankment 5 miles long in 25 days, how much overtime must 60 men work to finish an embankment 2 miles long in 32 days, 12 hours being an ordinary day's work?

(16) If the cost of printing a book of 320 pages, with 21 lines on each page, and on an average 11 words in each line, be £19, find that of printing a book with 297 pages, 28 lines on each page and 10 words in each line.

(17) If 200 men in 12 days of 8 hours each can dig a trench 160 yds. long, 6 yds. wide, and 4 yds. deep, in how many days of 10 hours will 90 men dig a trench 450 yds. long, 4 yds. wide, and 3 yds. deep?

(18) If 18 men working 9 hours a day can dig a trench 108 ft. long, 8 ft. broad, and 12 ft. deep in 21 days, how many days of 10 hours must 24 men work to dig a trench 120 ft. long, 9 ft. wide, and 16 ft. deep?

(19) If 5 men working 8 hours a day dig a trench 105 ft. long, 6 ft. wide, and 1 foot deep in 3 days, how many hours a day must 32 men work in order to dig a trench 126 ft. long, 22 ft. wide, and 10 ft. deep in 15 days?

(20) A man walking 8 hours a day at a uniform speed walked 780 miles in 26 days; in how many days, walking 10 hours a day at the *same speed*, could he walk 450 miles?

(21) If 16 men can do a piece of work in $8\frac{1}{2}$ days, working 5 hours a day, how many men working 11 hours a day can do twice as much in 10 days?

(22) If 2 men can earn £10 in 4 weeks by working 6 days a week, how much should 5 men earn in 16 days?

(23) If the rent of 14 houses for 5 weeks amounts to £36. 15s., what will be the total rent of 35 houses of the same kind for 52 weeks?

(24) The freight for goods on a railway being at the rate of 25s. per ton for 120 miles, what should be the charge for 168 lb. carried 288 miles?

(25) If 3 tons are carried 54 miles for £2. 0s. 6d., how many miles can 11 tons be carried for £5. 10s.?

(26) If 12 men earn £75. 12s. in 12 days working 9 hours a day, how many hours a day must 42 men work in order to earn £205. 16s. in the same number of days?

(27) If £7. 17s. 6d. be paid for the use of a field for 4 months, for how many months should a field of the same quality but of double the size be leased for 30 guineas?

(28) If the wages of 11 labourers for 15 weeks be £103. 2s. 6d., in how many weeks will the wages of 13 labourers amount to £170. 12s. 6d. at the same rate per week?

(29) A railway company charges each of a number of excursionists two-fifths of the ordinary fare; the ordinary fare is 6s. 3d. for 50 miles; how much should 250 excursionists pay to travel 120 miles?

(30) If 15 carts, making a journey every 40 minutes, take 8 days of 12 hours each to remove a certain number of bricks, how many carts of double the capacity, making a journey every hour, would be required to remove the same number of bricks in 5 days of 9 hours each?

PROPORTIONATE DIVISION.

204. When we know the proportion between the parts of a given quantity, however many the parts may be, we can easily determine the parts themselves.

Example 1. Divide 35s. between Tom and Dick, so that Tom may have 3 shares and Dick 4 shares.

We can do this by dividing 35s. into (3 + 4) equal parts, i.e., 7 equal parts, and taking 3 of them for Tom and 4 of them for Dick.

Each equal part — $35s. \div 7 = 5s.$;

\therefore Tom's share — $5s. \times 3 = 15s.$,

and Dick's share — $5s. \times 4 = 20s.$

Example 2. Divide 3 guineas between 3 boys and 2 girls, giving each girl twice as much as each boy.

$$\begin{aligned}
 &1 \text{ girl's share} = 2 \text{ boys' shares;} \\
 &\therefore 2 \text{ girls' shares} = 4 \text{ boys' shares;} \\
 &\therefore 3 \text{ boys' shares} + 4 \text{ boys' shares} = 63s.; \\
 &\therefore 7 \text{ boys' shares} = 63s.; \\
 &\therefore 1 \text{ boy's share} = \underline{9s.}, \\
 &\text{and } 1 \text{ girl's share} = \underline{18s.}
 \end{aligned}$$

Example 3. Divide £6. 5s. between 5 boys and 4 girls, giving each boy 2s. 6d. more than each girl.

It is evident that as the 5 boys will receive 12s. 6d. more than the girls, this amount must be first taken away, and the result of dividing the remainder into 9 equal parts will give the share of each girl.

$$\begin{aligned}
 \text{The share of each girl} &= \frac{1}{9} \text{ of } (£6. 5s. - 12s. 6d.) \\
 &= \frac{1}{9} \text{ of } £5. 12s. 6d. \\
 &= \underline{12s. 6d.};
 \end{aligned}$$

$$\text{and the share of each boy} = \underline{15s.}$$

Example 4. A bill of £19. 16s. 6d. was paid by giving a certain number of half-crowns and twice as many florins; how many of each coin were there?

$$\begin{aligned}
 &1 \text{ half-crown} + 2 \text{ florins} = 2s. 6d. + 4s. = 6s. 6d.; \\
 \therefore \text{the number of half-crowns} &= \frac{£19. 16s. 6d.}{6s. 6d.} \\
 &= \frac{396\frac{1}{2}}{6\frac{1}{2}} \\
 &= \frac{793}{13} \\
 &= \underline{61}; \\
 \text{and the number of florins} &= \underline{122.}
 \end{aligned}$$

EXERCISE 97.

(1) Divide £5. 12s. 6d. between two boys, giving to one thrice as much as to the other.

(2) A yacht and its fittings cost £2011. 4s.; the yacht itself cost five times as much as the fittings; what was the cost of the fittings?

(3) A sum of £2228. 16s. was left to a boy and two girls; the boy was to have 3 parts, and each daughter 2 parts; what did each receive?

(4) Divide £35. 5s. 1½d. among A, B and C, so that B may get twice, and C three times as much as A.

(5) Three persons, A, B, C, agree to pay their hotel bill in the proportions of 4, 5, and 6; if the bill amounts to £10, what is the share of each?

(6) Two hundred and seven apples were divided among three boys; the first gets five times as many as the third, and the second gets three times as many as the third; how many did each boy get?

(7) Divide £3798 among three brothers, so that the eldest may have three times as much as the youngest, and the youngest half as much as the second.

(8) Divide £97. 7s. 2d. among Tom, Dick and John, so that Tom may get twice as much as Dick, and Dick twice as much as John.

(9) Divide £10. 10s. among 5 men and 6 women, so that each man may have three times as much as each woman.

(10) Divide £5049 between two persons, so that one shall have £155 more than the other.

(11) Divide £95. 15s. 7d. between two persons, giving one £3. 14s. 2d. more than the other.

(12) Twenty guineas have to be divided among three persons; the second gets 17s. more than the first, and the third gets 26s. more than the second; how much does each get?

(13) A bag contains £73. 7s., made up of equal numbers of shillings and sixpences; how many are there of each coin?

(14) A bill of £13. 17s. 6d. was paid by giving a certain number of sixpences and twice as many shillings; how many of each coin were there?

(15) A bill of £8. 3s. 4d. was paid with a certain number of sovereigns, three times as many shillings, and four times as many pennies; find the number of each coin used.

(16) A bag contains £18. 15s. 10d., made up of crowns, shillings and pennies, there are three times as many crowns as pennies and twice as many shillings as pennies; find the number of each coin.

(17) A bag contains 10 guineas in half-crowns, florins, and shillings; there are twice as many florins and three times as many shillings as half-crowns; find how many of each coin the bag contains.

(18) An equal number of postmen and telegraph messengers received £109. 4s. for 4 weeks' services; the men were paid 30s. a week, and the boys had 15s. 6d.; how many were there of each?

(19) Divide $2\frac{1}{2}$ cwt. of wood between 10 men and 8 women, giving to each man twice the weight given to each woman.

(20) Divide 3 cwt. of cake between 60 boys and 104 girls, giving to each boy twice as much as to each girl.

BANKRUPTCIES.

205. A man becomes bankrupt when the money which he owes is more than that which he possesses.

What he owes is called his **liabilities**, or **debts**; what he possesses is called his **assets**, or his **estate**.

He is the **debtor**; those to whom he owes anything are his **creditors**.

Each creditor is supposed to receive the same fraction of the assets that the money due to him is of the debtor's whole liabilities.

If the assets amount, for example, to $\frac{2}{3}$ of the debts, each creditor receives $\frac{2}{3}$ for each £1 that is due to him; and the debtor is said to pay a dividend of 13s. 4d. in the £.

Example 1. If a bankrupt's debts amount to £1500, and his assets to £950, how much in the £ can he pay?

$$\begin{aligned} \text{Dividend on a debt of } £1500 &= £950, \\ \therefore \dots\dots\dots £1 &= £\frac{950}{1500} = £\frac{19}{30} = 152d., \\ \therefore \dots\dots\dots £1 &= \underline{12s. 8d.} \end{aligned}$$

Example 2. If a bankrupt pays 12s. 8d. in the £, how much will a creditor receive to whom he owes £190?

$$\begin{aligned} \text{Dividend on debt of } £1 &= £\frac{19}{30}, \\ \therefore \dots\dots\dots £190 &= £\frac{19}{30} \times 190 \\ &= £\frac{361}{3} \\ &= \underline{£120. 6s. 8d.} \end{aligned}$$

NOTE.—Such examples as *Example 2* can also be worked by Practice.

EXERCISE 98.

(1) If a bankrupt's liabilities amount to £1875, and his assets to £910. 3s. 1½d., how much in the £ can he pay?

(2) A bankrupt, whose effects are worth £1181. 12s. 6d., owes £4726. 10s.; how much in the £ can he pay?

(3) A bankrupt fails for £12500, and his estate realises only £3906. 5s.; what dividend does he pay, and what will a creditor receive whose claim is for £798. 10s.?

(4) A bankrupt pays 13s. 4d. in the £; how much will a creditor receive to whom he owes £3276. 15s.?

(5) A bankrupt paid 4s. 4½d. in the £; what was the loss on a claim of £865. 4s.?

(6) A bankrupt pays a dividend of 11s. 5d. in the £; what do his creditors lose on debts amounting to £4360?

(7) A bankrupt, whose liabilities are £2480, has assets consisting of goods worth £1077. 5s., and book-debts which amount to £930, but are valued at only 17s. 6d. in the £; what dividend in the £ will he be able to pay?

(8) A bankrupt owes £5174. 15s., including £1552. 8s. 6d. for rent, taxes and wages, which have to be paid in full. His assets being £4269. 3s. 4½d., what dividend will his other creditors receive?

(9) A bankrupt, whose assets are worth £227, owes his creditors £185, £301, and £422 respectively; how much will each receive?

(10) A bankrupt, whose estate is worth £201, owes his creditors £133, £212, and £325 respectively; what will each receive?

RATES AND TAXES.

206. Rates and taxes are reckoned at so much in the pound.

If, for example, the income-tax is 8*d.* in the £, a tax of 8*d.* has to be paid on each £1 of income, i.e., $\frac{1}{20}$ of an income has to be paid to the Government.

The value of a person's property, as estimated by the authorities, is called the **assessment**, or **rateable value**, of the property.

Example 1. A corporation needs £416 13*s.* 4*d.* for public purposes; the assessment being £5000, find how much in the £ the rate will be.

On an assessment of £5000 the rates—£416. 13*s.* 4*d.*;

$$\begin{array}{r} \therefore \dots\dots\dots \text{£1} \dots\dots\dots \\ \qquad \qquad \qquad \text{£416. 13*s.* 4*d.*} \\ \qquad \qquad \qquad \hline \qquad \qquad \qquad 5000 \\ \qquad \qquad \qquad = \frac{8333\frac{1}{4}}{5000} \text{ *s.* } = 1\frac{16}{1000} \text{ *s.* } = \frac{1}{62} \text{ *s.* } \\ \qquad \qquad \qquad \hline \qquad \qquad \qquad \text{—1*s.* 8*d.*} \end{array}$$

Example 2. Find the tax, at 10*d.* in the £, on property assessed at £760.

The tax on assessment of £1 = £ $\frac{1}{20}$;

$$\therefore \dots\dots\dots \text{£760} = \text{£}\frac{1}{20} \times 760 = \text{£}\frac{1}{4} \times 95 \\ \qquad \qquad \qquad \hline \qquad \qquad \qquad \text{—£31. 13*s.* 4*d.*}$$

EXERCISE 99.

(1) A school board needs £715. 12*s.* 6*d.* for purposes of education; the assessment being £13740, find how much in the £ the rate will be.

(2) The rateable value of a parish being £5676. 13*s.* 4*d.*, and £745. 1*s.* 3*d.* being required for public purposes, how much in the £ will the rate be?

(3) If a man has to pay £18. 15*s.* as income-tax on an income of £1500, how much in the £ is the tax?

(4) Find the amount of the local rates at 13*d.* in the £ on property assessed at £16241. 5*s.*

(5) What is the amount of rates to be paid on an assessment of £74. 10*s.* at 3*s.* 4*d.* in the £?

(6) How much will a poor-rate of 2*s.* 8*d.* in the £ produce in a parish where the rateable value of the whole property is £4736. 5*s.*?

(7) A man pays a district-rate of 1*s.* 6*d.* in the £ on his rental, a water-rate of 1*s.* in the £, and a poor-rate of 1*s.* 10½*d.* in the £, if the rent and taxes amount to £85. 6*s.* 3*d.*, what is the rent?

(8) After paying income tax at the rate of 4*d.* in the £, a man has £491 13*s.* 4*d.* remaining; what was his gross income?

(9) The taxes paid by a certain householder amount to £11 15*s.* 7½*d.* at 3*s.* 7½*d.* in the £; what is the rent of his house?

(10) If a tax on £75 is £4. 10*s.* 7½*d.*, how much should it be on £48?

PERCENTAGES.

207. In many cases we express the fraction which one quantity is of another by a symbol which has 100 for its denominator.

The first quantity is thus estimated in *hundredths*, or as a **percentage**, of the other.

For example, $3s. = \frac{3}{20}$ of $\pounds 1 = \frac{15}{100}$ of $\pounds 1$;

i.e., 3s. is 15 *hundredths*, or 15 **per cent.**, of $\pounds 1$.

Also 6 pence $= \frac{1}{40}$ of $\pounds 1 = \frac{2\frac{1}{2}}{100}$ of $\pounds 1$;

i.e., 6 pence $= 2\frac{1}{2}$ *per cent.* of $\pounds 1$.

208. The *numerator*, which expresses the number of hundredths, is called the **rate per cent.** For example, in the cases just considered the *rates per cent.* are 15 and $2\frac{1}{2}$.

The words *per cent.* are expressed briefly by the symbol $\%$, or by the letters p.c. Thus $3\frac{1}{2}$ *per cent.* may be written $3\frac{1}{2}\%$, or $3\frac{1}{2}$ p.c.

EXERCISE 100.

What fractions are denoted by the following rates per cent.?

- | | | | | |
|-----------------------|-----------------------|---------------------|---------------------|----------------------|
| (1) 3. | (2) 5. | (3) 40. | (4) 75. | (5) $2\frac{1}{2}$. |
| (6) $12\frac{1}{2}$. | (7) $33\frac{1}{3}$. | (8) $\frac{1}{2}$. | (9) $\frac{1}{4}$. | (10) $\frac{1}{5}$. |

EXERCISE 101.

What rates per cent. do the following fractions denote?

- | | | | | |
|----------------------|----------------------|----------------------|----------------------|----------------------|
| (1) $\frac{1}{2}$. | (2) $\frac{1}{5}$. | (3) $\frac{1}{20}$. | (4) $\frac{1}{25}$. | (5) $\frac{1}{40}$. |
| (6) $\frac{1}{12}$. | (7) $\frac{1}{30}$. | (8) $\frac{2}{5}$. | (9) $\frac{7}{10}$. | (10) $\frac{3}{4}$. |

209. CASE I. To find the value of a percentage of a given quantity.

To do this we find the corresponding vulgar or decimal fraction of it.

Example. Find the value of $12\frac{1}{2}$ per cent. of $\pounds 1$.

$$\begin{aligned}
 12\frac{1}{2} \text{ per cent. of } \pounds 1 &= \frac{12\frac{1}{2}}{100} \text{ of } \pounds 1 = \frac{25}{200} \text{ of } \pounds 1 \\
 &= \frac{1}{8} \text{ of } \pounds 1 \\
 &= \underline{\underline{2s. 6d.}}
 \end{aligned}$$

EXERCISE 102.

Find the value of the following percentages:—

- | | |
|--|---|
| (1) 5 per cent. of $\pounds 20$, $\pounds 70$, $\pounds 150$. | (2) 3 per cent. of $\pounds 50$, $\pounds 150$, $\pounds 200$. |
| (3) $2\frac{1}{2}$ per cent. of $\pounds 10$, $\pounds 40$, $\pounds 60$. | (4) 8 per cent. of $\pounds 50$, $\pounds 100$, $\pounds 75$. |

- (5) 10 per cent. of £20, £80, £175. (6) 20 per cent. of £10, £50, £150.
 (7) $12\frac{1}{2}$ per cent. of £460, £750. (8) 3 per cent. of £123, 6s. 8d.
 (9) 3 per cent. of 7 lb 13 oz. (10) 5 per cent. of 3 tons 17 cwt 21 lb.
 (11) $6\frac{1}{2}$ per cent. of 2 qrs. 14 lb. (12) 45 p. c. of 12 cwt. 2 qrs. 20 lb.
 (13) $2\frac{1}{2}$ per cent. of £1. 13s. 4d. (14) $5\frac{1}{2}$ per cent. of £20. 8s. 4d.
 (15) $7\frac{1}{2}$ per cent. of £52. 10s. (16) 17 per cent. of £7. 3s. 9d.
 (17) 5s. per cent. of £1780. (18) 2s. 6d. per cent. of £1520.
 (19) 4s. per cent. of £1475. (20) 7s. 6d. per cent. of £2880.
 (21) 15s. per cent. of £1850. (22) 1s. 6d. per cent. of £1950.
 (23) 1s. 8d. per cent. of £3330. (24) 3s. 9d. per cent. of £2670.

210. CASE II. To find how much per cent. one quantity is of another.

We express the former quantity as a vulgar fraction of the latter, and we convert this fraction to another having 100 for its denominator.

Example. What per cent. of £62. 10s. is £16?

$$\begin{aligned} \text{The fraction} &= \frac{£16}{£62\frac{1}{2}} = \frac{32}{125} = \frac{256}{1000} = \frac{256}{1000} \times \frac{100}{100} = \frac{256}{10} = 25\frac{6}{10} \\ &\therefore \text{rate per cent.} = 25\frac{6}{10} \text{ or } \underline{25\cdot6}. \end{aligned}$$

EXERCISE 103.

What per cent.

- (1) of £28 is £14? (2) of £35 is £7?
 (3) of 75 lb. is 45 lb.? (4) of £7. 10s. is £5?
 (5) of £6260 is £313? (6) of £180 is £6. 15s.?
 (7) of £450 is £50. 12s. 6d.? (8) of £637. 10s. is £19. 2s. 6d.?
 (9) of £402 is £2 10s. 3d.? (10) of £1040. 17s 6d is £312. 5s. 3d.?
 (11) of £62. 10s. is £9. 7s. 6d.? (12) of £27. 10s. is £3. 6s.?
 (13) of £250 is £8. 15s.? (14) of £650 is £40. 12s. 6d.?

How much per cent. is

- (15) 2d. in the 1s.? (16) 3d. in the 1s.? (17) 6d. in the £1?
 (18) 1s in the £1? (19) 2s. 6d. in the £1? (20) 3s. 4d. in the £1?

211. CASE III Given the *percentage* and also the *rate per cent.*, to find the original quantity.

We multiply the percentage by the inverse of the vulgar fraction which the rate per cent. denotes.

NOTE.—Notice the distinction between *percentage* and *rate per cent.* If, for example, £20 be the given quantity and 10 the *rate per cent.*, the *percentage* is £2.

Example. Find the amount of which £18 is 24 per cent.

$$£18 = \frac{24}{100} \times \text{the required amount};$$

$$\therefore \text{the required amount} = £\frac{100}{24} \times 18 = £\frac{300}{4} = \underline{\underline{£75.}}$$

EXERCISE 104.

Of what amount is

- | | |
|-----------------------------------|-------------------------------------|
| (1) £96, 6 per cent.? | (2) £960, 33½ per cent.? |
| (3) £114, 4½ per cent.? | (4) £7. 16s., ¾ per cent.? |
| (5) £117. 15s., 3½ per cent.? | (6) £103. 2s. 6d., ¾ per cent.? |
| (7) £2. 10s. 3d., ⅝ per cent.? | (8) £45. 7s. 3½d., ⅓ per cent.? |
| (9) £3. 13s. 9d., ⅓ per cent.? | (10) £1. 13s. 4½d., ⅓ per cent.? |

MISCELLANEOUS EXAMPLES.

Percentages.

EXERCISE 105.

(1) A man died and bequeathed £4275 to his wife and two children. His wife received 33½ per cent. of the whole; what amount did each child get?

(2) A house is bought for £2425. Twenty per cent. of the money is paid at once; how much remained?

(3) How many persons are engaged in agriculture, if they constitute 24 per cent. of a population of 30725?

(4) A cask contained 42 gallons, of which 14 gallons escaped; what per cent. was this loss?

(5) A car proprietor had 160 horses in 1897, and 172 in 1898; what was the increase per cent.?

(6) The population of a town was 5600 in 1881, and 4802 in 1891; what per cent. was the decrease?

(7) A man spends 65 per cent. of his income, and saves £434; what is the amount of his income?

(8) If 8½ per cent. of the workpeople in a factory is 792, find the number employed.

(9) If a grocer mixes 12 cwt. 3 lb. of chicory with 3 tons 8 cwt. 17 lb. of coffee; what percentage of the whole is chicory?

(10) It requires £264. 7s. 6d. to pay one week's wages at a factory; if the wages increase 5 per cent., what amount will then be required?

(11) A box containing 500 oranges is broken and 32 per cent. of its contents stolen; how many oranges are left in the box?

(12) Of the oranges in a case 7½ per cent. are unsound and 370 are good; how many oranges does the case contain?

(13) In a town formerly of 252000 inhabitants, 231000 are now living; what is the decrease per cent.?

(14) The population of a town in 1887 was 15780, and in 1897 it was 22092; what was the increase per cent.?

(15) If $2\frac{1}{2}$ per cent. of 4250 tons of coal be lost in carting, what quantity is left?

(16) A cask containing originally 630 gallons was found after a time to contain only 525 gallons; what percentage had been drawn off?

(17) If I pay $2\frac{1}{2}$ per cent. of a bill of £83. 12s. 6d., how much do I still owe?

(18) A poulterer bought 350 birds, of which 119 were ducks and the rest geese; what percentage of the whole number were geese?

(19) A man's income is £980. 10s., of which £20. 8s. 6½d. is paid as income tax; what percentage of his income is this?

(20) A dealer sells a sack of coals weighing only 196 lb and charges for 224 lb.; of how much per cent. does he defraud his customer?

COMMISSION, BROKERAGE, PREMIUM, AND DISCOUNT FOR CASH.

212. When a person employs an agent to buy or sell property of any kind he pays the agent for his trouble by giving him a *percentage* on the value of the property bought or sold.

The money thus paid to the agent is called his **commission**.

The agent is sometimes called a **broker**, and his commission is sometimes called **brokerage**.

In the case of an *insurance company*, which, in return for certain payments, undertakes to make good a loss from fire or shipwreck, or to pay a certain sum of money after a man's death to his representatives, the person who insures his property or his life pays annually to the company a certain percentage of the sum insured. The percentage is in this case called a **premium**. The agreement is called a **policy**.

A percentage allowed for ready-money payment for goods bought is called **discount for cash**.

Commission, Brokerage, Premium and Discount for Cash are therefore names given to a percentage in particular cases.

Example 1. An estate agent buys a house for £5250, and receives a commission of 5s. per cent.; what does he get?

$$\begin{aligned}\text{Commission} &= \frac{5}{100} \times £5250 = £ \frac{5250}{40} = £13\frac{1}{4} \\ &= \underline{\underline{£13. 2s. 6d.}}\end{aligned}$$

Example 2. A gentleman pays an annual premium of £2. 14s. per cent. on a life insurance policy for £12550; what does his annual payment amount to?

$$\begin{aligned}\text{Annual payment} &= \text{£} \frac{2\frac{14}{20}}{100} \times 12550 = \text{£} \frac{27}{100} \times 1255 \\ &= \text{£} \frac{27 \times 251}{20} = 6777 \text{ shillings} \\ &= \underline{\text{£}338. 17s.}\end{aligned}$$

EXERCISE 106.

(1) A broker purchases £20000 worth of goods; what is his commission at $\frac{1}{4}$ per cent.?

(2) An agent collects debts to the amount of £481. 10s.; what is his commission at $2\frac{1}{2}$ per cent.?

(3) A broker buys £11000 worth of goods; what is his commission at $\frac{7}{8}$ per cent.?

(4) What is the cost of insuring a cargo valued at £6750, the premium being 3 per cent.?

(5) Find the commission on the sale of 736 barrels of flour for £1. 5s. per barrel, the rate of commission being $2\frac{1}{2}$ per cent.

(6) An agent received 5 per cent. for buying cotton; his commission amounted to £432. 10s.; what was the value of the cotton bought?

(7) A ship is worth £34000, and is insured for $\frac{3}{4}$ of its value; what does the premium amount to at 3 per cent.?

(8) A house is insured for $\frac{3}{4}$ of its value at $1\frac{1}{2}$ per cent., and the premium is £18; what is the value of the house?

(9) A tradesman allows a discount of 15 per cent. for cash; find the cash prices of articles marked respectively (i) £2. 15s., (ii) £5. 17s. 6d., (iii) £10. 12s. 6d.

(10) Instead of allowing 4 per cent. discount for cash, a tradesman gives $\frac{1}{2}$ d. in the shilling. Find the amount of his error on a bill of £30. 12s. 6d.

PROFIT AND LOSS.

213. When a thing is sold for more than it cost, it is said to be sold **at a profit**: when it is sold for less, it is said to be sold **at a loss**.

Profit or loss is often expressed as a *percentage* of the money originally expended.

For example, a man who buys goods for £100, and sells them for £110, makes a *profit* of £10 on an outlay of £100, i.e., a profit of 10 per cent.

And if he buys them for £100 and sells them for £90, he *loses* £10 on an outlay of £100, i.e., he loses 10 per cent.

It must be noticed carefully that the percentage is reckoned on the outlay, and not on the amount for which the goods are sold.

214. CASE I. Given the cost price and the selling price; to find the gain or loss per cent.

Example 1. A grocer buys coffee at 10d. per lb., and sells it at 1s. 4d. per lb.; what is the gain per cent.?

$$\begin{aligned}\text{Gain on 10d.} &= 6d.; \\ \therefore \text{gain on 1d.} &= \frac{6}{10}d.; \\ \therefore \text{gain per cent.} &= \frac{6}{10} \text{ of } 100 \\ &= 60.\end{aligned}$$

Example 2. A grocer sells tea at 1s. 6d. per lb. which cost him 2s. per lb.; what is the loss per cent.?

$$\begin{aligned}\text{Loss on 24d.} &= 6d.; \\ \therefore \text{loss on 1d.} &= \frac{6}{24}d.; \\ \therefore \text{loss per cent.} &= \frac{6}{24} \text{ of } 100 \\ &= 25.\end{aligned}$$

EXERCISE 107.

What is the gain or loss per cent. in the following transactions?

- (1) If a brooch which cost £3 is sold for £3. 15s.
- (2) If a dealer bought a gross of toys for 10s., and sold them at a penny each.
- (3) If coal is bought at 9d. per cwt., and sold at 22s. 6d. per ton.
- (4) If a newspaper-boy buys papers at 9d per dozen (13 to the dozen), and sells them at a penny each.
- (5) If sugar, costing £1. 5s. per cwt., is sold at 3½d. per lb.
- (6) If sugar, bought at £18. 13s. 4d. per ton, is sold at 1½d. per lb.
- (7) If I buy 6 ponies for £170 and 4 cows for £38, and sell the ponies at £38 each and each cow for £8.
- (8) If tea is bought at 1s. 5½d. per lb., and sold at 1s. 9½d. per lb.
- (9) If cigars are bought at £2. 1s. 8d. per 100, and sold at 6d. each.
- (10) If 46 yards of silk are bought for 10 guineas, and sold at the rate of £3. 7s. 3d. for 8 yards.
- (11) If 2 cwt. of chicory, bought at £1. 9s. 2d. per cwt., is mixed with 5 cwt. of coffee, bought at £8. 4s. 6d. per cwt., and the mixture is sold at 1s. 4d. per lb.
- (12) If 16 lb. of tea bought at 2s. 8d. per lb., 28 lb. at 2s. 6d. per lb., and 11 lb. at 2s. 4d. per lb. are mixed together, and the mixture is sold at 3s. 9d. per lb.

215. CASE II. Given the cost price and the gain or loss per cent.; to find the selling price.

Example 1. If a house was bought for £9600, and is sold at a profit of $12\frac{1}{2}$ per cent., what does the profit amount to, and for how much is the house sold?

$$\text{Profit} = \frac{12\frac{1}{2}}{100} \text{ of } £9600$$

$$= \frac{1}{8} \text{ of } £9600$$

$$= £1200;$$

$$\therefore \text{selling price of house} = £9600 + £1200$$

$$= \underline{£10800.}$$

Example 2. If a house was bought for £1350, and is sold at a loss of 20 per cent., what does the loss amount to, and for how much is the house sold?

$$\text{Loss} = \frac{20}{100} \text{ of } £1350$$

$$= £270;$$

$$\therefore \text{selling price of house} = £1350 - £270$$

$$= \underline{£1080.}$$

EXERCISE 108.

(1) A house is bought for £4250, and is sold at a profit of 12 per cent.; how much was gained, and for how much was the house sold?

(2) Cloth is bought at 1s. 6d. a yard; at what price per yard must it be sold to gain 25 per cent.?

(3) A tradesman, selling off, reduces his prices to 10 per cent. below cost price; what did he get for goods for which he gave £65?

(4) A man bought a horse for £45, and sold it again at a loss of 7 per cent.; for how much was the horse sold?

(5) A merchant buys a fifty-gallon cask of wine for £62. 10s., and sells it at a profit of 4 per cent.; at what price per gallon does he sell the wine?

(6) A grocer buys 10 cwt. 3 qrs. 21 lb. of sugar for £30, and pays 12s. 6d. for expenses; at what rate must he sell it per lb. to gain 25 per cent.?

(7) A farmer rents $305\frac{1}{2}$ acres of land at £4. 17s. 6d. per acre, but the landlord allows an abatement of 20 per cent.; what does the tenant pay?

(8) A plumber bought 4 tons 16 cwt. of lead for £1. 0s. $2\frac{1}{2}$ d. per cwt., and sold the whole so as to clear $12\frac{1}{2}$ per cent.; what did he receive from the sale?

(9) A grocer mixes two kinds of sugar at 4d. and $6\frac{1}{2}$ d. per lb., taking 3 lb. of the first to 2 lb. of the second; at what price per lb. must he sell the mixture to make a profit of 20 per cent. on his outlay?

(10) A spirit merchant mixes 80 gallons of whisky at 15s. 6d. per gallon with 96 gallons at 17s. 1d., and sells the mixture so as to make a profit of 10 per cent. on his outlay; at what price per gallon does he sell it?

(11) A grocer buys coffee at £8. 10s. per cwt., and chicory at £2 10s. per cwt.; he mixes them in the proportion of 5 parts chicory to 7 coffee; at what price per lb. must he sell the mixture to gain $16\frac{1}{2}$ per cent. on his outlay?

(12) A grocer buys one kind of tea at 8 guineas per cwt., and another kind at £14 per cwt.; he mixes them in the proportion of 5 lb. of the former kind to 3 lb. of the latter; at what price per lb. must he sell the mixture to gain 20 per cent. on his outlay?

216. CASE III. Given the selling price and the gain or loss per cent.; to find the cost price.

Example 1. Goods are sold for £11500, at a profit of 15 per cent. on the cost price; what did they cost?

$$£11500 = \text{cost price} + \frac{15}{100} \text{ of cost price}$$

$$= \frac{115}{100} \text{ of cost price}$$

$$= \frac{23}{20} \text{ of cost price;}$$

$$\therefore \text{cost price} = \frac{20}{23} \text{ of } £11500$$

$$= \underline{\underline{£10000.}}$$

Example 2. A cow is sold for £19. 4s., at a loss of 20 per cent. on the cost price; what did it cost?

$$£19. 4s. = \text{cost price} - \frac{20}{100} \text{ of cost price}$$

$$= \frac{80}{100} \text{ of cost price;}$$

$$\therefore \text{cost price} = \frac{5}{4} \text{ of } £19. 4s.$$

$$= \underline{\underline{£24.}}$$

EXERCISE 109.

(1) Find the cost price of goods sold for £27. 19s., at a gain of $7\frac{1}{2}$ per cent.

(2) A house is sold for £2030, at a profit of 12 per cent.; what did it cost?

(3) A horse was sold for £28, at a gain of $16\frac{1}{2}$ per cent.; what did it cost?

(4) A horse is sold for £102, at a loss of 15 per cent.; what did it cost?

(5) Goods were sold for £216, at a loss of 10 per cent.; what did they cost?

(6) What is the cost of lead per cwt., if the sale of 48 cwt. for £45. 11s. 3d. gives a profit of £12. 10s. per cent.?

(7) A grocer by selling sugar at $3\frac{1}{2}$ d. per lb. loses 2 per cent. on his outlay; what did the sugar cost him per cwt.?

(8) A merchant sells cigars at 1s. each, and makes a profit of 42½ per cent. on his outlay; what is the cost price of 100 cigars?

(9) By selling ten houses for £3059. 10s., a man gained 5½ per cent.; what did each house cost him originally?

(10) Find the cost of goods per ton, if the sale of 16 tons for £21. 8s. 9d. yields a profit at the rate of 5 per cent.

(11) Merchandise is sold at the rate of 10s. 8½d. per cwt., and a profit is made at the rate of 20 per cent.; find the cost price of 3 tons.

(12) I sell 512 yards of silk for £89. 16s. 8d., and thereby realise a profit of 9½ per cent.; at what price per yard did I buy the silk?

217. CASE IV. Given two selling prices, to compare the gain or loss per cent.

Example. By disposing of goods for £9. 10s., a man loses 5 per cent. What would be his loss or gain per cent., if he had sold them for £11. 17s. 6d.?

$$£9. 10s. = \frac{95}{100} \times \text{cost price};$$

$$\therefore \text{cost price} = £9 \frac{10}{10} \times \frac{100}{95} \\ = £10;$$

$$\therefore \text{at } £11. 17s. 6d., \text{ the gain on } £10 = £1 \frac{17}{2};$$

$$\therefore \text{the gain on } £100 = £18 \frac{1}{2};$$

$$\therefore \text{gain per cent.} = \underline{\underline{18 \frac{1}{2}}}.$$

EXERCISE 110.

(1) By selling a house for £4162. 10s. a man lost 7½ per cent.; how much per cent. would he have lost or gained if he had sold it for 4500 guineas?

(2) If by selling goods for £136 I lose 16 per cent., how much per cent. would I have lost or gained if I had sold them for 160 guineas?

(3) Goods were sold for £225. 10s. at a gain of 12½ per cent.; what would have been gained or lost per cent. by selling them for £187. 10s.?

(4) A person selling an article for £2. 12s. 6d. gains 5 per cent.; what would he have gained or lost per cent. by selling it for £2. 7s. 6d.?

(5) If 3 per cent. be gained by selling cloth at 12s. 10½d. per yard, what would have been the gain per cent. if it had been sold for 13s. 6d. per yard?

(6) By selling goods for £116. 17s. I lose 5 per cent.; what would I have gained per cent. by selling them for £127. 12s. 3d.?

(7) A grocer sells cheese at £3. 6s. 8d. per cwt., and gains 10 per cent.; how much would he gain per cent. by selling it at 7½d. per lb.?

(8) By selling 6 lb. of tea for 15s. 9d. a grocer gains 5 per cent.; how much will he gain per cent. by selling 50 lb. of the same tea for £6. 9s. 2d.?

(9) If $3\frac{1}{2}$ per cent. be gained by selling butter at £5. 3s. 6d. per cwt., what would be the gain or loss per cent., if it were sold at £7 per cwt.?

(10) A draper, by selling cloth at 7s. per yard, gains 5 per cent.; what would be his rate of profit or loss per cent. if he were to sell the same cloth at 5s. $7\frac{1}{2}$ d. per yard?

218. CASE V. When the gain or loss per cent. varies, to compare selling prices.

Example By selling hay at £3 a ton there is a loss of 10 per cent.; at what price must it be sold to gain 15 per cent.?

$$£3 = \frac{90}{100} \times \text{cost price};$$

$$\therefore \text{cost price} = £3 \times \frac{100}{90}$$

$$= £3. 6s. 8d.;$$

at 15 per cent., the gain on £3. 6s. 8d. = $\frac{15}{100}$ of £3. 6s. 8d.

$$= £\frac{9}{20} \times \frac{1}{2}$$

$$= £\frac{1}{2}$$

$$= 10s.,$$

$$\therefore \text{price at which hay must be sold} = £3. 6s. 8d. + 10s. = \underline{\underline{£3. 16s. 8d.}}$$

EXERCISE 111.

(1) A person, by disposing of goods for £182, loses at the rate of 9 per cent.; what ought they to have been sold for to realise a profit of 7 per cent.?

(2) A silver cup was sold for 12 guineas, at a loss of $5\frac{1}{2}$ per cent.; for what should it have been sold to gain 26 per cent.?

(3) Goods were sold for £30 at a loss of 10 per cent. They ought to have been sold to gain 30 per cent.; how much were they sold under their proper price?

(4) If by selling sugar at $5\frac{1}{2}$ d. per lb. one loses 16 per cent.; at what price per cwt. should one sell it to gain 8 per cent.?

(5) If by selling wine at 15s. per gallon I lose 6 per cent., at what price must I sell it to gain $17\frac{1}{2}$ per cent.?

(6) By selling an article for £9. 12s. 6d. a tradesman loses $3\frac{3}{4}$ per cent.; at what price should he sell it to gain $2\frac{1}{2}$ per cent.?

(7) If 11 per cent. be lost by selling 230 yards of cloth for £85 5s. 10d., at what price must the cloth be sold per yard to gain 17 per cent.?

(8) If $5\frac{1}{2}$ per cent. would be gained by selling 121 yards of silk for £26. 11s. $10\frac{3}{4}$ d., at how much per yard must it be sold to gain 12 per cent.?

(9) A grocer sells cheese at £3. 13s. 4d. per cwt., and thereby gains 10 per cent.; at what price per lb. must he sell it so as to gain 12 per cent.?

(10) By selling 12 lb. of tea for £1. 11s. 6d., a grocer gains 5 per cent.; for how much must he sell 50 lb. of it to gain $3\frac{1}{2}$ per cent.?

MISCELLANEOUS EXAMPLES.

Profit and Loss.

EXERCISE 112.

(1) A dealer buys 36 sheep for £45, and sells 10 of them at a loss of 16 per cent.; at what price must he sell each of the remainder to gain 10 per cent. on his outlay?

(2) By selling goods at £12 per ton, 6 per cent. is gained on the outlay; what would be lost per cent. by selling at the rate of 7 lb. for 6d.?

(3) I was doubting whether to sell a horse at a price which would have ensured my losing $8\frac{1}{2}$ per cent., when I got another bid of 218 guineas, and thereby cleared 9 per cent.; what was the first bid made?

(4) A refiner buys sugar at the rate of £25 per ton; the cost of refining is £1. 1s. 6d. per cwt. He sells the refined sugar at the rate of $5\frac{1}{2}$ d. per lb.; how much per cent. does he gain?

(5) A farmer buys 70 sheep for £2. 5s. each, and sells 10 of the worst at once for £1. 17s. 6d. each; at what price must he sell each of the remainder so as to clear 20 per cent. on the purchase?

(6) A butcher contracts to supply a regiment with 100 cwt. of beef at $5\frac{1}{2}$ d. per lb.; what will be his gain per cent., if he can procure the meat for £2 per cwt.?

(7) A dairyman bought 63 gallons of milk at 2s. per gallon, but 15 gallons were lost by leakage; at what price per gallon must he sell the remainder, so as to gain 25 per cent. on the whole prime cost?

(8) By selling 48 yards at 1s. 8d. per yard, a draper gains 4s.; what was the cost price per yard, and what was his gain per cent.?

(9) A confectioner gained 60 per cent. by selling sugar-candy at $1\frac{1}{4}$ d. per oz.; at what rate per cent. would he have gained or lost by selling it at 8d. per lb.?

(10) A timber-merchant sold a quantity of mahogany logs for £1000, losing $33\frac{1}{3}$ per cent.; for what sum ought he to have sold the whole in order to gain $\frac{1}{4}$ of the price they cost him?

SIMPLE INTEREST.

219. Money paid for the use of money lent is called **Interest**.

It is usual to reckon the charge at so much per £100 for a year, i.e., at a certain **Rate per cent. per annum**.

Thus, if the rate be 5 per cent. per annum, it follows that the interest on £100 for 1 year is £5; for two years, £10; for $\frac{1}{2}$ year, £2. 10s., and so on.

220. The sum of money upon which interest is reckoned is called the **Principal**. When we add the interest to the principal we get the **Amount**, i.e., the sum to which the principal will amount at the end of a given time.

When it is agreed between the lender and the borrower that the interest shall be paid over periodically, as soon as it becomes due, the money is said to be lent at **Simple Interest**.

221. Questions involving interest can be worked out by the Unitary Method.

222. CASE I. Given the Principal, the Time, and the Rate per Cent.; to find the Simple Interest.

Example. Find the simple interest on £3265. 12s. 6d. for $5\frac{1}{2}$ years at 4 per cent. per annum.

$$\begin{aligned}
 \text{Interest on £100 for } 5\frac{1}{2} \text{ years} &= £5\frac{1}{2} \times 4; \\
 \therefore \quad \quad \quad \text{£3265}\frac{1}{2} \text{ for } 5\frac{1}{2} \text{ years} &= £3265\frac{1}{2} \times \frac{1}{100} \times 5\frac{1}{2} \times 4 \\
 &= £221\frac{3}{8} \times \frac{1}{8} \times 11 \\
 &= £104\frac{11}{16} \times 11 \\
 &= £1149\frac{9}{16} \\
 &= \frac{1}{4} \text{ of } £2873. 15s. \\
 &= \underline{\underline{£718. 8s. 9d.}}
 \end{aligned}$$

Hence it will be seen that to find the simple interest we multiply the principal by the rate per cent., and then by the number of years, and we divide the product by 100. These operations may be performed very conveniently in either of the following ways:—

£	s.	d.	
3265	12	6	
			4 = rate p. c.
13062	10	0	
			$5\frac{1}{2}$ = no. of yrs.
65312	10	0	
6531	5	0	
100) 718,43	15	0	
	20		
	8,75		
	12		
	9,00		
			Interest = <u>£718. 8s. 9d.</u>

(2) Since $\frac{4 \times 5\frac{1}{2}}{100} = \frac{11}{16}$, we may proceed thus:—

£	s.	d.
3265	12	6
		11
5,359	21	6
10	718	4
Interest = <u>£718. 8s. 9d.</u>		

EXERCISE 113.

Find the simple interest on

- | | |
|--|--|
| (1) £650 for 1 year at 5 per cent. | (2) £735 for 1 year at $2\frac{1}{2}$ per cent. |
| (3) £215 for 1 year at $6\frac{1}{2}$ per cent. | (4) £1829 for 1 year at $3\frac{3}{4}$ per cent. |
| (5) £3265 for 1 year at $2\frac{3}{4}$ per cent. | (6) £2055 for 1 year at $5\frac{1}{2}$ per cent. |
| (7) £245 for 2 years at 3 per cent. | (8) £356 for 3 years at $2\frac{1}{2}$ per cent. |

- (9) £525 for 5 years at $3\frac{1}{2}$ per cent.
- (10) £960 for 12 years at $2\frac{1}{4}$ per cent.
- (11) £325. 10s. for $2\frac{1}{2}$ years at 4 per cent.
- (12) £774. 11s. 3d. for 4 years at 5 per cent.
- (13) £886. 10s. 9d. for $4\frac{1}{2}$ years at $5\frac{1}{2}$ per cent.
- (14) £1545 for $5\frac{1}{2}$ years at 4 per cent.
- (15) £2763 for $4\frac{1}{2}$ years at 5 per cent.
- (16) £1465 for $7\frac{1}{4}$ years at $3\frac{1}{2}$ per cent.
- (17) £2367. 10s. for 3 years at $3\frac{1}{2}$ per cent.
- (18) £1715. 1s. 8d. for 5 years at 3 per cent.
- (19) £670. 15s. for 8 months at 5 per cent.
- (20) £3460 for 8 months at $3\frac{1}{2}$ per cent.
- (21) £850 for 3 years 3 months at $4\frac{1}{2}$ per cent.
- (22) £8240 for 14 months at 3 per cent.
- (23) £8245 for 4 months at 3 per cent.
- (24) £41. 13s. 4d. for 8 months at $4\frac{1}{2}$ per cent.

223. When simple interest is to be calculated from one day in the year to another, the first of the days mentioned is not reckoned.

Example. Find the simple interest on £543. 17s. 6d. from June 23rd to August 5th at $3\frac{1}{2}$ per cent. per annum.

The number of days for which interest is reckoned

$$= 7 \text{ in June} + 31 \text{ in July} + 5 \text{ in August} = 43;$$

$$\therefore \text{the interest} = £543. 17s. 6d. \times \frac{3\frac{1}{2}}{100} \times \frac{43}{365}$$

$$= £ \frac{4351}{8} \times \frac{7}{200} \times \frac{43}{365}$$

$$= £ \frac{1309651}{584000}$$

$$= \underline{\underline{£2. 4s. 10d.}} \text{ to the nearest penny.}$$

NOTE.—In banks time is saved by the use of books containing interest tables.

EXERCISE 114.

Find the simple interest on

- (1) £533. 6s. 8d. for 73 days at 3 per cent.
- (2) £2160. 12s. 6d. for 1 year 73 days at 5 per cent.
- (3) £130 from March 5th to October 10th at $3\frac{1}{2}$ per cent.
- (4) £243. 6s. 8d. from May 15th to November 7th at $3\frac{3}{4}$ per cent.
- (5) £380. 4s. 2d. from January 1st, 1896, to April 30th at $4\frac{1}{2}$ per cent.

- (6) £325 from January 10th, 1898, to June 5th at $3\frac{1}{2}$ per cent.
 (7) £760. 8s. 4d. from January 3rd, 1896, to May 2nd at 5 per cent.
 (8) £684. 7s. 6d. from March 1st to December 1st at $5\frac{1}{2}$ per cent.
 (9) £547. 10s. from February 5th, 1898, to March 5th at $6\frac{1}{2}$ per cent.
 (10) £1368. 15s. from December 30th, 1898, to February 13th, 1899, at 2 per cent.

224. CASE II. Given the Interest (or the Amount), the Time, and the Rate per Cent. ; to find the Principal.

Example 1. What sum will amount to £300. 16s. in 4 years at 5 per cent.?

$$\begin{aligned}
 \text{Amount of £100 in 4 years at 5 per cent.} &= £120, \\
 \therefore \text{principal which in 4 yrs. will amount to £120} &= £100, \\
 \therefore \text{.....} &\text{.....} \quad \text{£1} = £1\frac{2}{5} = £\frac{7}{5}, \\
 \therefore \text{.....} &\text{.....} \quad £300\frac{1}{2} - \frac{1}{5} \times £300. 16s. \\
 &= \frac{1}{5} \times £1504 \\
 &= \underline{\underline{£250. 13s. 4d.}}
 \end{aligned}$$

Example 2. What sum will produce £113. 8s. interest in $4\frac{1}{2}$ years at $4\frac{1}{2}$ per cent.?

$$\begin{aligned}
 \text{Principal which produces £4}\frac{1}{2} \text{ in 1 yr.} &= £100, \\
 \therefore \text{.....} \quad \text{£113}\frac{1}{2} \text{ in } 4\frac{1}{2} \text{ yrs.} &= £\frac{113\frac{1}{2}}{4\frac{1}{2}} \times \frac{1}{4\frac{1}{2}} \times 100 \\
 &= £\frac{227}{9} \times \frac{2}{9} \times \frac{1}{4\frac{1}{2}} \times 100 \\
 &= £7 \times 80 \\
 &= \underline{\underline{£560.}}
 \end{aligned}$$

EXERCISE 115.

What principal will amount to

- (1) £411. 5s. in 5 years at $3\frac{1}{2}$ per cent.?
 (2) £814. 10s. in $5\frac{1}{2}$ years at $2\frac{1}{2}$ per cent.?
 (3) £188. 2s. 6d. in $1\frac{1}{2}$ years at 5 per cent.?
 (4) £424. 2s. 3d. in $3\frac{1}{2}$ years at $4\frac{1}{2}$ per cent.?

What principal will produce as interest

- (5) £551. 18s. 6d. in $3\frac{1}{2}$ years at $2\frac{1}{2}$ per cent.?
 (6) £344. 16s. 8d. in $2\frac{1}{2}$ years at $4\frac{1}{2}$ per cent.?
 (7) £908. 11s. 6d. in $4\frac{1}{2}$ years at $3\frac{1}{2}$ per cent.?
 (8) £529. 18s. 5d. in $3\frac{1}{2}$ years at $4\frac{1}{2}$ per cent.?
 (9) £124. 6s. 3d. in 3 years 3 months at $4\frac{1}{2}$ per cent.?
 (10) £129. 12s. 9d. in 1 year 73 days at 5 per cent.?

225. CASE III. Given the Principal, Rate per Cent. and Interest (or Amount); **to find the Time.**

If we find the interest on the Principal for 1 year, and determine how many times this interest is contained in the given interest, the quotient will be the number of years required.

Example. In what time will £2833. 6s. 8d. amount to £3215. 16s. 8d. at 3 per cent.?

Amount of £2833. 6s. 8d. in reqd. time = £3215. 16s. 8d.;
 \therefore interest on = £382. 10s. 0d.;
 but interest on £2833. 6s. 8d. for 1 yr at 3% = £85;

$$\begin{aligned}\therefore \text{required time} &= \frac{382\frac{1}{2}}{85} \text{ years} \\ &= 4\frac{5}{17} \text{ years} \\ &= \underline{4\frac{1}{2} \text{ years.}}\end{aligned}$$

EXERCISE 116.

In what time will

- (1) £2175 amount to £2338. 2s. 6d. at $2\frac{1}{2}$ per cent.?
- (2) £3745 amount to £3932. 5s. at $2\frac{1}{2}$ per cent.?
- (3) £1260 amount to £1496. 5s. at $3\frac{3}{4}$ per cent.?
- (4) £1650 amount to £1891. 6s. 3d. at $4\frac{1}{2}$ per cent.?
- (5) £4500 amount to £5141. 5s. at 3 per cent.?
- (6) £4550 amount to £6597. 10s. at 6 per cent.?
- (7) £4550 amount to £5573. 15s. at 3 per cent.?
- (8) £1300 amount to £1493. 7s. 6d. at $3\frac{1}{2}$ per cent.?
- (9) £1440 amount to £1831. 10s. at $7\frac{1}{4}$ per cent.?
- (10) £4420 amount to £6121. 14s. at $5\frac{1}{2}$ per cent.?

226. CASE IV. Given the Principal, Time, and Interest (or Amount); **to find the Rate per Cent.**

If we find the interest on the Principal at the Rate of 1 per cent. for the given time, and determine how many times this is contained in the given interest, the quotient will be the rate per cent. required.

Example. At what rate per cent. will £1260 amount to £1496. 5s. in 3 years 9 months?

Interest on £1260 for $3\frac{3}{4}$ years at reqd. rate = £1496. 5s. - £1260
 = £236. 5s.,
 and interest on £1260 for $3\frac{3}{4}$ years at 1 per cent. = £47. 5s.;
 \therefore required rate per cent. = $\frac{£236. 5s.}{£47. 5s.}$
 = 5.

EXERCISE 117.

At what rate per cent. will

- (1) £325 amount to £379. 3s. 4d. in 5 years?
- (2) £250 amount to £302. 10s. in 2 years?

- (3) £1650 amount to £1784. 1s. 3d. in $2\frac{1}{2}$ years?
- (4) £1250 amount to £1962. 10s. in 6 years?
- (5) £3825 amount to £4972. 10s. in 3 years?
- (6) £3745 amount to £3932. 5s. in 2 years?
- (7) £1315 amount to £1545. 2s. 6d. in $3\frac{1}{2}$ years?
- (8) £1035 amount to £1071 4s. 6d. in $10\frac{1}{2}$ months?
- (9) £2416 amount to £3050. 4s. in 5 years?
- (10) £1250 amount to £1446. 17s. 6d. in $4\frac{1}{2}$ years?

COMPOUND INTEREST.

227. We have seen that if money be lent at Simple Interest the interest is paid over periodically.

It may sometimes be agreed that the interest shall not be paid over, but shall be retained by the borrower, and shall be considered as an additional loan upon which he must subsequently pay additional interest.

The loan thus becomes increased at regular periods, and with it is increased the sum upon which interest must be paid.

For example, suppose £100 be lent in this way, the rate of interest being $2\frac{1}{2}$ per cent. and the interest being payable yearly. At the end of one year the interest is £2. 10s. This is retained by the borrower, and the loan becomes £102. 10s. For the second year the interest is £2. 11s. 3d. This again is retained, and the loan becomes £105. 1s. 3d.

Money lent upon these conditions is said to be lent at **Compound Interest**.

228. Given the Principal, Time and Rate per Cent.; to find the Amount, and hence the Increase.

When the rate is 5 per cent. per annum, the Interest on any Principal for one year is $\frac{5}{100}$, i.e. $\frac{1}{20}$, of the Principal. The quotient can in this case be obtained at sight, and the whole calculation may be arranged as in the following example.

Example. Find what £634. 9s. 4d. will amount to in 3 years at 5 per cent. per annum compound interest.

Principal	—£634. 9s. 4d.
Interest for 1st year ($\frac{1}{20}$)	= £31. 14s. 5.60d.,
∴ amount in 1 year	= £666. 3s. 9.60d.
Interest for 2nd year ($\frac{1}{20}$)	= £33. 6s. 2.28d.,
∴ amount in 2 years	= £699. 9s. 11.88d.
Interest for 3rd year ($\frac{1}{20}$)	= £34. 19s. 5.99d.,
∴ amount in 3 years	= £734. 9s. 5.87d.,
and Increase in 3 years	= £100. 0s. 1.87d.

NOTE. -We may proceed in this way whenever the rate per cent. leads to an easy divisor. If, for example, the rate per cent. be 10, 12½,

$2\frac{1}{2}$, etc., the corresponding fractions would be $\frac{1}{10}$, $\frac{1}{8}$, $\frac{1}{40}$, etc., and therefore the divisors would be 10, 8, 40, etc. In such cases as these the quotients can be written down as in the above example.

EXERCISE 118.

Find the amount and the increase, at compound interest, of

- (1) £1234 in 3 years at 5 per cent.
- (2) £625. 10s. in 2 years at 5 per cent.
- (3) £923. 17s. 6d. in 3 years at 5 per cent.
- (4) £1724. 15s. in 4 years at 10 per cent.
- (5) £2794 in 3 years at $12\frac{1}{2}$ per cent.
- (6) £2618. 5s. in 3 years at $12\frac{1}{2}$ per cent.
- (7) £8075 in 3 years at $2\frac{1}{2}$ per cent.
- (8) £4096 in 3 years at $1\frac{1}{4}$ per cent.
- (9) £842. 12s. 6d. in 3 years at $8\frac{1}{2}$ per cent.
- (10) £374. 16s. 7d. in 3 years at $8\frac{1}{2}$ per cent.

229. When the rate of interest is not one of those which lead to an easy divisor, we adopt the following method.

Example. Find the amount and the increase of £345. 10s. for $2\frac{3}{4}$ years at 4 per cent. compound interest.

We first express the given sum as pounds and a decimal of a pound.

$$\text{£}345. 10s. = \text{£}345.5$$

We divide by 100
by moving the decimal
point *two* places to the
left.

$$\begin{array}{r}
 13.820 = \text{interest for 1st year} \\
 345.5 \\
 \hline
 359.32 = \text{amount in 1 year} \\
 4 \\
 \hline
 14.3728 = \text{interest for 2nd year} \\
 359.32 \\
 \hline
 373.6928 = \text{amount in 2 years} \\
 3 = \text{rate for } \frac{3}{4} \text{ year} \\
 \hline
 11.210784 = \text{interest for last } \frac{3}{4} \text{ year} \\
 373.6928 \\
 \hline
 384.903584 = \text{amount in } 2\frac{3}{4} \text{ years} \\
 20 \\
 \hline
 18.071680 \\
 12 \\
 \hline
 0.860160
 \end{array}$$

$$\begin{aligned}
 \therefore \text{Amount} &= \text{£}384. 18s. 0.86d., \\
 \text{and Increase} &= \text{£}39. 8s. 0.86d.
 \end{aligned}$$

NOTE.—If the interest is to be added half-yearly we may obtain the result by the preceding method if we double the number of years, halve the rate of interest, and then suppose that the interest is added *yearly*.

EXERCISE 119.

Find the amount and the increase, at compound interest, of

- (1) £3600 in 2 years at 3 per cent.
- (2) £3752 in 2 years at 4 per cent.
- (3) £6845 in 3 years at 4 per cent.
- (4) £4500 in 3 years at 6 per cent.
- (5) £3650 in 3 years at $5\frac{1}{2}$ per cent.
- (6) £3695 in $2\frac{1}{2}$ years at $4\frac{1}{2}$ per cent.
- (7) £6500 in $3\frac{1}{2}$ years at 3 per cent.
- (8) £1050 in $3\frac{3}{4}$ years at 4 per cent.

In the next examples, add the interest half-yearly:—

- (9) £1000 in 2 years at 8 per cent.
- (10) £10 in $2\frac{1}{2}$ years at $4\frac{1}{2}$ per cent.
- (11) £250 in 2 years at $4\frac{1}{2}$ per cent.
- (12) £880 in $2\frac{1}{2}$ years at $8\frac{1}{2}$ per cent.

MISCELLANEOUS EXAMPLES.

Interest (Simple and Compound).

EXERCISE 120.

(1) How much money invested in a bank at 4 per cent. per annum simple interest will produce £30 a year?

(2) A banker charged £18. 12s. for the loan of £744 for 3 months; find the rate of interest per cent. per annum.

(3) A man borrows £1250 at 4 per cent.; he repays the loan when the principal and interest amount to £1666. 13s. 4d.; for how long did he keep the money?

(4) The yearly rent of a house being £130, what should be the price of the house that the purchaser may get $6\frac{1}{2}$ per cent. on his investment?

(5) What is the rate of interest per cent. per annum, if £2. 10s. 7½d. is paid for the use of £150 for 2 months?

(6) A man sails from England on January 1st 1895, leaving a sum of money at his banker's at $3\frac{1}{2}$ per cent. simple interest. On July 1st 1899 he returns and finds that £549 16s. 3d. is the amount standing to his credit. How much of this sum is interest, and how much principal?

(7) A youth puts £30 in the bank on his 17th birthday, and the money accumulates at $2\frac{1}{2}$ per cent. per annum compound interest; find, to the nearest penny, the amount standing to his credit on his 21st birthday.

(8) A man puts £40 in a bank at the end of each year; how much will he have at the end of 5 years, reckoning compound interest at 3 per cent. per annum?

(9) Find the difference between the simple and the compound interest on £2000 for 2 years at $6\frac{1}{2}$ per cent. per annum.

(10) Find, to the nearest penny, the difference between the simple and the compound interest on £1500 for 3 years at $4\frac{1}{2}$ per cent. per annum.

TRUE DISCOUNT.

230. It is a common custom for a merchant to pay for goods, not in money, but in what are called **Bills of Exchange**, each of which contains a promise that an amount stated shall be paid after a certain interval, which may be three days, or a week, or three months, or any other period whatever.

231. If now a creditor *A* holds a bill of exchange and wants ready money before the time when *B* will pay him, it will be sufficient if he can find a third person *C* willing to give ready money for the right to receive from *B*, when that time comes, the amount stated in the bill. They will only have to determine *how much* money *C* shall give *A* in exchange for this right.

The *theoretical* ready-money value of a bill is called its **True Present Worth**, and will clearly be less than the amount of the bill itself.

The difference between the true present worth and the amount of the bill is called the **True Discount**.

232. In determining the present worth of a bill we must consider that *C* loses the use of his money until he receives the amount of the bill from *B*. The money which he receives from *B* at the stated time should therefore repay to him the money which he gave *A*, and also the money which this might have earned.

Hence we see that the **true present worth of a bill is that sum which, with interest on itself for the time until the bill is paid, would produce the amount stated in the bill.** This would place *A* and *C* on exactly equal terms. It follows that

True Discount = Interest on True Present Worth.

When a bill is exchanged for ready money it is said to be **discounted**.

233. In questions involving discount there are *four* quantities; (1) the Amount of the Bill, (2) the Rate of Interest, (3) the Time which the Bill has to *run*, (4) the Present Worth.

These quantities correspond exactly to the four quantities involved in questions on simple interest. The *Time* and the *Rate of Interest* appear in both; the *Present Worth* corresponds to the *Principal*; and the *Amount of the Bill* corresponds to the *sum to which the Principal will amount*.

Example. Find the present worth of £1356. 13s. 4d., due 3 months hence, reckoning simple interest at 7 per cent.

This corresponds to Case II. of Simple Interest.

$$\begin{aligned}
 &\text{Interest on } £100 \text{ for 3 mos. at 7 per cent.} = £\frac{1}{4} \times 7 = £1\frac{3}{4}, \\
 \therefore &\text{Amount of } £100 \text{ in } \dots \dots \dots = £101\frac{3}{4}, \\
 \therefore &\text{P. W. of } £101\frac{3}{4} \text{ due in 3 mos.} \dots \dots \dots = £100, \\
 \therefore &\dots \dots \dots £1356\frac{1}{4} \dots \dots \dots = £\frac{100}{101\frac{3}{4}} \times 1356\frac{1}{4} \\
 &\qquad \qquad \qquad = £\frac{100}{101\frac{3}{4}} \times \frac{1356\frac{1}{4}}{1} \\
 &\qquad \qquad \qquad = £\frac{1356\frac{1}{4}}{101\frac{3}{4}} \\
 &\qquad \qquad \qquad = £1333. 6s. 8d.
 \end{aligned}$$

$$\therefore \text{ the true discount on } £1356. 13s. 4d. = £23. 6s. 8d.$$

The discount could also be found directly, since it is $\frac{1\frac{3}{4}}{101\frac{3}{4}}$ of the bill.

EXERCISE 121.

Find, to the nearest penny, the present worth and the true discount of the following bills:—

- (1) £177 5s. 6d., due 1 year hence, interest at 5 per cent.
- (2) £5747, due 9 months hence, interest at $3\frac{1}{2}$ per cent.
- (3) £225. 11s., due 6 months hence, interest at 8 per cent.
- (4) £4120. 8s. 7d., due 9 months hence, interest at 4 per cent
- (5) £1842. 15s., due 3 months hence, interest at 5 per cent.
- (6) £377. 12s. 6d., due in 18 months, interest at 4 per cent.
- (7) £550. 13s. 4d., due in 15 months, interest at 4 per cent.
- (8) £249. 1s., due 15 months hence, interest at $2\frac{1}{2}$ per cent
- (9) £152. 12s. 8d., due 4 months hence, interest at 4 per cent.
- (10) £276. 10s. 5d., due 219 days hence, interest at $3\frac{1}{2}$ per cent.

BANKER'S DISCOUNT.

234. We have shown how to determine the discount on bills of exchange so that buyer and seller may stand on equal terms, but that principle is theoretical only, and is not adopted by business men. A merchant holding a bill of exchange may take it to a banker, or to a bill discounter, who will pay him the amount of the bill less a charge at a certain rate per cent. per annum, which is the fee for discounting the bill before it is legally due. The amount retained by the bill-broker is the interest on the amount of the bill for the time the bill has to run, instead of the interest on the true present worth of the bill.

The banker or bill-broker has therefore the advantage.

235. From the bill itself one may determine on what day it is nominally due, but it is not customary to require payment until the third day after, the three extra days being called **Days of Grace**.

For example, a bill which is dated January 1st 1898, and which contains a promise to pay a certain amount 3 months after that date, is not legally due until April 4th.

If therefore a bill of £100, due nominally on April 1st, be taken to a banker on February 12th, he will reckon that between that day and April 4th, when the bill will be paid, there are $16 + 31 + 4$, or 51 days; and he will pay over what remains of the £100 after the *interest* for 51 days has been subtracted.

Example. Find the cash value of a bill for £328. 10s., which is dated January 1st, 1898, is payable in 3 months, and is discounted on February 12th; the rate of discount being $2\frac{1}{2}$ per cent.

It will be noticed that 1898 is not a leap year, and that there are therefore only 28 days in February. Also the bill becomes payable, *nominally* on April 1st, *legally* on April 4th.

The number of days between Feb. 12th and April 4th = 51.

Interest on £1 for 365 days = $\text{£}2\frac{1}{2} \times \frac{1}{100}$,

$$\begin{aligned} \therefore \dots\dots\dots \text{£}328\frac{1}{2} \text{ for } 51 \text{ days} &= \text{£}328\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{100} \times \frac{51}{365} \\ &= \text{£}\frac{657}{2} \times \frac{5}{2} \times \frac{1}{100} \times \frac{51}{365} \\ &= \text{£}\frac{657}{4} \times \frac{1}{100} \times \frac{51}{73} \\ &= \text{£}\frac{8}{100} \times 51 \\ &= \text{£}\frac{408}{100}; \end{aligned}$$

\therefore banker's discount = £1. 2s. 11d.;

\therefore cash value of the bill = £328. 10s. - £1. 2s. 11d.
= £327. 7s. 1d.

NOTE.—Bankers and bill-brokers use books containing interest tables which give the interest on various amounts at various rates, thus facilitating calculations.

EXERCISE 122.

(In every case give the result to the nearest penny.)

(1) What will a banker retain on discounting a bill of £1275, drawn on the 4th of March at 10 months, and discounted on the 14th of August at 5 per cent.?

(2) What will a banker retain on discounting a bill of £666. 13s. 4d., drawn on the 2nd of March at 8 months, and discounted on the 8th of June at $3\frac{1}{2}$ per cent.?

(3) A bill of £170. 12s. 6d., drawn on the 22nd Dec., 1895, at 8 months, is discounted on the 29th of April at $3\frac{3}{4}$ per cent.; how much will the banker retain?

(4) A bill of £463. 10s., drawn on May 3rd at 6 months, is discounted on May 18th; find the discount at $2\frac{1}{2}$ per cent.

(5) Find the banker's discount on a bill of £696. 10s. 10d., drawn on June 12th at 7 months, and discounted on Sept. 26th at $6\frac{1}{2}$ per cent.

(6) What will a banker give on the 1st of July for a bill of £1931. 9s. 2d., drawn on the 10th of May at 6 months, interest being reckoned at $5\frac{1}{2}$ per cent.?

(7) On the 27th of December, 1897, a bill of £123 7s. 6d. was discounted at $4\frac{1}{2}$ per cent.; the bill was drawn on the 31st of July at 12 months; what was the discount?

(8) A bill of £1231. 17s. 6d., drawn on April 6th at 9 months, is discounted on June 4th; find the banker's discount at $3\frac{1}{2}$ per cent.

(9) A bill for £152. 1s. 8d. is drawn on the 5th of April at 4 months, and is discounted by a banker on the 8th of May at $3\frac{1}{2}$ per cent.; how much does the holder receive?

(10) A bill for £310. 5s. is drawn on the 2nd of April at 4 months, and is discounted by a banker on the 6th of June at 5 per cent.; how much does the holder receive?

STOCKS AND SHARES.

236. The money used in carrying on a business is called **Capital**. When the capital is very large, say £10,000,000, as in the case of the construction of a new railway, the capital required is divided into **shares** of definite amount, say of £10 or £100 each.

237. The people who own the shares are called **Shareholders**, and form what is called a **Company**. The business of a company is managed by a certain number of elected shareholders called **Directors**.

238. When a company is established, the shares are often converted into **stock**. The distinction between shares and stock is this:—when the capital is in the form of *stock*, any odd amount of stock may generally be obtained; but when it is divided into *shares*, only whole shares can usually be obtained, parts of a share not generally being dealt in.

239. At regular intervals, usually half a year, the accounts are made up and, after all expenses are paid, the profits are divided among the shareholders at so much per share, or so much for each £100 stock. What each receives is called his **Dividend**.

240. Supposing that when a company is fairly started it does a good business, the dividend on each share or on each £100 stock increases, and the value of each share or of each £100 stock will also increase; if however the company is not prosperous, the value of each share or of each £100 stock will become less.

The amount of dividend paid to a shareholder depends on the *nominal* value of a share, and not on the price at which it was bought.

We must be careful therefore to remember the distinction between the *nominal* value of a share, which is only the amount of *capital* which it represents, and its *market* value, which is the money which the holder could get if he sold it.

241. When the market value of a share is *greater* than the nominal value, the share is said to be **at a premium**. When the market value is *less* than the nominal value, the share is said to be **at a discount**. When the two values are the same, the share is said to be **at par**.

Similarly, when the market value of £100 stock in a particular company is £125, the stock is said to be at £25 *premium*; and when the market value is £85, the stock is said to be at £15 *discount*.

The market value is called the **price** of the share or stock.

242. The English Government owes a very large sum of money, which is called the National Debt. It is represented chiefly by what are called **Consols**, which may be treated as a kind of *stock*, on which interest is paid at a fixed rate. This rate is at present (1899) $2\frac{3}{4}$ per cent. per annum.

243. CASE I. To find the income derived from a given amount of stock.

Example. What annual income will be derived from £2875 of 4 per cent. stock?

By 4 per cent. stock is meant a stock on £100 of which is paid a regular dividend of 4 per cent. per annum.

Income from £100 stock = £4;

$$\begin{aligned}\therefore \dots\dots\dots \text{£}2875\dots\dots &= \text{£}\frac{2875}{100} \times 4 \\ &= \underline{\underline{\text{£}115.}}\end{aligned}$$

EXERCISE 123.

What annual income will be derived from

- (1) £3300 of 5 per cent. stock?
- (2) £4275 of 4 per cent. stock?
- (3) £800 of $3\frac{1}{2}$ per cent. stock?
- (4) £10775 of 3 per cent. stock?
- (5) £7650 of $4\frac{1}{2}$ per cent. stock?
- (6) £8225 of $2\frac{1}{2}$ per cent. stock?
- (7) £3241. 17s. 6d. of 6 per cent. stock?
- (8) £4326. 10s. of $2\frac{1}{2}$ per cent. stock?
- (9) 3072 shares bearing a half-yearly dividend of 2s. 6d. each?
- (10) 1875 shares bearing a half-yearly dividend of 7s. 6d. each?

244. CASE II. To find how much stock at a given price can be bought (or sold) for a given sum of money.

Example. What amount of $2\frac{1}{2}$ per cent. stock at $92\frac{1}{8}$ can be bought for £1434. 17s. 10d.?

The words "at $92\frac{1}{8}$ " mean that £92 $\frac{1}{8}$ is the market price of £100 stock.

Amount of stock obtained for £92 $\frac{1}{8}$ = £100 ;

$$\begin{aligned} \therefore \dots\dots\dots \text{£}1434\frac{17}{8} &= \text{£} \frac{1434\frac{17}{8}}{92\frac{1}{8}} \times 100 \\ &= \frac{\text{£}1721\frac{181}{8}}{125} \times \frac{8}{8} \times 100 \\ &= \text{£}1377\frac{181}{10} \\ &= \text{£}1377\frac{181}{10} \\ &= \text{£}1553\frac{1}{8}; \end{aligned}$$

\therefore amount of stock bought = £1553. 6s. 8d.

EXERCISE 124.

What amount of

- (1) $3\frac{1}{2}$ per cent. stock at 98 can be bought for £4900?
- (2) $4\frac{1}{2}$ per cent. stock at 105 can be bought for £2394?
- (3) 3 per cent. stock at $85\frac{1}{2}$ can be bought for £3591?
- (4) 4 per cent. stock at $97\frac{1}{4}$ can be bought for £14076?
- (5) 3 per cent. stock at $92\frac{1}{2}$ can be bought for £3445. 12s. 6d.?
- (6) 4 per cent. stock at $90\frac{1}{2}$ can be sold for £3249?
- (7) $2\frac{1}{2}$ per cent. stock at $87\frac{1}{2}$ can be sold for £787. 10s.?
- (8) $2\frac{1}{4}$ per cent. stock at $82\frac{1}{2}$ can be sold for £821. 5s.?
- (9) $2\frac{3}{4}$ per cent. stock at $92\frac{1}{2}$ can be sold for £1202. 10s.?
- (10) 3 per cent. stock at 90 can be sold for £3249?

245. CASE III. To find the value of a given quantity of stock at a given price.

Example. What must be paid for £1553 6s. 8d. of 3 per cent. stock at $92\frac{1}{8}$?

$$\begin{aligned} \text{Cost of £100 stock} &= \text{£}92\frac{1}{8}; \\ \therefore \dots\dots\dots \text{£1} &\dots\dots = \text{£}\frac{1}{8}\frac{1}{8}; \\ \therefore \dots\dots\dots \text{£}1553\frac{1}{8} \text{ stock} &= \text{£}\frac{1553\frac{1}{8}}{92\frac{1}{8}} \times \frac{8}{8} \times 100 \\ &= \text{£}1377\frac{181}{10} \\ &= \text{£}1377\frac{181}{10} \\ &= \text{£}1434. 17s. 10d.; \\ \therefore \text{money required} &= \text{£}1434. 17s. 10d. \end{aligned}$$

Instead of the 3rd and following lines we might multiply £1553. 6s. 8d. by 739, and divide by 800 in the ordinary way.

EXERCISE 125.

What must be paid for

- (1) £1500 of 3 per cent. stock at 90?
- (2) £4550 of 3 per cent. stock at 90?
- (3) £4500 of 5 per cent. stock at $112\frac{1}{2}$?
- (4) £2137. 10s. of 4 per cent. stock at 80?
- (5) £3340 of 4 per cent. stock at 98?

What will be received on selling

- (6) £8500 of 3 per cent. stock at $97\frac{3}{4}$?
- (7) £1245 of 4 per cent. stock at 84?
- (8) £7440 of 3 per cent. stock at $92\frac{7}{8}$?
- (9) £2050 of 4 per cent. stock at 90?
- (10) £3725 of 5 per cent. stock at 113?

246. CASE IV. To find the income derived from the investment of a given amount of money in a given stock at a given price.

Example. What income will be derived from the investment of £4537. 10s. in 3 per cent. stock at $90\frac{3}{4}$?

$$\begin{aligned}
 &\text{Income from } £100 \quad \text{stock} = £3; \\
 &\quad \text{but price of } £100 \quad \text{stock} = £90\frac{3}{4}; \\
 &\therefore \text{income from } £90\frac{3}{4} \quad \text{money} = £3; \\
 &\therefore \dots\dots\dots £1 \quad \dots\dots = £ \frac{3}{90\frac{3}{4}} = £ \frac{12}{363} = £ \frac{4}{121}; \\
 &\therefore \dots\dots\dots £4537\frac{1}{2} \quad \dots\dots = £ \frac{4}{121} \times \frac{2075}{2} \\
 &\qquad\qquad\qquad = £2 \times 75 \\
 &\qquad\qquad\qquad = \underline{\underline{£150.}}
 \end{aligned}$$

Instead of the last three lines we might multiply £4537. 10s. by 4, and divide by 121 in the ordinary way.

EXERCISE 126.

What income will be derived from investing

- (1) £21000 in $2\frac{1}{2}$ per cent. consols at 99?
- (2) £1274 in $3\frac{1}{2}$ per cent. stock at 91?
- (3) £25935 in 3 per cent. stock at 90?
- (4) £3519 in $3\frac{1}{2}$ per cent. stock at $97\frac{3}{4}$?
- (5) £7560 in 3 per cent. stock at $94\frac{1}{2}$?
- (6) £4788 in $3\frac{1}{2}$ per cent. stock at 105?
- (7) £3220 in $3\frac{1}{2}$ per cent. stock at $80\frac{1}{2}$?
- (8) £13000 in 4 per cent. stock at 96?
- (9) £13000 in $3\frac{1}{2}$ per cent. stock at 91?
- (10) £8505 in 3 per cent. stock at $101\frac{1}{4}$?

247. CASE V. To find how much money must be invested in a given stock at a given price to yield a given income.

Example. What sum must be invested in a $3\frac{1}{2}$ per cent. stock at 104 to obtain an income of £329?

Amount invested to obtain £329 income = £104;

$$\begin{aligned} \therefore \dots \dots \dots \text{£1} \dots \dots &= \text{£} \frac{104}{3\frac{1}{2}}; \\ \therefore \dots \dots \dots \text{£329} \dots \dots &= \text{£} \frac{104}{3\frac{1}{2}} \times 329 \\ &= \text{£} \frac{104 \times 329}{3\frac{1}{2}} \\ &= \text{£} 104 \times 94 \\ &= \underline{\underline{\text{£} 9776.}} \end{aligned}$$

EXERCISE 127.

How much money must be invested in

- (1) $3\frac{1}{2}$ per cent. stock at $80\frac{1}{2}$ to obtain an income of £140?
- (2) 3 per cent. stock at $94\frac{1}{2}$ to obtain an income of £25. 10s.?
- (3) 4 per cent. stock at 120 to produce annually £800?
- (4) 3 per cent. stock at 90 to obtain an annual income of £864. 10s.?
- (5) 3 per cent. stock at $85\frac{1}{2}$ to obtain an income of £151. 10s.?
- (6) $3\frac{1}{2}$ per cent. stock at $82\frac{1}{2}$ to obtain an income of £46. 13s. 4d.?
- (7) 4 per cent. stock at 90 to obtain an income of £320?
- (8) 3 per cent. stock at $85\frac{1}{2}$ to obtain an income of £126?
- (9) $4\frac{1}{2}$ per cent. stock at $91\frac{1}{2}$ to obtain an income of £42. 15s.?
- (10) 3 per cent. stock at 80 to obtain an income of £100?

248. CASE VI. To compare the investment of money in various stocks at various prices.

Example. Which is the better investment— $5\frac{1}{2}$ per cent. stock at 105, or $4\frac{1}{2}$ per cent. stock at $87\frac{1}{2}$ assuming the two stocks to be equally sound?

In former case, income from £105 money = £5 $\frac{1}{2}$,

$\dots \dots \dots \text{£1} \dots \dots = \text{£} \frac{11}{16}$;

in latter case, $\dots \dots \dots \text{£} 87\frac{1}{2} \dots \dots = \text{£} 4\frac{1}{2}$,

$\dots \dots \dots \text{£1} \dots \dots = \text{£} \frac{9}{16}$.

It will be found in the usual way that $\frac{11}{16}$ is greater than $\frac{9}{16}$; and therefore the former investment is the better.

EXERCISE 128.

Which is the better investment, assuming them to be equally sound,

- (1) Bank stock paying 10 per cent. at $234\frac{1}{2}$, or 3 per cent. railway stock at $92\frac{1}{2}$?
- (2) $3\frac{1}{2}$ per cent. stock at $98\frac{1}{2}$, or $3\frac{1}{2}$ per cent. stock at par?

- (3) 3 per cent. stock at 85, or 4 per cent. at 96?
 (4) 3 per cent. stock at $83\frac{1}{2}$, or $3\frac{1}{2}$ per cent. at 97?
 (5) $3\frac{1}{2}$ per cent. stock at 88, or 4 per cent. at 91?
 (6) 4 per cent. stock at 96, or 5 per cent. at 112?
 (7) $1\frac{3}{4}$ per cent. stock at $47\frac{1}{4}$, or $3\frac{3}{4}$ per cent. at 99?
 (8) $2\frac{1}{2}$ per cent. stock at $71\frac{1}{4}$, or 2 per cent. at $56\frac{3}{4}$?
 (9) 3 per cent. stock at $91\frac{1}{2}$, or $3\frac{1}{2}$ per cent. at $108\frac{1}{2}$?
 (10) Railway shares, paying £6 per share, when the £100 share is at 132, or 3 per cent. railway stock at 93?

249. CASE VII. To find the rate per cent. obtained on the money invested, when the price of the stock and the rate per cent. paid on it are given.

Example. What rate of interest is obtained by investing in 4 per cent. stock at $92\frac{1}{2}$?

The rate per cent. required is to be reckoned on the money invested.

Income from £ $92\frac{1}{2}$ money = £4;

$$\begin{aligned}\therefore \dots\dots\dots \text{£}100 \dots\dots\dots &= \text{£} \frac{100}{92\frac{1}{2}} \times 4 \\ &= \text{£} \frac{400}{92\frac{1}{2}} \\ &= \text{£} \frac{160}{37} \\ &= \text{£} 4\frac{1}{3} \frac{2}{3};\end{aligned}$$

\therefore rate of interest = $4\frac{1}{3}\frac{2}{3}$ per cent.

EXERCISE 129.

What rate of interest is obtained by investing in

- | | |
|---|--|
| (1) 3 per cent. stock at 85? | (2) 5 per cent. stock at 130? |
| (3) 4 per cent. stock at 104? | (4) $2\frac{3}{4}$ per cent. stock at 99? |
| (5) $3\frac{1}{2}$ per cent. stock at 105? | (6) 4 per cent. stock at $92\frac{1}{2}$? |
| (7) 3 per cent. stock at 84? | (8) 4 per cent. stock at $93\frac{1}{2}$? |
| (9) $3\frac{1}{4}$ per cent. stock at $97\frac{3}{4}$? | (10) 3 per cent. stock at 90? |

250. Stocks and shares are bought and sold at a special market called the Stock Exchange, the business of which is carried on by stock-brokers, who charge a certain sum for each £100 stock, and for each share, bought or sold. This charge is called **brokerage**.

Brokerage is calculated on the *nominal* value.

Thus if the price of Consols be $110\frac{1}{2}$, and the broker charge 2s. 6d. per cent., a buyer would have to pay £($110\frac{1}{2} + \frac{1}{8}$), and a seller would receive £($110\frac{1}{2} - \frac{1}{8}$) for each £100 stock.

251. CASE VIII. Questions in which brokerage must be taken account of.

Example. A man has £7220 of 3 per cent. stock; he sells out at $102\frac{1}{2}$, and invests the proceeds in $2\frac{1}{2}$ per cent. stock at $90\frac{1}{2}$. Find the change in his income, allowing brokerage at $\frac{1}{2}$ per cent.

Income derived from 3 per cents.	$= £7220 \times 1\frac{1}{2}\%$
	$= £216. 12s.;$
money derived from sale of £100 of 3% stock	$= £(102\frac{1}{2} - \frac{1}{2}) = £102\frac{1}{2};$
money derived from sale of £7220 stock	$= £ \frac{7220 \times 102\frac{1}{2}}{100}$
	$= £7400. 10s.;$
money paid for £100 of $2\frac{1}{2}\%$ stock	$= £(90\frac{1}{2} + \frac{1}{2}) = £90\frac{1}{2};$
income derived from $2\frac{1}{2}$ per cents.	$= £ \frac{7400\frac{1}{2} \times 2\frac{1}{2}}{90\frac{1}{2}}$
	$= £205;$
\therefore diminution of income	$= £11. 12s.$

EXERCISE 130.

- (1) How much stock can be obtained by investing £3772. 6s. 3d. in $2\frac{1}{2}$ per cents. at $88\frac{1}{2}$, brokerage being $\frac{1}{2}$ per cent.?
- (2) What sum will be obtained by selling out £8975 stock in the $2\frac{1}{2}$ per cents at $89\frac{1}{2}$, brokerage being $\frac{1}{2}$ per cent.?
- (3) What sum of money must be paid to purchase £2600 of $3\frac{1}{2}$ per cent. stock at $93\frac{1}{2}$, allowing $\frac{1}{2}$ per cent. brokerage?
- (4) What income will be obtained by investing £10098 in 3 per cent stock at $93\frac{1}{2}$, brokerage being $\frac{1}{2}$ per cent.?
- (5) How much money must be invested in 3 per cent. stock at $95\frac{1}{2}$, brokerage being $\frac{1}{2}$ per cent., to obtain an income of £1000?
- (6) What interest per cent. is obtained by investing in 5 per cent. stock at $119\frac{1}{2}$, brokerage being $\frac{1}{2}$ per cent.?
- (7) What is gained by investing £1950 at $97\frac{1}{2}$ and selling out at 104; brokerage being $\frac{1}{2}$ per cent. on each transaction?
- (8) A man invests £19040 in 5 per cent. stock at $74\frac{1}{2}$; the stock rising 2 per cent., he sells out; what does he gain, brokerage being $\frac{1}{2}$ per cent. on each transaction?
- (9) Find the alteration in income occasioned by transferring £3200 stock from 3 per cent stock at $86\frac{1}{2}$ to 4 per cent. stock at $114\frac{1}{2}$, the brokerage being $\frac{1}{2}$ per cent. on each transaction.
- (10) If £10425 of $3\frac{1}{2}$ per cent. stock be sold out at $102\frac{1}{2}$, and the proceeds be invested in $2\frac{1}{2}$ per cent. stock at $69\frac{1}{2}$, what will be the change of income, brokerage being $\frac{1}{2}$ per cent. on each transaction?

MISCELLANEOUS EXAMPLES.

Stocks and Shares.

EXERCISE 131.

(In every case brokerage is allowed for in the price.)

(1) If *A* invest £1210 in $3\frac{1}{2}$ per cents. at $97\frac{1}{2}$, and a like sum in 5 per cent stock at $90\frac{3}{4}$, what will be his income?

(2) *A* invests £1695. 16s. 8d. in $3\frac{1}{2}$ per cents. at $101\frac{1}{2}$, and *B* invests £1767. 3s. 9d. in 3 per cents. at $94\frac{1}{2}$; what is the difference between the incomes thence obtained?

(3) What half-yearly dividend is derived from an investment of £3000 in $3\frac{1}{2}$ per cents. at 105?

(4) Which is the better investment—railway shares paying £6 per share when the £100 share is at 132, or a 3 per cent. stock at 93?

(5) A person transfers £1000 stock from the 4 per cents. at 90 to the 3 per cents. at 72; how much of the latter stock will he obtain, and what will be the change in his income?

(6) A man has £6600 stock in the 4 per cents. at 98; when the price of this stock has fallen 2 per cent., he transfers his capital to the $3\frac{1}{2}$ per cents. at $83\frac{1}{2}$; find the alteration in his income.

(7) A person whose annual income is £450 transfers his property from the 3 per cents. at 84 to the $3\frac{1}{2}$ per cents. at $87\frac{1}{2}$; what change does he make in his income?

(8) The sum of £1245 is invested in the 4 per cents. at 84; how much money must be invested in the $3\frac{1}{2}$ per cents. at 98 to produce the same income?

(9) If railway stock bought at 28 per cent. premium pay $7\frac{1}{2}$ per cent. on the investment, how much per cent. would it pay if it were bought at 10 per cent. discount?

(10) If I buy 10 shares of £20 each at $27\frac{1}{2}$, and sell out at $37\frac{1}{2}$ after receiving a dividend of 15 per cent., how much shall I gain in all?

(11) A person invests £1365 in the 3 per cents. at 91, he sells out £1000 stock when they have risen to $95\frac{1}{2}$, and the remainder when they have fallen to 85; how much does he gain or lose by the transaction?

(12) A man buys £700 of 3 per cent. stock at $94\frac{1}{2}$, and also invests £585 in Russian 5 per cent. stock at $97\frac{1}{2}$; how much stock has he standing in his name? If he sells out of the 3 per cents. at 95, and out of the 5 per cents. at $96\frac{1}{2}$, how much does he gain or lose by the transaction?

(13) How much $3\frac{1}{2}$ per cent. stock at $97\frac{1}{2}$ can be purchased for £3519, and what income will be thence derived?

(14) What must I invest in the $3\frac{1}{2}$ per cents. at 96 in order to obtain an income of £711. 4s. $8\frac{3}{4}$ d.? How much per cent. do I get from the investment?

(15) What half-yearly dividend is due upon an investment of £2500 in 3 per cent. stock at $87\frac{1}{2}$, after deducting 7d. in the £ for income-tax?

(16) When the 3 per cents. are at $87\frac{1}{2}$, and a 5 per cent. stock is at $130\frac{1}{2}$, which is the more profitable investment? And what sum does a person invest if the difference of the incomes resulting from the two investments would be £561?

(17) How much must one invest in a $3\frac{1}{2}$ per cent. stock at 91 in order to receive £590 per annum, after paying 4d. in the £ income-tax?

(18) What sum must be invested in a $\frac{1}{2}$ per cent. stock at 105, in order to produce an income of £312, after deducting income-tax at the rate of 6d. in the £?

(19) How much is gained by investing £3445 in a $3\frac{1}{2}$ per cent. stock at $86\frac{1}{2}$ and selling out at $91\frac{1}{2}$?

(20) A person has £2000 in a $3\frac{1}{2}$ per cent. stock; how much must he also have in a 3 per cent. stock, that his whole income may be £400 per annum?

SQUARE ROOT.

252. When a number is multiplied by itself the result is called the *square* of the number (Art. 55).

The square of a number is denoted by placing a small figure at the right hand of the upper part of the number; for example, the square of 8, or 8×8 , is represented by 8^2 .

253. The *square root* of a given number is that number which multiplied by itself will produce the given number. For example, 8 is the *square root* of 64.

254. The square root of a number is denoted by the sign $\sqrt{}$ placed before the number; thus, the square root of 64 is denoted by $\sqrt{64}$.

255. Since $\sqrt{100} = 10$, and $\sqrt{10000} = 100$, and $\sqrt{1000000} = 1000$, etc., it follows that the square root of any number between 100 and 10000 lies between 10 and 100, and consists of *two* digits; that the square root of any number between 10000 and 1000000 lies between 100 and 1000, and consists of *three* digits, and so on.

Hence we see that *two* additional digits in a number lead to *one* additional digit in the square root.

If therefore we mark off the digits of a number in periods of *two*, beginning at the right hand, the number of these periods will be the same as the number of digits in the square root.

The left-hand period will sometimes contain only *one* digit.

256. We will consider the number 1444, obtained by multiplying 38 by 38. Its square root is 38.

In ascertaining this square root we determine first the number of *tens*, and then the number of *units*.

If we write 38 in the form $30 + 8$, and multiply it by $30 + 8$, thus,

$$\begin{array}{r}
 30 + 8 \\
 30 + 8 \\
 \hline
 30 \times 8 + 8^2 \\
 30^2 + 30 \times 8 \\
 \hline
 30^2 + 2 \times 30 \times 8 + 8^2
 \end{array}$$

we see that $1444 = 30^2 + (2 \times 30 + 8) \times 8$.

We can find the number of *tens* in the square root by ascertaining what multiple of 10 has its square *next less than* 1444; this is clearly 30, for $40^2 = 1600$ and is too great.

Having found the *tens* we next find the *units*.

Subtracting 30^2 or 900 from 1444, the remainder is 544; hence 544 must be equal to $(2 \times 30 + 8) \times 8$.

If we could divide 544 by $2 \times 30 + 8$ we should get the number of units 8 at once. But, since the divisor itself involves this unascertained number 8, the plan adopted is to use the part 2×30 as a **trial divisor**, to find by means of it a **trial quotient**, and then to see whether

$$(2 \times 30 + \text{trial quotient}) \times \text{trial quotient} = 544.$$

The first trial quotient may not prove correct; if it proves to be too great, we try the number next less, and so on.

In this particular case, if we divide 544 by 2×30 , i.e., by 60, we get a trial quotient 9. But $(60 + 9) \times 9 = 621$, and is too great; we therefore try 8, and we find that $(60 + 8) \times 8 = 544$.

The digits of the square root 38 are thus found in succession.

The above operations may be stated concisely thus:—

$$\begin{array}{r}
 14,44 \overline{) 30 + 8,} \quad \text{or thus,} \quad 14,44 \overline{) 38.} \\
 \underline{900} \\
 60 + 8 \overline{) 544} \\
 \underline{544} \\
 0
 \end{array}$$

In the latter form, which is the practical one, the process is briefly stated as follows:—

- (1) We mark off the digits in twos, beginning at the right hand.
- (2) The greatest square in 14 is 9, and its square root is 3. We place 3 in the root place. We multiply 3 by 3, and subtract the product from 14. The remainder is 5.
- (3) To this remainder we annex the period 44, and our dividend becomes 544. Twice the root digit 3 is 6, so we put 6 in the divisor.

(4) Instead of 60 we take 6 as a *trial divisor*, and we take 54 as a *trial dividend*. Proceeding as explained above, we find that 8 is the true quotient. We annex 8 to the 3 in the root place, and also to the divisor. We multiply 68 by 8, and, subtracting the product from 544, we get no remainder, and the operation is completed.

The square root is thus 38.

257. In this way a square root can be obtained, however many digits the number may have.

Example. Find the square root of 56644.

1st. Mark off the digits in periods of *two*, beginning at the right hand.

2nd. The root nearest to that of the first period 5 is 2. The square of 2 is 4. Subtracting 4 from 5, the remainder is 1. Bringing down the second period, 166 is the next dividend.

3rd. The *trial divisor* is 2×2 or 4, and the *trial dividend* is 16. The true quotient is 3. Place 3 as the second digit in the root, and also annex 3 to the *trial divisor*.

4th. The next *trial divisor* is 46, obtained by doubling the 23 in the root; and the *trial dividend* is 374.

$$\begin{array}{r} 25,66,44 \quad | \quad 238 \\ \underline{4} \\ 43 \, 166 \\ \underline{129} \\ 468 \, 3744 \\ \underline{3744} \end{array}$$

EXERCISE 132.

Find the square root of

- | | | | |
|-----------------|-----------------|-----------------|-----------------|
| (1) 361. | (2) 3481. | (3) 6889. | (4) 8649. |
| (5) 22201. | (6) 14161. | (7) 64009. | (8) 226576. |
| (9) 253009. | (10) 546121. | (11) 14837904. | (12) 39740416. |
| (13) 64432729. | (14) 10588516. | (15) 22099401. | (16) 25836889. |
| (17) 420391001. | (18) 930677049. | (19) 676208016. | (20) 951228964. |

To find the Square Root of a Decimal.

258. It follows from the nature of decimals that the method just explained is applicable to pure and to mixed decimals in exactly the same way as to integers. One or two points, however, must be noticed.

(1) We mark off the periods not from the *end* of the decimal, but from the *decimal point* both ways, right and left, thus—3,21,54,8.

(2) We put a decimal point in the square root immediately before the digit obtained from using the first period in the decimal part of the number.

(3) If the given decimal has an *odd* number of decimal places, we annex a 0 in order to make an exact number of pairs; and we may then annex as many additional pairs of 0's as we please.

(4) If the square root of an integer does not come out exactly, we may approximate to it by affixing a decimal point and 0's, and then continuing the process.

Example. Find the square root of 1127·6164.

Before we bring down the period 61, which comes next after the decimal point in the number, we put a decimal point in the root.

The required square root is 33·58.

$$\begin{array}{r}
 3 \overline{) 11, 27, 61, 64} \quad \underline{33 \cdot 58} \\
 \underline{9} \\
 63 \overline{) 227} \\
 \underline{189} \\
 665 \overline{) 3861} \\
 \underline{3325} \\
 6708 \overline{) 53664} \\
 \underline{53664}
 \end{array}$$

EXERCISE 133.

Find the square root of

- (1) 1·96. (2) 86·49. (3) 28·8369. (4) 331·24.
 (5) 1015·6969. (6) 170·3025. (7) 100·861849. (8) 676·208016.
 (9) ·042849. (10) ·00000961. (11) ·00010201. (12) ·0000001849.
 (13) ·00005476. (14) ·00047961. (15) ·0000005184. (16) ·0000021904.
 (17) ·0000063001. (18) ·0003272481. (19) ·00014161. (20) 8264·446281.

To find the Square Root of a Vulgar Fraction.

259. We know that $\frac{2}{3} \times \frac{3}{2} = 1$; conversely, the square root of $\frac{2}{3}$ is $\frac{\sqrt{2}}{\sqrt{3}}$; and generally, whenever the numerator and denominator are both squares, the root of the fraction is a fraction whose numerator is the root of the original numerator, and denominator the root of the original denominator.

The fraction whose square root is required must first be reduced to its lowest terms.

Example. Find the square root of $\frac{49}{256}$.

$$\sqrt{\frac{49}{256}} = \frac{\sqrt{49}}{\sqrt{256}} = \frac{7}{16}.$$

In *all* cases we can express the fraction as a decimal, and we can then find its square root as in Art. 258.

260. To determine the square root of a mixed number, we may express the number in the form of an improper fraction, and then, if the denominator is an exact square, we may determine the square root of numerator and denominator separately by the method of Art. 257. We can in all cases, however, express the mixed number in the decimal form and then proceed as in Art. 258.

Example. Find the square root of $20\frac{3}{4}$.

$$\begin{aligned}
 \text{(i)} \quad \sqrt{20\frac{3}{4}} &= \sqrt{\frac{83}{4}} = \frac{\sqrt{83}}{2} = \frac{9.1104\dots}{2} = 4.5552\dots; \\
 \text{(ii)} \quad \sqrt{20\frac{3}{4}} &= \sqrt{20.75} = 4.5552\dots
 \end{aligned}$$

EXERCISE 134.

Find the square root of

- | | | | |
|--------------------------|--------------------------|--------------------------|-------------------------|
| (1) $11\frac{1}{4}$. | (2) $10\frac{1}{4}$. | (3) $39\frac{1}{4}$. | (4) $210\frac{1}{4}$. |
| (5) $37\frac{1}{4}$. | (6) $65\frac{1}{4}$. | (7) $42\frac{1}{4}$. | (8) $9\frac{1}{4}$. |
| (9) $4\frac{9}{16}$. | (10) $516\frac{1}{16}$. | (11) $514\frac{1}{16}$. | (12) $3\frac{25}{16}$. |
| (13) $2352\frac{1}{4}$. | (14) $37\frac{1}{4}$. | (15) $17\frac{9}{16}$. | (16) $\frac{1}{4}$. |
| (17) $\frac{9}{16}$. | (18) $\frac{9}{16}$. | (19) $\frac{1}{4}$. | (20) $4\frac{9}{16}$. |

261. The following is an example of a problem involving the application of square root.

Example. Find the number of yards of fence required to enclose a square park containing 109 ac. 3 ro. 8 per. 9 sq. yds.

We first reduce the area to sq. yards.

109 ac. 3 ro. 8 per. 9 sq. yds.	
4	
$\overline{439}$	7 $\overline{53,14,41\ 729}$
40	49
$\overline{17568}$	142 414
304	284
$\overline{527040}$	1449 $\overline{13041}$
4892	13041
9	
531441 sq. yds.	

Area of park = 531441 sq. yds.,

\therefore length of side = $\sqrt{531441}$ yds.

= 729 yards;

\therefore length of fence = 729 \times 4 yards

= 2916 yards.

EXERCISE 135.

(1) A number of boys spent as many farthings each as there were boys, and the sum spent was £1 10s. 1d., how many boys were there?

(2) A square pavement contains 20736 square stones, all of the same size; how many rows of stones are there, and how many stones are there in each row?

(3) The area of a square garden is 44100 square feet; find the length of a side in yards.

(4) The area of a square field is $122\frac{1}{4}$ acres; find the length of a side in yards.

(5) How many yards of fencing are required to enclose a square park containing 832 ac. 2 ro. 25 per.?

- (6) The area of a square field is $2\frac{1}{2}$ acres; what would it cost to fence it round at $6\frac{1}{2}d.$ per foot?
- (7) How many yards long is a square lawn if the cost of levelling and turving it at $3s. 4\frac{1}{2}d.$ per square yard is £851. 18s. $4\frac{1}{2}d.$?
- (8) If the cost of turving a square tennis-court at $6d.$ per square yard is £11. 0s. $6d.$, find the cost of surrounding it with a fence at $2s. 6d.$ per yard.
- (9) A square field contains 13 ac. 81 sq. yds.; how long would it take a man to run round the boundary at the rate of $7\frac{1}{2}$ miles an hour?
- (10) How many times must a boy run round a square field covering 10 acres to run a mile?

MENSURATION OF RECTANGLES.

262. A **rectangle** is a four-sided figure whose opposite sides are parallel, and whose angles are right angles.

The floor, ceiling and walls of a room are generally rectangular.

263. The *area* of a figure is the surface included within the lines which bound it.

264. Suppose that a rectangular surface, such as the floor of a room, is 4 yds. long and 3 yds. wide; we can divide the sides into 4 equal parts and 3 equal parts respectively, each part being 1 yard. If now through the points of division we draw lines parallel to the sides in the manner shown in the margin, it is clear that we divide the surface into 4 rows of smaller areas, and that there are 3 of the smaller areas in each row. Moreover each of the smaller areas is a square yard, and therefore the area of the surface is 3×4 square yards.



Hence, to find the number of *square yards* in a rectangular area, we multiply the number of *yards* in the length by the number of *yards* in the breadth. If the length and breadth be given in *feet* or in *inches*, the product will give the number of *square feet* or *square inches*.

265. Using the words, *length*, *breadth*, and *area* to denote the *number of units* in each respectively, the relation between them may be stated thus:—

$$\text{area} = \text{length} \times \text{breadth};$$

therefore . $\text{length} = \text{area} \div \text{breadth},$

and $\text{breadth} = \text{area} \div \text{length}.$

NOTE.—Artificers generally use the term *width* instead of *breadth*.

Example. Find the area of the floor of a room 12 ft. 8 in. long and 10 ft. 9 in. wide.

$$\begin{aligned}\text{Length of room} &= 12\frac{2}{3} \text{ ft.} = 12\frac{2}{3} \text{ ft.}, \\ \text{and width} &= 10\frac{3}{4} \text{ ft.} = 10\frac{3}{4} \text{ ft.}; \\ \therefore \text{area} &= 12\frac{2}{3} \times 10\frac{3}{4} \text{ sq. ft.} = 136\frac{1}{2} \text{ sq. ft.} \\ &= 136 \text{ sq. ft. } 24 \text{ sq. in.}\end{aligned}$$

EXERCISE 136.

Find the area of a room whose dimensions are

Length.	Width.	Length.	Width.
(1) 13 ft. 3 in.,	12 ft. 10 in.	(2) 24 ft. 5 in.,	21 ft. 8 in.
(3) 14 ft. 6 in.,	13 ft. 2 in.	(4) 16 ft. 8 in.,	12 ft. 6 in.
(5) 18 ft. 10 in.,	15 ft. 7 in.	(6) 8 ft. 7½ in.,	11 ft. 4½ in.
(7) 19 ft. 3 in.,	13 ft. 5 in.	(8) 20 ft. 6 in.,	15 ft. 9 in.
(9) 59 ft. 4 in.,	48 ft. 9 in.	(10) 34 ft. 9 in.,	26 ft. 6 in.

(11) If 2240 stones, each 9 inches square, will pave a court, find the area of the court.

(12) If a school-room be 42 ft. long and 20 ft. wide, how many boys will it accommodate, allowing 8 sq. ft. to each boy?

(13) How many panes of glass, each 2 ft. long and 1 ft. 5 in. wide, will glaze a window 8 ft. high and 5 ft. 8 in. wide?

(14) In a row of 25 houses each house has 17 windows, each window 4 panes, and each pane measures 18 in. by 9 in. What will be the cost of glazing all these windows at 6d. per sq. foot?

(15) Find the cost of laying a plot of ground, 40 yds. long and 100 ft. wide, with turfs each 1 yard long and 1 foot wide; the turfs when laid costing 6s. 9d. per hundred.

(16) How many marble slabs 1 ft. long and 9 in. broad would pave a hall 50 yds. long and 50 ft. broad? What will the cost be if the price of the slabs is 5s. per dozen?

(17) How many planks, each 13½ ft. long and 10½ in. wide, will be required for the construction of a platform 54 yds. long and 21 yds. broad? What will be the cost at 5½d. per sq. foot?

(18) Find the cost of paving a court, 59 ft. 4 in. long and 48 ft. 9 in. wide, at 4s. 6d. per sq. yard.

Carpeting Floors.

266. Carpet, as manufactured, is in long strips, of uniform width. The length of the strip required to cover a floor will necessarily depend upon the area of the floor and upon the width of the carpet.

Example 1. What length of carpet, 27 inches wide, will cover a floor $10\frac{1}{2}$ ft. by $15\frac{1}{2}$ ft.?

$$\begin{aligned}\text{Area of room} &= 10\frac{1}{2} \times 15\frac{1}{2} \text{ sq. ft.} \\ &= 2\frac{1}{2} \times 3\frac{1}{2} \text{ sq. ft.}; \\ \therefore \text{area of carpet} &= 2\frac{1}{2} \times 3\frac{1}{2} \text{ sq. ft.}; \\ \text{and width of carpet} &= \frac{9}{4} \text{ ft.}; \\ \therefore \text{length of carpet} &= 2\frac{1}{2} \times 3\frac{1}{2} \div \frac{9}{4} \text{ ft.} \\ &= 2\frac{1}{2} \times 3\frac{1}{2} \times \frac{4}{9} \text{ ft.} \\ &= 2\frac{1}{3} \text{ ft.} \\ &= \underline{72\frac{1}{3} \text{ ft.}}\end{aligned}$$

Example 2. Find the cost of the carpet in Ex. 1 at 4s. 6d. a yard.

$$\begin{aligned}\text{Length of carpet} &= 72\frac{1}{3} \times \frac{1}{3} \text{ yards}; \\ \therefore \text{cost of carpet} &= \frac{1}{3} \times 2\frac{1}{3} \times \frac{9}{2} \text{ shillings} \\ &= 108\frac{1}{3} \text{ shillings} \\ &= \underline{\underline{\pounds 5. 8s. 6d.}}\end{aligned}$$

EXERCISE 137.

Find (i) the length of carpet required, (ii) the cost of covering a floor whose length and breadth are respectively

- (1) 22 ft. 6 in. and 16 ft. 4 in.; carpet, $5\frac{1}{2}$ ft. wide, at 5s. 6d. per yd.
- (2) 8 yds. and $7\frac{1}{2}$ yds.; carpet, 2 ft. wide, at 4s. 11d. per yd.
- (3) 27 ft. and $17\frac{1}{2}$ ft.; carpet, 27 in. wide, at 6s. 3d. per yd.
- (4) 15 ft. and 14 ft. 7 in.; carpet, 2 ft. 11 in. wide, at 2s. $11\frac{1}{2}$ d. per yd.
- (5) 21 ft. 8 in. and 16 ft. 6 in.; carpet, $\frac{3}{4}$ yd. wide, at 3s. $4\frac{1}{2}$ d. per yd.
- (6) 12 ft. 9 in. and 16 ft. 6 in.; carpet, 33 in. wide, at 4s. 6d. per yd.
- (7) 24 ft. 6 in. and 18 ft.; carpet, 27 in. wide, at 5s. 6d. per yd.
- (8) 19 ft. 6 in. and 11 ft. 3 in.; carpet, $\frac{3}{4}$ yd. wide, at 4s. $7\frac{1}{2}$ d. per yd.
- (9) 19 ft. 7 in. and 18 ft. 9 in.; carpet, 25 in. wide, at 5s. 6d. per yd.
- (10) 17 ft. 4 in. and 14 ft. 3 in.; carpet, $23\frac{3}{4}$ in. wide, at 5s. $2\frac{1}{2}$ d. per yd.

Area of Walls, Painting and Papering.

267. To find the area of a rectangular surface we multiply the length by the breadth (Art. 264), and to find the area of a wall we multiply the length of the wall by the height.

In such rooms as we can consider there are 4 walls, those which are opposite to one another being equal in area.

The area of each of two walls = length of room \times height,
and the area of each of the other two = breadth of room \times height;

therefore the total area of the four walls

$$\begin{aligned} &= 2 \times \text{length of room} \times \text{height} + 2 \times \text{breadth of room} \times \text{height} \\ &= 2 \times (\text{length} + \text{breadth}) \times \text{height} \\ &= \text{perimeter of room} \times \text{height}; \end{aligned}$$

the **perimeter** being the distance round the room

This expression gives the area of the walls, including that occupied by windows, doors, and fire places. To find how much *paper* would be wanted to cover the walls we must subtract the area occupied by these from the whole area.

NOTE.—Wall papers are sold in *pieces*, each 12 yards long and 21 inches wide.

Example 1. Find the area of the walls of a room, 12 ft. 6 in. long, 10 ft. 6 in. wide, and 10 ft. high.

$$\begin{aligned} \text{Perimeter} &= 2(12\frac{1}{2} + 10\frac{1}{2}) \text{ ft.} \\ &= 46 \text{ ft.}; \\ \text{height} &= 10 \text{ ft.}; \\ \therefore \text{area of walls} &= \underline{460 \text{ sq. ft.}} \end{aligned}$$

Example 2. If in the room in Ex. 1 there is one door, 7 ft. by 4 ft., and one window, 6 ft. by 3½ ft., how many pieces of paper (each 12 yards in length), 21 in. wide, would be required?

$$\begin{aligned} \text{Area of walls} &= 460 \text{ sq. ft.}, \\ \text{area of door} &= 7 \times 4 \text{ sq. ft.} = 28 \text{ sq. ft.}, \\ \text{and area of window} &= 6 \times 3\frac{1}{2} \text{ sq. ft.} = 21 \text{ sq. ft.}; \\ \therefore \text{area of paper required} &= (460 - 28 - 21) \text{ sq. ft.} = 411 \text{ sq. ft.}; \\ \text{width of paper} &= 1\frac{1}{2} \text{ ft.}; \\ \therefore \text{length of paper} &= 411 \div 1\frac{1}{2} \text{ ft.} \\ &= 411 \times \frac{2}{3} \text{ ft.} \\ &= 274\frac{2}{3} \text{ feet} \\ &= 78\frac{2}{3} \text{ yards}; \end{aligned}$$

$$\therefore \text{number of pieces required} = 78\frac{2}{3} \div 12 = \underline{7} \text{ (say).}$$

NOTE.—Generally one piece in ten must be allowed for waste in fitting the pattern.

Example 3. Find the cost of the paper in Ex. 2 at 5s. a piece.

$$\begin{aligned} \text{Cost of paper} &= 5 \times 7 \text{ shillings} \\ &= \underline{\underline{£1. 15s.}} \end{aligned}$$

EXERCISE 138.

Find the area of the walls of a room whose dimensions are

	Length.	Width.	Height.
(1)	24 ft.,	18 ft.,	15 ft.
(2)	18 ft.,	15 ft.,	13 ft.
(3)	21 ft. 5 in.,	18 ft. 7 in.,	10 ft.
(4)	23 ft. 8 in.,	15 ft. 10 in.,	11 ft. 11 in.
(5)	22 ft. 5 in.,	18 ft. 4 in.,	10 ft. 9 in.
(6)	14 ft. 9 in.,	9 ft. 3 in.,	10 ft. 6 in.
(7)	23 ft. 7 in.,	15 ft. 5 in.,	11 ft. 6 in.
(8)	25 ft. 7 in.,	16 ft. 9 in.,	13 ft. 6 in.
(9)	27 ft. 5 in.,	14 ft. 7 in.,	12 ft. 10 in.
(10)	18 ft. 9 in.,	13 ft. 3 in.,	14 ft. 6 in.

EXERCISE 139.

(1) What length of wall paper, 27 in. wide, will be required for a room 18 ft. long, 12 ft. wide, and 11 ft. high?

(2) A room is 23 ft. 6 in. long, 18 ft. 9 in. wide, and 12 ft. high; what length of paper, 1 ft. 9 in. wide, will be needed to cover the walls?

(3) How much paper, $\frac{3}{4}$ yd. wide, will be sufficient to paper a room 22 ft. 5 in. long, 12 ft. 1 in. wide, and 11 ft. 3 in. high? What will it cost at $4\frac{1}{2}$ d. per yard?

(4) Find the cost of papering a room 25 ft. long, 18 ft. 6 in. wide, and 10 ft. high, with paper, 2 ft. wide, at 3d. a yard.

(5) Find the cost of papering a room 24 ft. 10 in. long, 13 ft. 5 in. wide, and 11 ft. 4 in. high, with paper, 1 ft. 11 in. wide, at $7\frac{1}{2}$ d. per yd.; allowing for an area of 108 sq. ft. occupied by doors and windows.

(6) A room is 21 ft. 4 in. long, 15 ft. 9 in. wide, and 14 ft. high. The door and windows occupy 65 sq. ft. Find the cost of papering the remaining surface with paper, 25 in. wide, at 3s. 9d. per piece of 12 yards.

(7) A room measures 16 ft. by 21 ft., and is 11 ft. high. There is a door 7 ft. by 3 ft., and 2 windows 8 ft. by 4 ft. Find the cost of papering the room with paper, 2 ft. wide, at $2\frac{1}{2}$ d. a yard.

(8) A room 25 ft. 7 in. long, 16 ft. 9 in. wide and 13 ft. 6 in. high, has 3 windows 5 ft. by 3 ft. 6 in., a door 7 ft. by 4 ft., and a fireplace 5 ft. by 4 ft. Find the cost of papering the room with paper, $\frac{3}{4}$ yd. wide, at 4s. 6d. per piece of 12 yards.

length. Find the cost of painting the walls of a room 24 ft. 9 in. long, 18 ft. 6 in. wide, and 11 ft. 6 in. high, at 3s. 9d. per sq. yard.

In 8. Find the cost of painting the walls of a room 30 ft. 6 in. long, 18 ft. 6 in. wide, and 10 ft. 6 in. high, at 1s. 8d. per sq. yard.

(11) A room is 45 ft. 6 in. long, 24 ft. 10 in. wide, and 13 ft. 4 in. high. The walls are to be painted at $10\frac{1}{2}d.$ per sq. yard, and the ceiling whitewashed at $1\frac{1}{2}d.$ per sq. yard. Find the cost.

(12) Find the cost of painting a room 21 ft. long, 15 ft. wide, and 11 ft. 4 in. high; the walls and doors being painted at $1s. 3d.$, and the ceiling at $9d.$ per sq. yard, allowance being made for two windows 7 ft. high by 3 ft. wide, and a fireplace 6 ft. wide by $4\frac{1}{2}$ ft. high.

MENSURATION OF RECTANGULAR SOLIDS.

268. A rectangular solid has six faces, each of which is a rectangle. If such a solid be 4 feet long, 3 feet wide and 2 feet high, we can divide the edges respectively into 4, 3, and 2 equal parts, each of which parts will be 1 foot; and if we draw planes through the points of division as in the diagram, the solid will be separated into a number of smaller blocks each



of which is a *cubic foot*; and since there are two layers, in each of which there are 3×4 blocks, we see that there are $2 \times 3 \times 4$ blocks altogether, and that the solid therefore contains $2 \times 3 \times 4$ cubic feet.

We can thus find the **volume**, or **cubical content**, of any rectangular solid by expressing the length, breadth, and height in terms of a common unit of measurement, and multiplying the three numbers together. The product will give the volume of the solid in terms of the *corresponding* unit in cubic measure.

269. Using the words volume, length, breadth and height to denote the *number of units* in each respectively, the relation between them may be stated thus:—

$$\begin{aligned} \text{volume} &= \text{length} \times \text{breadth} \times \text{height}; \\ \text{therefore} \quad \text{length} &= \text{volume} \div (\text{breadth} \times \text{height}), \\ \text{breadth} &= \text{volume} \div (\text{length} \times \text{height}), \\ \text{and} \quad \text{height} &= \text{volume} \div (\text{length} \times \text{breadth}). \end{aligned}$$

Example Find the cubic capacity of a rectangular block of wood 16 ft. 6 in. long, 9 in. wide, and 4 in. thick.

$$\begin{aligned} \text{Length} &= 16\frac{1}{2} \text{ ft.}, \\ \text{width} &= \frac{3}{4} \text{ ft.}, \\ \text{thickness} &= \frac{1}{3} \text{ ft.}; \\ \therefore \text{cubic capacity} &= 16\frac{1}{2} \times \frac{3}{4} \times \frac{1}{3} \text{ cu. ft.} \\ &= 10\frac{1}{4} \text{ cu. ft.} \\ &= \underline{4\frac{1}{4} \text{ cu. ft.}} \end{aligned}$$

EXERCISE 140.

Find the cubical content of a rectangular solid whose dimensions are

	Length.	Width.	Depth.		Length.	Width.	Depth.
(1)	10½ ft.,	5½ ft.,	4 ft.	(2)	6½ ft.,	2½ ft.,	3½ ft.
(3)	3 ft. 5 in.,	4 ft. 9 in.,	8 ft. 7 in.	(4)	3 ft. 4 in.,	2 ft. 5 in.,	1 ft. 7 in.
(5)	7 ft. 6 in.,	5 ft. 8 in.,	10 in.	(6)	10 ft. 9 in.,	6 ft. 4 in.,	4 ft. 2 in.
(7)	17 ft. 3 in.,	14 ft. 7 in.,	11 ft. 2 in.				
(8)	45 ft. 8 in.,	19 ft. 1½ in.,	16 ft. 9 in.				

(9) Find the height of a room which is 12 ft. long, 10 ft. wide, and contains 1440 cu. ft. of air.

(10) If 17 cu. ft. 594 cu. in. of oak are required to floor a room 13 ft. 6 in. long, and 12 ft. 4 in. wide, what is the thickness of the boards?

(11) Find the cost of excavating a cellar whose length, breadth and depth are respectively 6 yds., 16 ft., and 7 ft., at 9d. per cu. yard.

(12) How many bricks each 9 in. long, 4½ in. wide, and 3 in. deep, will be needed for a wall 25 yds. long, 15 ft. high, and 1 ft. 10½ in. thick?

(13) A room, 42 ft. by 20 ft., accommodates 105 children. What must be the height of the room if each child has 83½ cu. ft. of air?

(14) A cu. foot of wood weighs 20 lb. Find the weight of 10 planks, each 30 ft. long, 1 ft. wide, and 1 in. thick.

(15) What is the value of a block of stone 5 ft. 3 in. long, 2 ft. 4 in. wide, and 1 ft. 2 in. thick, at 4 guineas per cu. foot?

(16) A cubic yard of gravel weighs 30 cwt. A cartload is a ton. How many cartloads of gravel can be dug out of a hole 10 ft. 6 in. long, 6 ft. wide, and 8 ft. deep?

(17) A number of blocks measuring 9 in. long, 5½ in. broad, and 1½ in. deep, exactly fill a box whose length is 1½ ft., breadth 11½ in., and depth 13½ in. How many blocks are there?

(18) Two thousand packets, each 4 in. long, 3 in. broad, and 3 in. deep, are required to be packed in a box of which the inside dimensions are 4 ft. 3 in. in length, 3 ft. 4 in. in breadth, and 2 ft. 6 in. in depth. How many must be left out?

THE COINAGE OF CERTAIN COUNTRIES.

270. In several countries the coinage has been arranged to suit the decimal notation.

In France, Belgium, and Switzerland the principal coin is called a franc, and this amount is divided into 100 equal parts, called centimes.

In consequence of the relation between the centime and the franc, all operations with money are much simplified.

For example, 513 centimes = 5·13 francs,
 59609 = 596·09 francs,
 13 = 13 of a franc,
 2 = ·02 of a franc.

Again, 5 francs 69 centimes = 5·69 francs = 569 centimes.

271. The reduction of centimes to francs is therefore effected by inserting a decimal point to the left of the second figure from the end, and the reduction of francs to centimes by removing the point.

EXERCISE 141.

Reduce to francs

- | | | | |
|--------------|--------------|-------------|-------------|
| (1) 35010 c. | (2) 3161 c. | (3) 4035 c. | (4) 6234 c. |
| (5) 15601 c. | (6) 34001 c. | (7) 15 c. | (8) 57 c. |
| (9) 4 c. | (10) 3 c. | (11) 1 c. | (12) 2·5 c. |

Reduce to centimes

- | | | | |
|------------------|------------------|-----------------|-----------------|
| (13) 56 f. 34 c. | (14) 48 f. 95 c. | (15) 4 f. 15 c. | (16) 3 f. 82 c. |
| (17) 5 f. 9 c. | (18) 25 f. 8 c. | (19) 36 f. | (20) 50 f. |
| (21) 62 f. | (22) 120 f. | (23) 205 f. | (24) 400 f. |

272. The principal monetary systems are the following ; most of them are now decimal.

Gold-Standard Countries.

Name.	Standard coin.	Grains of fine gold.	Exchange value.	Subdivisions.
United Kingdom	Sovereign	113·0016	£1	20 shillings
Latin Union	Franc	4·4803	9·51 <i>d.</i>	100 centimes
Germany	Mark	5·5313	11·745 <i>d.</i>	100 pfennige
Austria-Hungary	Florin	9·4099	19·985 <i>d.</i>	100 kreutzers
Holland	Florin	9·3459	19·84 <i>d.</i>	100 cents
Norway, Sweden } and Denmark }	Krone	6·2227	13·2 <i>d.</i>	100 ore
United States	Dollar (gold)	23·2199	49·29 <i>d.</i>	100 cents
Turkey	Pound	102·0804	18 <i>s.</i> 0 8 <i>d.</i>	100 piastres

The States which form the "Latin Monetary Union" are France, Italy, Switzerland, Belgium, and Greece, and their coins are alike in quality and weight. The same system has been adopted in part by Spain, but it has not joined the Union. The *francs* and *centimes* of France are however called *lire* and *centesimi* in Italy, *pesetas* and *centimos* in Spain, *drachmas* and *lepta* in Greece.

Silver-Standard Countries.

<i>Name.</i>	<i>Standard coin.</i>	<i>Grains of fine silver.</i>	<i>Subdivisions.</i>
India	Rupee	165·0	16 annas
Russia	Rouble	277·7221	100 copeks
Mexico	Dollar (silver)	377·168	100 cents

Exchange value.

At the present time (Jan. 1899) the exchange value of a rouble is about 2s. 1d., and of a silver dollar about 1s. 11½d., but these values vary with the market price of silver. A rupee is subject to special conditions and exchanges for about 1s. 4d.

EXCHANGE.

273. In finding the equivalent in the monetary system of one country of a given amount expressed according to that of another, we determine what is called the **rate of exchange**.

274. When English money is exchanged for French money, the *nominal* rate is 25 francs (or, more accurately, 25·22½ francs) for £1; and in the case of American money, one dollar (gold) for 4s. 2d. These rates are called the **par of exchange**, and depend upon the relative weights of pure gold.

The use of the term *par*, to express the equality between the nominal rate and the actual rate, is similar to the use of the same word to express that the nominal value of a share or of stock is the same as its marketable value.

EXERCISE 142.

Exchange

France.

- (1) £246. 13s. 10d. at 25 fr. 18 c. per £1.
 (2) £256. 18s. 4d. at 25 fr. 30 c. per £1.

Italy.

- (3) £335 at 25 l. 20 c. per £1. (4) £460 at 25 l. 22 c. per £1.

Greece.

- (5) £370 at 25 dr. 10 l. per £1. (6) £149 at 25 dr. 30 l. per £1.

Spain.

- (7) £85. 10s. at 9½d. per peseta. (8) £92. 11s. at 9d. per peseta.

Portugal.

- (9) £450 at 4 mil. 110 reis per £1.
 (10) £565 at 4 mil. 130 reis per £1.

NOTE.—1000 reis = 1 milreis.

Germany.

- (11) £632. 10s. at 20 mk. 36 pf. per £1.
 (12) £742. 15s. at 20 mk. 64 pf. per £1.

Austria.

- (13) £320 at 11 fl. 54 kr. per £1. (14) £860 at 10 fl. 3 kr. per £1.

Holland.

- (15) £850 at 12 fl. per £1. (16) £222 at 11 fl. 93 c. per £1.

Norway.

- (17) £1250 at 18 krone 17 ore per £1.
 (18) £3574 at 18 krone 22 ore per £1.

United States.

- (19) £150. 10s. at 47½d. per dollar. (20) £480. 5s. at 49d. per dollar.

Russia.

- (21) £370 at 6 r. 25 co. per £1. (22) £645 at 6 r. 45 co. per £1.

India.

- (23) £198 at Rs. 9. 10 aa. per £1. (24) £279 at Rs. 10. 4 aa. per £1.

FOREIGN BILLS OF EXCHANGE.

275. Business transactions between one country and another are commonly carried on with what are called **Foreign Bills of Exchange**, or briefly, **Foreign Bills**, by means of which large accounts may be settled without sending from one country to another any large quantities of gold or silver.

276. The medium between a merchant who wants to buy a bill and a merchant who wants to sell one is generally a **bill-broker**, who sells to the one and buys from the other.

For example, suppose *A* in London sends goods to *B* in Paris, and suppose *C* in Paris sends goods of exactly the same value to *D* in London; *C* then draws a bill on *D*, which bill *D* accepts.

This is a case in which *D* in London owes *C* in Paris exactly as much as *B* in Paris owes *A* in London, and the whole might be settled by *D* paying *A*, and *B* paying *C*. For this purpose *B* in Paris may buy the bill which *C* has drawn on *D* who is in London, and may send it by post to *A* who is also in London. *A* then gets the money from *D* and the matter is at an end, without any gold or silver having passed between the two places.

When Paris owes London more than London owes Paris, bills on London are sought after in Paris, and a merchant there is willing to pay more for them than their nominal value; in Paris bills on London are then at a **premium**, and in London at the same time bills on Paris are at a **discount**.

277. The rate of exchange, which varies from day to day, is called the **course of exchange**.

278. The following are examples of direct rates of exchange between two countries.

In actual business transactions, brokerage, stamp duty, interest or discount, and commission, if any, would have to be taken into consideration.

The usual charges of bill-brokers for buying bills are $\frac{1}{8}$ to $\frac{1}{16}$ per cent.

Example 1. A merchant in London buys a bill on Paris for 8534·4 francs; how much will have to be paid when the course of exchange on Paris is 25·4 francs?

The cost of a bill on Paris for 25·4 fr. = £1,

$$\begin{aligned} \therefore \dots\dots\dots 8534\cdot4 \text{ fr.} &= \text{£} \frac{8534\cdot4}{25\cdot4} \\ &= \text{£}336. \end{aligned}$$

Example 2. A merchant in Paris buys a bill on London when the course of exchange on London is 25·2 francs; for how much must the bill be drawn to be worth 8534·4 francs in Paris?

The face value of a bill on London worth 25·2 fr. = £1.

$$\begin{aligned} \therefore \dots\dots\dots 8534\cdot4 \text{ fr.} &= \text{£} \frac{8534\cdot4}{25\cdot2} \\ &= \text{£}338. 13s. 4d. \end{aligned}$$

Example 3. A debt of 8534·4 francs is owed by a London merchant to one in Paris. The London rate of exchange is 25·4, the Paris rate is 25·2. Find which is cheaper; (i) to buy direct bills on Paris, or (ii) to instruct his creditor to draw upon him in London.

From the solutions to Ex. 1 and 2 it will be seen that by the first method there is a saving of £2. 13s. 4d.

EXERCISE 143.

Find, to the nearest penny, the cost of a bill of exchange in London for

- (1) 65000 francs on Paris at 25·30 (francs per £1).
- (2) 86000 marks on Berlin at 20·60 (marks per £1).
- (3) 52500 florins on Vienna at 12·15 (florins per £1).
- (4) 68000 florins on Amsterdam at 12·1½ (12 fl. 1½ stivers per £1; 1 fl. = 20 stivers).
- (5) 75000 kronas on Stockholm at 18·40 (kronas per £1).
- (6) 62500 lire on Genoa at 26·82½ (lire per £1).
- (7) 40800 dollars on New York at 4·85 (\$ per £1).
- (8) 7528 rupees on Bombay at 1s. 4d. (for 1 rupee).
- (9) 9250 roubles on St Petersburg at 25½ (pence for 1 rouble).
- (10) 8360 milreis on Lisbon at 35½ (pence for 1 milreis).
- (11) 4070 taels on Shanghai at 2s. 5d. (for 1 tael).
- (12) 5080 dollars on Hong Kong at 1s. 10d. (for 1 dollar).

(13) If £1 in France exchanges for 25·2 francs, and if 20 francs in England exchange for 15s. 9d., how much do you lose in £100 by the two exchanges?

(14) A London merchant wishes to pay a debt of 4500 francs to one in Paris, when exchange is quoted at 25·4 in London and 25 45 in Paris. What gain is there to the former if, instead of remitting, he allows himself to be drawn upon?

(15) A London merchant wishes to pay a Berlin merchant for goods received, and finds the rates of exchange are London on Berlin, 20·53 marks for £1; Berlin on London, 20·27 marks for £1. Which will be the more advantageous—for him to remit bills to Berlin, or allow himself to be drawn upon in London?

THE METRIC SYSTEM OF WEIGHTS AND MEASURES.

279. A decimal system of weights and measures based on the principle that each unit in it is *ten* times as great as the next smaller has been adopted in France and most European countries. In England it is used chiefly for scientific measurements, but is now legal for general use.

The unit of length is the **metre**, whence the system is called the **Metric System**. The metre can be readily compared with the English yard, for its length is a little more than 3 feet $3\frac{1}{8}$ inches.

Multiples of the metre are distinguished by prefixes derived from the Greek:—

deca = 10,	kilo = 1000,
hecto = 100,	myria = 10000.

Sub-multiples are distinguished by prefixes derived from the Latin:

deci = $\frac{1}{10}$ th,	centi = $\frac{1}{100}$ th,	milli = $\frac{1}{1000}$ th.
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Thus the TABLE OF LENGTH is as follows:—

10 metres (m.) = 1 deca-metre (Dm.)	$\frac{1}{10}$ of a metre = 1 deci-metre (dm.)
10 decametres = 1 hecto-metre (Hm.)	$\frac{1}{100}$ of a metre = 1 centi-metre (cm.)
10 hectometres = 1 kilo-metre (Km.)	$\frac{1}{1000}$ of a metre = 1 milli-metre (mm.)
10 kilometres = 1 myria-metre (Mm.)	

The abbreviations usually employed are given in the brackets.

280. The reduction of weights and measures expressed in the

metric system can therefore be performed very simply by inserting, or omitting, or changing the position of, the decimal point.

For example, the length which is represented by 6 *kilometres* 5 *hectometres* 2 *decametres* 7 *metres* 8 *decimetres* 3 *centimetres* may be expressed as 6·52783 kilometres, or 65·2783 hectometres, or 652·783 decametres, or 6527·83 metres, or 65278·3 decimetres, or 652783 centimetres, or 6527830 millimetres.

Example 1. Reduce 46·451 Km. to metres.

Moving the decimal point 3 places to the right gives the result at once.

Thus $46\cdot451 \text{ Km.} = \underline{46451 \text{ metres.}}$

Example 2. Express 5343·258 cm. in kilometres.

Moving the decimal point 5 places to the left gives the result at once.

Thus $5343\cdot258 \text{ cm.} = \underline{.05343258 \text{ Km.}}$

EXERCISE 144.

Express in kilometres

- | | | |
|-----------------|------------------|---------------------|
| (1) 753·452 Hm. | (2) 34·4325 Dm. | (3) 352·565 metres. |
| (4) 4200·75 dm. | (5) 3524·256 cm. | (6) 4503·0275 mm. |

Express in metres

- | | | |
|----------------|----------------|----------------|
| (7) 63246 Km. | (8) 73245 Dm. | (9) 42369 Hm. |
| (10) 75298 cm. | (11) 93452 dm. | (12) 96452 mm. |

Express in centimetres

- | | | |
|-------------------|--------------------|-------------------|
| (13) 5247·565 Km. | (14) 40·130635 Hm. | (15) 325·0075 Dm. |
| (16) 427·523 dm. | (17) ·00275 m. | (18) ·00075 Hm. |

Express in millimetres

- | | | |
|-------------------|-------------------|-------------------|
| (19) ·0017562 m. | (20) 385725·2 dm. | (21) 5275·65 cm. |
| (22) 7·425675 Dm. | (23) ·0000375 Hm. | (24) ·5437575 Km. |

281. The unit of *area* is a square of which each side is a metre in length. It is therefore a **square metre**.

In measuring land on a small scale it is customary to take as unit a *square decametre*, or a square whose side is 10 metres in length. This is called an **are**.

Only two other denominations are in common use; viz.,

100 *ares* = 1 *hectare* ($2\frac{1}{2}$ acres nearly),

and $\frac{1}{100}$ of an *are* = 1 *centiare*.

The areas of farms are expressed in *hectares*; the areas of countries and departments are expressed in *square kilometres*.

282. The unit of *volume* is a **cubic metre**. In measuring wood this is called a **stère**. It is equal to $35\frac{1}{2}$ cub. feet nearly.

283. The unit of *capacity* in liquid and dry measure is called a **litre**, and is equal in volume to a cube of which each edge is a decimetre in length. It is therefore a **cubic decimetre**.

Its multiples and sub-multiples are

10 litres = 1 decalitre (Dl.)	$\frac{1}{10}$ of a litre = 1 decilitre (dl.)
100 litres = 1 hectolitre (Hl.)	$\frac{1}{100}$ of a litre = 1 centilitre (cl.)
1000 litres = 1 kilolitre (Kl.)	$\frac{1}{1000}$ of a litre = 1 millilitre (ml.)
A litre = $1\frac{1}{4}$ pints nearly; 1 kilolitre = $27\frac{1}{2}$ bushels nearly.	

284. The unit of *weight* is the weight of a cubic centimetre of pure water. This weight is called a **gramme**.

For ordinary purposes only the gramme (g.) and the kilogramme (Kg.), are used. The latter is commonly called a *kilo*.

A kilogramme = $2\frac{1}{2}$ lb. avoirdupois nearly.

EXERCISE 145.

(1) If a kilo. be equal to $2\frac{1}{2}$ lb., how many grammes make 14 stone (avoi.)?

(2) How many kilos. are there in 720 lb., if 100 kilos. are equal to 1·9684 cwt.?

(3) How many kilometres may be measured from 393·708 miles, a metre being taken to be equal to 39·3708 inches?

(4) Express $89\frac{1}{2}$ miles in metres, 32 metres being taken to be equal to 35 yards.

(5) A kilometre being 1093·638 yards, find, to two places of decimals, how many metres there are in 5 miles.

(6) Find the value of 1 7625 metres of cloth at 3·78 fr. per metre.

(7) Find the cost of 82·125 kilos. of butter at 3 fr. 44 c. per kilo. (1 franc = 100 centimes.)

(8) Give, in English money, the value of 2 yards of silk worth $6\frac{1}{2}$ francs per metre. (1 metre = 39·37 inches; 1 franc = 9·4 pence.)

(9) How many hours will a train, going at the rate of 25 miles per hour, take in travelling from Paris to Madrid, a distance of 1450 kilometres? (1 metre = 39·3708 inches.)

(10) The annual rent of a hectare of land is 130 francs; find the rent of one acre in English money. (25 francs = £1; 100 hectares = 247 acres.)

THE CALCULATION OF PRICES.

Mental Rules.

285. The following rules should be known for the mental calculation of certain prices.

It is not quite desirable that a pupil should burden his memory with all these disconnected rules, but he should endeavour to grasp the principles which underlie them. If he can do that, he will soon be able to invent a rule to suit each case as it occurs.

(1) To find the cost of a dozen.

Reckon the given cost of one article in pence, and call the pence shillings.

Thus the cost of 1 dozen @ 1s. 4d. = 16s. ;

and the cost of 1 dozen @ 2s. 6½d. = 30s. 6d. = £1. 10s. 6d.

(2) To find the cost of a gross.

NOTE.—A gross = a dozen times a dozen.

Thus the cost of 1 gross @ 1s. 4d. = cost of 12 at 16s. = £9. 12s.

(3) To find the cost of 48.

Reckon as many shillings as there are farthings in the cost of one article.

NOTE.—This method can be applied to numbers such as 384, 432, and 528, when the number of dozens is large.

(4) To find the cost of a score.

Reckon the given cost of one article in shillings, and call the shillings pounds.

Thus the cost of 1 score @ 7s. 9d. = £7. 15s. ;

and the cost of 1 score @ £1. 2s. 7d. = £22½ = £22. 11s. 8d.

(5) To find the cost of 240.

Reckon as many sovereigns as there are pence in the cost of one article.

Thus the cost of 240 @ 1s. 5d. = £17.

(6) To find the cost of 100.

Reckon as many pence and twice as many shillings as there are farthings in the cost of one article.

Thus the cost of 100 @ 8½d. = 34d. + 68s. = £3. 10s. 10d.

(7) To find the cost of 365.

Reckon as many sovereigns, as many half-sovereigns, and five times as many pence as there are pence in the cost of one article.

Thus the cost of 365 @ 1s. 5d. = £17 + £8. 10s. + 85d. = £25. 17s. 1d.

(8) To find the cost of 313.

Reckon as many sovereigns, crowns, shillings, and pence as there are pence in the cost of one article.

Thus the cost of 313 (*a* 6s. 9d. = £81 + (5s. × 81) + 81s. + 81d.
 = £81 + £20. 5s. + £4. 1s. + 6s. 9d.
 = £105. 12s. 9d.

(9) To find the cost of any number of articles.

Find the cost of the nearest multiple of any of the above numbers, and add or subtract the cost of the remainder

Thus (by Rule 5) the cost of 239 articles (*a* 1s. 5d.
 = £17 - 17d. = £16. 18s. 7d.,

and the cost of 241 articles (*a* 1s. 5d. = £17 + 17d. = £17. 1s. 5d.

(10) To find the cost of a hundredweight, when the price per lb. is given.

Multiply 9s. 4d. by the number of pence in the price.

(11) To find the cost of any number of articles at 6s. 8d. and other aliquot parts of £1.

Multiply the number of articles by the fraction which expresses the relation that the cost of one article bears to £1.

At 6s. 8d. the number of £'s in the cost is one-third of the number of articles.

NOTE — We proceed in a similar manner when the prices are such as 4s., 5s., 1s. 8d., 1s. 4d., 1s. 3d., 13s. 4d., 15s., etc.

(12) The above rules must be modified for such prices as the following. —

(a) At 4s. 2d. Since 4s. 2d. = £ $\frac{2}{3}$ = £ $\frac{1}{3}$,
 add 0 to the number of articles and divide by 48.

(b) At 6s. 3d. Since 6s. 3d. = £ $\frac{5}{8}$ = £ $\frac{1}{2}$ + £ $\frac{1}{8}$, add 0 and divide by 32.

(c) At 16s. 8d. Since 16s. 8d. = £ $\frac{2}{3}$ = £ $\frac{1}{3}$, add 0 and divide by 12.

(d) At £1. 13s. 4d. Since £1. 13s. 4d. = £ $\frac{5}{3}$ = £ $\frac{1}{3}$,
 add 0 and divide by 6.

(e) At £16. 13s. 4d. Since £16. 13s. 4d. = £ $\frac{5}{3}$ = £ $\frac{1}{3}$,
 add 00 and divide by 6.

(f) At 12s. 6d. Since 12s. 6d. = £ $\frac{1}{2}$ = £ $\frac{1}{4}$, add 0 and divide by 16.

(g) At 8s. 4d. Since 8s. 4d. = £ $\frac{2}{3}$, add 0 and divide by 24.

(h) At 2s. 7½d. Since 2s. 7½d. = 2s. 6d. + $\frac{1}{16}$ of 2s. 6d., find the cost at 2s. 6d., and add a shilling for every sovereign in that cost.

NOTE — We proceed in a similar manner when the prices are such as 3s. 6d., 10s. 6d., and 7s.

(i) At 2s. 4½d. Since 2s. 4½d. = 2s. 6d. - $\frac{1}{8}$ of 2s. 6d., find the cost at 2s. 6d., and subtract a shilling for every sovereign in that cost.

NOTE. We proceed in a similar manner for such prices as 3s. 2d., 9s. 6d., and 10½d.

EXAMINATION PAPERS.

I. COLLEGE OF PRECEPTORS. JUNIOR FORMS.

1. If in 15 years 1196521 emigrants left England for Canada, 3048206 for the United States, 708225 for Australia and New Zealand, and 93113 for other places, how many emigrants were there in all?

Find the average number leaving each year.

2. From three millions three thousand and three take one million seventy thousand and seventeen. Explain each step of the working.

3. Divide 689436 by 33 by two methods.

4. Reduce 7482 halfpence to pounds; also, find how many three-penny-pieces would be worth 6742 half guineas.

5. How many ounces are there in 11 tons 25 lb.?

6. What is the value of 9753 sewing-machines at £3. 14s. 6 $\frac{1}{2}$ d. each?

7. If I bought 3 cwt. 19 lb. of tea for £35. 10s., what did I give per lb.?

8. Reduce $\frac{11224}{11224}$ to its lowest terms; also, bring to their least common denominator $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.

9. Reduce $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ to a simple fraction in its lowest terms, and then divide it by $\frac{1}{5}$.

10. Make out a bill for the following articles; show how it should be receipted, and say how much change there will be after payment with a £10 note:—3 $\frac{1}{2}$ cwt. of firewood, at 1s. 9d. per cwt.; 26 lb. of soap, at 7 $\frac{1}{2}$ d. per lb.; 5 $\frac{1}{2}$ lb. of tea, at 3s. 4d. per lb.; 34 lb. of sugar, at 5 $\frac{1}{2}$ d. per lb.; 3 $\frac{1}{2}$ yards of flannel, at 1s. 11 $\frac{1}{2}$ d. per yard; 29 yards of calico, at 10 $\frac{1}{2}$ d. per yard.

II. COLLEGE OF PRECEPTORS. JUNIOR FORMS.

1. What is the difference between six millions five hundred thousand and four, and two millions ninety thousand and nine?

2. Explain the meanings of Short Division and Long Division; and, by both methods, divide 1728576 by 576 (which equals $6 \times 8 \times 12$).

3. How many half-hours are there between 10 a.m. on March 8th and 11 p.m. on November 13th?

4. If an ounce of gold is worth £3. 17s. 10 $\frac{1}{2}$ d., what weight of the metal could be bought for £934. 10s.?

5. In a bicycle race the first man won by 175 yards 8 inches. How many turns of his front wheel (measuring 6 feet 4 inches round) was the second man behind?

6. In 7 cwt. 2 qrs. 14 lb., how many half-ounces are there?

7. Find the value of 1008 horses, each worth £37. 15s.

8. Add together $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$; and divide the sum by 7.

9. Multiply the difference between $\frac{1}{12}$ of $\frac{1}{2}$ and $\frac{1}{2}$ of $\frac{1}{12}$ by 12.
10. On the day commemorating the Queen's long reign a party of 44 people went for a holiday. Each paid 7s. 6½d. for railway fare. On reaching their destination, half of them went on the water, and were charged 1s. 2½d. each for boat hire. One-eleventh of them took a carriage, which cost each 5s. 9½d. One fourth of them hired bicycles, for which each paid 3s. 3½d., and the rest of the original party spent, in various ways, 6s. 8½d. each. How much did the whole outing cost?

III. COLLEGE OF PRECEPTORS. JUNIOR FORMS.

- Write in words 701011. What would the number become on removing the figure 1 on the extreme right?
- If the population of one town is one million seven hundred and seventy-six thousand five hundred and fifty-six, and that of another 203652, how many more people are there in the larger than in the smaller town?
- How many strokes does a clock (which strikes hours only) strike in 24 hours? And how many in the year 1896?
- A farmer bought 588 sheep at £1. 12s. 6d. each. What did the flock cost?
- How many men could be paid 16s. 4½d. each from a bag containing £265. 5s. 6d.?
- If a bicycle wheel measures 6 feet 8 inches round, how many times would it turn in 5500 yards?
- Out of a box containing 3 cwt. 2 qrs 14 lb of sugar, how many half-pound parcels could be made?
- How many minutes has a child lived in 5 years 3 weeks, reckoning one leap year?
- Find the value of $(\frac{2}{3} \text{ of } \frac{3}{4}) + \frac{1}{12} + (\frac{2}{3} \text{ of } \frac{1}{12})$, and subtract the number from 1.
- A man travelled 417 miles by steamer at a cost of 1½d. per mile; 914 miles by train at 3½d. per mile; 39 miles by coach at 6½d. per mile; 128 miles by palanquin, requiring 16 bearers, the total wage of each bearer being £1. 17s. 2½d. How much did the whole journey cost for conveyances?

IV. COLLEGE OF PRECEPTORS. THIRD CLASS.

- Multiply nine millions nine thousand and nine by 11, and write down in words the difference between the product and one hundred millions.
- Divide 12345678 by 567, using the factors 7, 9, 9; and explain how you obtain the remainder.
- How many sums of 14s. 5½d. may be paid out of £464. 1s. 1½d.?
- Reduce 11 million pounds to tons, and 9 million square inches to acres.

5. A cyclist travels one furlong in 25 seconds. At this rate what distance can he travel in an hour?

6. Find the prime factors of 1287 and 2145, and their greatest common measure (highest common factor); and the least common multiple of 7, 9, 14, 24, 27, 54.

7. Add together $3\frac{1}{2}$, $4\frac{1}{2}$, $5\frac{1}{2}$, $6\frac{1}{2}$, $7\frac{1}{2}$; and find the difference between $4\frac{1}{2}$ and $5\frac{1}{2}$.

8. Simplify $\frac{8\frac{1}{2} \text{ of } \frac{1}{11} \div 6\frac{1}{2} \text{ of } \frac{1}{7}}{3\frac{1}{2} \text{ of } 2\frac{1}{2}}$.

9. A man pays £6. 13s. 4d. income tax, when the tax is at 8d. in the pound, and £160 of his income is free from the tax. What is his income?

10. Find, by Practice, the value of a crop on 10 ac. 2 roods 27 poles of land at £12. 16s. 8d. per acre.

V. COLLEGE OF PRECEPTORS. THIRD CLASS.

1. Divide (i) 14835879 by 729, by Long Division; (ii) 47347 by 63, by repeated Short Division. Show how you get the remainder in the latter case.

2. Multiply £4327. 15s. 4½d. by 729.

3. How many sums of £27. 2s. 9d. may be taken out of £379. 4s. 9d.? If any sum be left, how many children may have sixpence each from it?

4. Reduce 104 tons 17 cwt. 3 qrs. 14 lb. 7 oz. to ounces (avoir.).

5. If 14 equal packets of tea weigh 3 qrs. 17½ lb., how much will 1047 similar packets weigh?

6. Obtain (i) the sum, (ii) the difference, (iii) the product of the mixed fractions $17\frac{1}{2}$ and $27\frac{1}{2}$; and express all your results in the form of mixed fractions.

7. Reduce $\frac{\frac{1}{2} - (\frac{1}{2} \text{ of } \frac{1}{4}) + \frac{1}{4}}{1\frac{1}{2} \times \frac{1}{2} \times \frac{1}{4} - 1}$ to a single fraction.

8. Obtain, by Practice, the value of 187 yds. 2 ft. 9 in. of gold braid, at £2. 15s. 6d. per yard.

9. In an orchard I notice there are 14 pear trees, twice as many apple trees, and 7 times as many other trees as there are apple and pear trees together. How many trees are there in all?

10. Determine the length of road between the 1st and 351st telegraph posts, any consecutive two of which are 1 fur. 24 po. $2\frac{3}{4}$ yds. apart.

11. A linear mile containing 1760 yards, prove that there are 640 acres of land in a square mile.

VI. COLLEGE OF PRECEPTORS. THIRD CLASS.

1. Multiply 14830201 by 3851, and verify your answer by division.

2. Divide 135 tons 4 cwt. 3 qrs. 14 lb. by 42; and reduce 9 miles 3 fur. 30 po. 1 yd. 2 ft. to feet.

3. From 271 times £35. 4s. 2d. take £9411. 6s. 8d., and divide the remainder by 89.

4. I buy 500 oranges at 2 for three-halfpence, and again 500 more at 2 for threepence, after which 60 of the better sort are eaten; I then sell the remainder at five farthings each. How much do I gain or lose?

5. Calculate, by Practice, the cost of 1896 boxes of cigars at £1. 19s. 5½d. each.

6. In what time will a careful workman save £45. 14s. 3d. who earns weekly £2. 10s. 9½d., and expends in five weeks £7. 12s. 4½d.?

7. Find the greatest common measure of 75582 and 42237, and the least common multiple of 12, 20, 24, 54, 81, 63.

8. Find the value of $6\frac{1}{2} + 7\frac{1}{2} + 8\frac{1}{2} + 9\frac{1}{2}$. Find also the difference between $6\frac{1}{2}$ and $4\frac{1}{2}$.

9. Reduce to their simplest values:—

$$(i) (3\frac{1}{2} \times 5\frac{1}{2} \times \frac{7}{5}) - (\frac{1}{2} \times \frac{1}{11}); \quad (ii) \frac{1}{12} \times \frac{4\frac{1}{2}}{6\frac{1}{2}} \times \frac{6\frac{1}{2}}{11\frac{1}{2}}.$$

10. A clock, which loses 6 minutes in 15 hours, is 5 minutes too fast at midnight on Monday. What time will it indicate on the following Thursday at 5 o'clock in the afternoon, true time?

VII. CAMBRIDGE LOCAL. PRELIMINARY.

1. Multiply sixty thousand and twenty-seven by seven thousand and nine, and state the result in words.

2. How many farthings are there in £159. 17s. 11½d.?

3. Find the total cost of three dozen pocket-handkerchiefs at 1s. 10d. each, 29 collars at 9½d. each, and 13 neckties at 1s. 11½d. each.

4. Reduce 735934 ounces to tons, etc.

5. Find the value of $4\frac{1}{2} + 6\frac{1}{2} + 11\frac{1}{2} + 1\frac{3}{4}$.

6. Simplify $4\frac{1}{2} \times 5\frac{1}{2} \div 3\frac{1}{2}$.

7. Express $\frac{7}{11}$ as a decimal; and find the value of 23·8 kilogrammes of copper at 2 francs 5 centimes per kilogramme (1 franc = 100 centimes).

8. Find the value of $\frac{1}{2}$ of £1 + $\frac{1}{4}$ of 10s. 6d. + $3\frac{1}{2}$ of £1. 1s. 4d.

9. If $\frac{7}{11}$ of an estate be worth £4655, what is the value of the remainder?

10. At what rate per cent. per annum will £12 amount to £13 in $2\frac{1}{2}$ years at simple interest?

VIII. CAMBRIDGE LOCAL. PRELIMINARY.

1. Subtract fourteen millions seven hundred and two thousand and fifteen from one hundred millions eighty thousand and twenty-four, and express the result in words.

2. Divide 8276181 by 723.
3. What sum of money will it take to pay the following bill:—
14 lb. of pork, at $6\frac{3}{4}d.$ per lb.; $10\frac{1}{2}$ lb. of beef, at $10\frac{1}{4}d.$ per lb.; $1\frac{1}{2}$ doz. kidneys, at $1\frac{3}{4}d.$ each; 9 lb. of sausages, at $8\frac{1}{2}d.$ per lb.?
4. Reduce 3 tons 2 cwt. 7 oz. to ounces.
5. From $4\frac{2}{5} + 3\frac{1}{3}$ take $2\frac{1}{2} + 3\frac{7}{8}$.
6. Simplify $3\frac{1}{3} \times 2\frac{2}{3} \times 1\frac{5}{6} \div 2\frac{1}{4}$.
7. Add together 29·024, 125·31, ·09063, and 571.
8. Find the sum of ·0975 of £6. 5s. and ·3125 of £1. 10s.; and reduce your result to the decimal of £3.
9. What will be the rental of 30 hectares 75 ares of land at 3 francs 5 centimes per are? (1 hectare=100 ares; 1 franc=100 centimes.)
10. Find the simple interest on £982. 7s. 6d. for 4 years at $3\frac{3}{4}$ per cent. per annum.

IX. OXFORD LOCAL. PRELIMINARY.

1. From one million one thousand and one subtract the third part of it, and express the remainder in words.
2. Divide 1 mile 1 fur. 5 po. 4 yds. by 27.
3. Simplify $5\frac{1}{4} - 2\frac{1}{3} + \frac{1}{3}$ of $5\frac{1}{2}$.
4. Reduce 3 qrs. 3 lb. 8 oz. to the decimal of 1 cwt.
5. Multiply ·1264 by 1·00875.
6. If a person whose income is £262 a year spends five-sixths of it, and gives away £21. 16s. 8d.; what fractional part of his whole income will he have left?
7. What is the value of an ingot of silver weighing 576 ounces, if 22 ounces of the silver are worth £2. 9s. $0\frac{1}{2}d.$?

X. OXFORD LOCAL. PRELIMINARY.

1. Divide thirty-three millions ninety-nine thousand and sixty-six by three thousand and six, and express the quotient in words.
2. Multiply 3 qrs. 10 lb. 8 oz. by 112.
3. Reduce 2 dwt. 12 grs. to the fraction of 2 oz. (troy).
4. Simplify the expressions:—
(i) $2\frac{1}{2} + 3\frac{1}{3} - 4\frac{1}{3}$; (ii) $2\frac{3}{4} \times 4\frac{4}{11} - 3\frac{1}{4}$.
5. Divide 3·9963 by ·21.
6. Find the cost of 6 miles 5 furlongs of wire at £1. 13s. 4d. a mile.
7. If it costs £1. 10s. 3d. to run a train 11 miles, how much will it cost to run it 150 miles under the same conditions?
8. Find the simple interest on £360. 10s. for 5 years at 4 per cent.

XI. OXFORD LOCAL. JUNIOR.

1. Simplify (i) $2\frac{1}{2} - 1\frac{1}{2} + 3\frac{1}{2} - 4\frac{1}{2}$; (ii) $7\frac{1}{2} \times 4\frac{1}{2} \div 5\frac{1}{2}$.
2. Reduce $\cdot 0375$ and $\cdot 0375$ to vulgar fractions in their lowest terms; and find the value of $\cdot 017$ of £2. 1s. 8d.
3. A so-called 9 gallon cask of beer contains less than the right quantity by 2 gills. How many half-pints of beer can be drawn from it?
4. Find the rent of 7 acres 2 roods 20 poles at £1. 11s. 6d. per acre.
5. If 18 lb. 3 oz. cost 6s. 0 $\frac{3}{4}$ d., what will be the cost of 2 cwt. 1 qr. 3 lb.?
6. Find the simple interest on £708. 6s. 8d. for $7\frac{1}{2}$ years at 6 per cent.
7. Find the square root of $\cdot 75\bar{3}$ correct to three places of decimals.
8. A man has to be at a certain place in a certain time. If he walks at the rate of 4 miles an hour, he will be 5 minutes late; if he walks at the rate of 5 miles an hour, he will be 10 minutes early. Find the distance he has to walk.
9. The debts of a bankrupt amount to £2134. 10s. 6d. His assets consist of property worth £916. 15s. 4d. and of a bill for £513 due 4 months hence, simple interest being reckoned at 4 per cent. per annum. How much in the pound can he pay his creditors?
10. What money must be invested in a 4 per cent. stock at 85, to bring in a net income of £529 per annum, when the income-tax is 10d. in the pound, brokerage being reckoned at $\frac{1}{2}$ per cent.?

XII. OXFORD LOCAL. JUNIOR.

1. Simplify
 (i) $\frac{2444}{1111}$; (ii) $4\frac{1}{2} \frac{2\frac{1}{2}}{1\frac{1}{2}} + 2\frac{1}{2}$.
2. Divide $\cdot 021$ by the product of $1\cdot 5$ and $\cdot 28$; and express as a decimal of a hundredweight the difference between $\cdot 063$ of a quarter and $\cdot 42$ of a pound.
3. Multiply $5\cdot 62\bar{1}$ by $3\frac{1}{2}$; and find the value of $\cdot 058\bar{3}$ of £1.
4. Find the cost of 5 oz. 6 dwt. 16 grs. at £3. 7s. 6d. per oz.
5. If $\frac{1}{7}$ of a ton of metal is worth £68, what is the value of $5\frac{1}{2}$ tons?
6. In how many days will the simple interest on £2433 6s. 8d. at 5 per cent. per annum amount to exactly £100?
7. A can do a piece of work in 2 days, which B can do in $3\frac{1}{2}$ days. If A's wages are 38s. 6d. a week, and B's wages 27s. 6d. a week, what would A receive for doing a piece of work which B would do for £5. 10s.?
8. A train takes 20 minutes longer to do a journey when it is running 27 miles an hour than when it is running 30 miles an hour. Determine the length of the journey.

9. On what sum of money would the compound interest for 3 years at 5 per cent. per annum exceed the simple interest by £38. 2s. 6d.?

10. A rectangular tank, with sides and a bottom 1 inch thick, is 21 ft. long, 7 ft. 8 in. broad, and 11 ft. 1 in. deep, when measured externally. How many ounces of water will it contain, if 1 cubic foot of water weighs 1000 oz.?

XIII. CAMBRIDGE LOCAL. JUNIOR.

1. An engineering firm consumed seventy thousand and ninety tons of coal in three hundred and sixty-five days. Find the average consumption per day, correct to the nearest pound.

2. Express as a simple fraction in its lowest terms $\frac{3\frac{1}{2} + \frac{1}{2}}{2\frac{1}{2} + 1\frac{3}{8}}$.

3. Find the cost of 3 tons 3 cwt. 3 qrs. 12 lb. at £12. 16s. 8d. per ton.

4. A merchant buys 485 metres 70 centimetres of silk, and pays for it 3764 francs 20 centimes. Find the average cost per metre in francs and centimes (the answer to be correct to the nearest centime).

5. Simplify $(.09 + 2.069) \div (.905 - .89103)$.

6. Find the simple interest on £389. 6s. 8d. for 75 days at $3\frac{1}{2}$ per cent. per annum.

7. Simplify $.6918$ of £8. 9s. 7d. - $.0581$ of £90. 4s. 8 $\frac{1}{2}$ d. + $.46875$ of 4 guineas.

8. Find in yards the length of fence required to enclose a square field containing 7 acres 2 roods 18 poles 19 $\frac{1}{2}$ sq. yds.

9. The rateable value of a parish is £35130. Find the lowest rate per £ at which a rate, expressed in pence and half-pence, must be made so that the total receipts may exceed £3000.

10. A man invests in $4\frac{1}{2}$ per cent. stock at 123 $\frac{1}{2}$. What percentage is he actually receiving upon his outlay, after a deduction of 8d. in the £ has been made for income-tax? (Brokerage to be neglected.)

XIV. CAMBRIDGE LOCAL. JUNIOR.

1. Multiply seven hundred and eighty-three millions twenty-four thousand and nine by eight hundred and seven.

2. A man's yearly income is £741. 12s. 6d., and his daily expenditure a guinea and a half. How much did he save in the year 1894?

3. Find the sum of 16.94, .00037, 1.79375, 2.185, and .090087; and divide 50.939735 by .473.

4. Find the cost of 8 kilomètres 25 mètres of silk at 5 francs 75 centimes per mètre.

(1 kilomètre = 1000 mètres ; 1 franc = 100 centimes)

5. Find, by Practice, the cost of 5 tons 3 cwt. 3 qrs. 27 lb. 12 oz. at £14 per cwt.

6. Find the simple interest on £2020. 10s. at $4\frac{1}{2}$ per cent. per annum for 15 months.

7. Simplify $\frac{6\frac{3}{4} - 1\frac{1}{2} \text{ of } \frac{3}{4}}{8\frac{1}{2} + 4\frac{1}{2} \text{ of } \frac{3}{4}}$; and also

$(\frac{1}{4} \text{ of } 1 \text{ m. } 5 \text{ fur. } + \frac{1}{2} \text{ of } 4 \text{ po. } 2 \text{ yds. } + \frac{1}{4} \text{ of } 2 \text{ yds. } 2 \text{ ft. } 6 \text{ in.}) \times \frac{1}{4}$.

8. A bag contains £26. 5s. in half-crowns, florins, and shillings. There are three times as many florins and four times as many shillings as half-crowns. Find how many coins of each kind the bag contains.

9. A carpet $19\frac{1}{2}$ ft. by $15\frac{1}{2}$ ft., and costing 8s. per square yard, is laid down in a room measuring 23 ft. by 17 ft., and the rest of the floor is covered with floor-cloth at $5\frac{1}{2}$ d. a square foot. Find the total cost correct to the nearest penny.

10. If a man gain 8 per cent. by selling eggs at 1s. 3d. a score, how much per cent. would he gain by selling them at 1s. a dozen?

11. A and B can run at the rate of $12\frac{1}{2}$ and $12\frac{1}{4}$ miles an hour respectively. If A give B ten yards' start, in what time will he overtake him? When A has run a mile will he be in front of or behind B?

12. A person invests £7878 in $2\frac{1}{2}$ per cent. consols at 101, and pays income-tax at 8d. in the pound; on the stock rising to 103 he sells out and invests the proceeds in railway stock at 156, yielding 5 per cent. free of income-tax. Find the increase in his net income. (Brokerage to be neglected.)

XV. COLLEGE OF PRECEPTORS. SECOND CLASS.

1. If Neptune be 2706920000 miles distant from the Earth, and if light travels at the rate of 185000 miles a second, in what time will it traverse the space from one planet to the other?

2. A shopkeeper sells a variety of articles at $\frac{1}{4}$ d., $\frac{3}{4}$ d., $1\frac{1}{2}$ d., $3\frac{3}{4}$ d., $7\frac{1}{2}$ d., 1s. $7\frac{3}{4}$ d., and 2s. respectively. What will his bill for 5 dozen of each amount to, allowing a fourth part off for ready money?

3. Simplify $\frac{\frac{1}{2} \text{ of } \frac{1^5}{1^5} - \frac{1^7}{1^5} \div \frac{1}{2}}{\frac{1}{2} + \frac{1^5}{1^5} - \frac{1^7}{1^5} \text{ of } \frac{1}{2}}$.

4. Divide .012003824 by .427.

5. Add together, without reducing to vulgar fractions, $\cdot\dot{7}\dot{2}$, $2\cdot8\dot{3}$, and $15\cdot9\dot{9}\dot{0}$, giving the result (i) in the form of a recurring decimal, (ii) in that of a vulgar fraction.

6. Obtain, by Practice, the value of 320 tons 7 cwt. 3 qrs. 25 lb. of coal at £1. 3s. 4d. a ton, to the nearest penny.

7. A large square court is paved with square tiles measuring 9 inches each way. Of these there are 167281. What is the length of each side of the court?

8. If 24 men, working 10 hours a day, dig a trench in $14\frac{1}{2}$ days, in how many days will 29 men, working 8 hours a day, dig one three times as large?

9. What will the simple interest on £10723. 10s. at $4\frac{1}{2}$ per cent. per annum amount to in 5 years?

10. What is the present value of £3420, due 4 years hence, reckoning interest at $3\frac{1}{2}$ per cent. per annum?

11. Express 9 grammes 8 centigrammes 8 milligrammes as a decimal of 4 kilogrammes 9 decagrammes.

12. The base of a certain cube contains .0625 square metres. Suppose the length of each of its sides increased by the fourth part of a metre, what will then be the cubical content of the resulting cube?

13. A farmer owns a certain number of horses worth 25 guineas each, the same number of ≈ 18 . 10s. each, twice as many pigs worth £2. 15s. 6d. each, and a lot of poultry, equal in number to all the rest, worth on the whole ≈ 34 . 2s. 6d. The value of all this stock is £486. 16s. 6d. How many animals of each kind has he?

XVI. COLLEGE OF PRECEPTORS. SECOND CLASS.

1. Determine the value of seven millions one hundred and twenty-eight trinkets at 1s. $4\frac{1}{2}$ d. per twelve dozen.

2. How many telegraph poles will be required for wires extending 73 miles 3 furlongs 16 rods, supposing the interval between any two poles to be 99 feet?

3. If 3 sheep per acre are sufficient for prairie land, how many sheep may feed upon a plot 240 miles long by 18 miles broad?

4. Reduce $6\frac{1}{2} - \{13\frac{2}{3} \times \frac{7}{10} \text{ of } \frac{5}{4} + \frac{1}{4} \text{ of } \frac{21}{10}\}$ to a single fraction in its lowest terms.

5. If, in a division sum, the dividend is 483.758 and the quotient is 99.95, what is the divisor?

6. Add together 4.5, 5.6, 7.16, and 8.279.

7. Find, by Practice, to the nearest penny, the value of 11 tons 17 cwt. 3 qrs. 21 lb. at £4. 17s. 6d. a ton. (Most easily done by Decimals.)

8. Obtain the amount of £72552. 10s., when put out at $4\frac{1}{2}$ per cent. simple interest for $5\frac{1}{2}$ years.

9. Square 11.111.

10. Obtain the square root of 49381.7284.

11. If a French kilometre be taken as containing 1093.4 English yards, what fraction will 7 decametres be of 49 miles?

12. If a litre equals a cubic decimetre and contains 1.76 pints, how many (i) hectolitres and (ii) pints, will be contained in a tank whose capacity is 22 cubic hectometres?

13. An exhibition has 17525 visitors. Of these a fifth hold season tickets, each costing a certain number of half-sovereigns. A fifth of the remainder purchase day tickets costing the like number of half-crowns; and the rest pay the same number of shillings for their evening tickets. The proceeds of the exhibition are £7991. 8s. What were the three entrance charges?

XVII. MINOR SCHOLARSHIPS. LONDON COUNTY COUNCIL, ETC.

Part 1. Rules.

1. Divide eight millions seven hundred and ninety-one thousand six hundred and five by eighty eight. (Use factors.)
2. Multiply 5 tons 13 cwt. 17 lb. 10 oz. by 9.
3. Reduce 120840 inches to furlongs.
4. Simplify $1\frac{1}{2}$ of $5\frac{1}{2} + \frac{4\frac{1}{2} - \frac{3}{15}}{1\frac{1}{15}} - \frac{6\frac{3}{4}}{1\frac{1}{8}}$.
5. Reduce $2\frac{1}{2}$ of £5. 11s. to the fraction of $2\frac{1}{16}$ of £4. 3s. 6d.
6. Express as decimal fractions $13\frac{3}{4}$ and $23\frac{1}{8}$. Divide .025 by 500.
7. What is the difference in pounds between $3\frac{1}{4}$ of 2 qr. 25 lb. and .25 of 4 cwt. 2 qr. 20 lb.?
8. Find, by Practice, the cost of 27 bush. 3 pks. 1 gall. at £4. 7s. 6d. per bushel.
9. Make out a bill for the following articles: 21 lb. at 1s. 11d. per lb.; 14 lb. at $2\frac{3}{4}$ d. per lb.; 7 lb. at 1s. $7\frac{1}{2}$ d. per lb.; 8 lb. at $3\frac{1}{2}$ d. per lb.

Part 2. Problems.

10. The smaller of two numbers is 782, and their difference is 156; what is their sum?
11. I bought 12 chairs at 8s. 9d. each, and broke two of them; at how much each must I sell the others so as to lose no money on the whole transaction?
12. A man wishes to pay three bills of £15, £9 15s., and £10. 17s. 6d. by means of one kind of coin; which is the largest coin he can use?
13. Find the cost of a carpet for a room 32 feet by $14\frac{1}{2}$ feet, the carpet being 27 inches wide, at 3s. per yard.
14. If 1 metre is equal to 1.1 yards, and 1100 yards equal to 1 verst, find how many versts there are in 2875 metres.
15. What sum of money will earn £100 in two years and twenty days at $2\frac{1}{2}$ per cent., simple interest?
16. Three men own a house worth £625; one has $\frac{2}{5}$ of it, and the second has $\frac{1}{5}$; what is the value of the third man's share?

17. If a litre is equal to .22 gallons, find to the nearest penny the value of a pint of liquid worth £2 a litre.

18. *A* can do a piece of work in 10 days, and *A* and *B* can do it together in 7 days; in what time can *B* alone do it?

19. If 10 compositors, each of whom sets 3 letters in 5 seconds, finish 27 pages in an hour and a half, how many compositors, each of whom sets 5 letters in 6 seconds, will complete 25 pages in an hour?

XVIII. MINOR SCHOLARSHIPS. LONDON COUNTY COUNCIL, ETC.

Part 1. Rules.

1. Divide £347. 1s. 3d. by 45. (Use factors.)
2. Give the answer to the following sum in addition :—

miles	furlongs	yards	feet	inches
3	6	128	0	9
7	7	88	2	3
10	3	25	2	6
18	5	205	1	11

3. Reduce two millions thirty thousand and sixty-four pints to bushels.

4. Simplify $\left(\frac{7\frac{1}{2} - 3\frac{1}{8}}{10\frac{3}{4}} \div \frac{5\frac{1}{2} + 1\frac{1}{2}}{6\frac{1}{2} - 2\frac{1}{3}}\right) \times \frac{7}{15}$.

5. Multiply 2 cwt. 2 qrs. 13 lb. by $3\frac{1}{4}$.

6. Multiply .003125 by .48, and divide the result by .000125.

7. Reduce £4. 17s. 6 $\frac{3}{4}$ d. to the decimal of £5.

8. Find, by Practice, the rent of 9 acres 3 roods 10 poles at £1. 17s. 8d. per acre.

9. Make out a bill for the following articles :—2 doz. pairs of gloves at 2s. 11 $\frac{1}{2}$ d. a pair; 37 yards of chintz at 9 $\frac{3}{4}$ d. a yard; one gross of buttons at 3 $\frac{1}{4}$ d. a dozen; 3 doz. yards of fringe at 1s. 3 $\frac{3}{4}$ d. a yard.

Part 2. Problems.

10. The smaller of two numbers is 347, and their difference is 58. What is their product?

11. Divide £183. 8s. 9d. among three persons, so that one of them may have half as much as each of the others.

12. Two cylinders measure 224 and 336 inches respectively in circumference. Find the shortest length of wire that can be wrapped round each an exact number of times.

13. How many bricks, each 9 inches long and 4 $\frac{1}{2}$ inches wide, will be required to pave the floor of a cellar 13 feet 6 inches square?

14. A manufacturer bought one ton of raw material for £24. 12s. 6d., and the cost of manufacture amounted to $\frac{1}{3}$ of that sum. He sold the whole of it when finished at the rate of 6½d. per lb. What was the amount of his gain?

15. If 189 persons occupy a room of which the dimensions are 84 feet, 27 feet, and 12½ feet, how many cubic feet of air are allowed to each?

16. The simple interest on a certain sum for 7½ years at 2½ per cent. is £107. 16s. 3d. What is the sum?

17. A foot-pound is the work done in lifting 1 lb. one foot high. What work (in foot-pounds) is done by a boy weighing 8 stone in lifting his own weight 25 times to a height of 3 feet 9 inches?

18. Taking a French metre to be equal to 39½ inches, find, approximately in yards, the difference between 5 miles and 8000 metres.

XIX. LONDON SCHOOL BOARD SCHOLARSHIPS.

1. The sum of two numbers is seventeen thousand and six, the smaller being five hundred and seventy; divide their difference by seven, and find the remainder.

2. Multiply 2 miles 3 furlongs 10 poles 3 yards by 103.

3. How many tons, etc., are there in 274553 ounces?

4. A greengrocer bought 420 oranges for £1. 1s.; he sold half of them at 9d. a dozen, and the rest at 7 for 6d.; what did he gain?

5. Two heaps of the same kind of shot weigh 150 tons 4 cwt. 2 qrs. and 124 tons 8 cwt. 2 qrs. respectively; what is the greatest possible weight of each shot?

6. Simplify $\frac{1}{17} \times 4\frac{1}{2} \cdot \frac{4\frac{1}{2}}{3} + \frac{1\frac{1}{2} - \frac{1}{17}}{\frac{2}{3} - \frac{1}{17}} \times \frac{5\frac{1}{2}}{9\frac{1}{2}}$.

7. Find the cost of covering a room, 34 ft. 9 in. long by 26 ft. 6 in. wide, with carpet at 2s. 3d. per square yard.

8. Simplify $\frac{.203 \times .003 \times .16}{.008 \times .0029}$.

9. How many men in 19 days could do a piece of work which 171 could do in 12 days?

10. Express in pounds the difference between .856 of 2 cwt. 26 lb., and 3.227 of 2 qrs. 10 lb.

11. If when meat is 9d. per lb. it costs £11. 16s. 3d. to supply 12 persons for 5 weeks, how much will it cost to supply 18 persons for 7 weeks, when meat is 6d. per lb.?

12. Find the difference between the simple and the compound interest on £6400 in 3 years at 3½ per cent. per annum.

13. If 15 quarters of wheat cost £46. 10s., what must they be sold for per quarter to gain 12½ per cent.?

14. Find the price of a 3 per cent. stock, when an investment of £434. 12s. 6d. produces an income of £14. 5s.

15. A vessel is full of a mixture of a spirit and water, in which there is found to be 17 per cent. by measure of spirit. Ten gallons are drawn off, and the vessel is filled up with water. The proportion of spirit is now found to be $15\frac{1}{2}$ per cent. How much does the vessel hold?

XX. PHARMACEUTICAL SOCIETY. PRELIMINARY.

1. A merchant bought 45 gallons of wine at 3s. $2\frac{1}{2}$ d per pint, and 18 gallons of wine for £15, and then mixing the whole with 22 quarts of water, sold it at the rate of 3s. 3d. per quart; how much did he gain or lose by the transaction?

2. Find the rent of 6 per. 4 sq. yds. 2 ft. 36 in. at 6s. 3d. per sq. yd.

3. Subtract $\frac{2\frac{1}{2}}{1\frac{1}{2}}$ of $\frac{3\frac{1}{2}}{6\frac{1}{2}}$ of $\frac{99\frac{1}{2}}{7\frac{1}{2}}$ from the sum of $9\frac{1}{2}$, $10\frac{1}{2}$, $17\frac{1}{2}$ and $1\frac{1}{2}$.

4. Reduce $3\frac{1}{2}$ d. to the decimal of 10s., and divide the result by 125.

5. A person is able to perform a journey of 1422 miles in $4\frac{1}{2}$ days when they are 10.164 hours long; how many days will he be in travelling 505.6 miles when the days are 8.4 hours long?

6. Divide 80 hectares of land between A, B, and C, so that A shall have 2 hectares 76 ares more than B, and C 11 hectares 12 ares more than B.

7. A stationer sold quills at 11s. per thousand, by which he cleared $\frac{1}{4}$ of his purchase money, but raised them to 13s. 6d. per thousand on their growing scarce; what did he clear per cent. at the latter price?

XXI. PHARMACEUTICAL SOCIETY. PRELIMINARY.

1. Multiply 98756 by 60324; and find what number added to the result will make it divisible by 90000 without remainder.

2. Multiply the difference between 11 cwt. 3 qrs. 17 lb. 10 oz. and 5 cwt. 2 qrs. 23 lb. 11 oz. by 528.

3. The captain and his two mates have respectively $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$ of the cargo allotted to them; the remainder belongs to 121 shareholders, and each of these receives £119. 10s.; what was the total value of the cargo?

4. Reduce 2.6 of £.877083 to the decimal of half a sovereign.

5. If with 34 kilo. of wool 25 m. of flannel 60 centim. wide can be made, what length of similar flannel 80 centim. wide can be made with 108 kilo. of wool?

6. The length of a street is 937 ft. 6 in., and its breadth 66 ft. 8 in.; find the cost of paving it at $8\frac{1}{2}$ d. per sq. yd.

7. A bankrupt has good debts to the amount of £456. 18s. 1d. and the following bad debts: £360. 7s. 10d., £120. 13s., and £19. 18s., for which he receives respectively 4s., 5s. and 9s. in the £; his own liabilities amount to £3408. 12s.; how much, to the nearest penny, can he pay in the £?

XXII. CIVIL SERVICE COMMISSION PRELIMINARY.

1. Multiply 3 miles 7 furlongs 18 yards by 73.
2. Divide 70 tons 9 cwt. 7 lb. by 45.
3. Reduce 3 qrs. 1 bushel 1 peck 1 quart to pints.
4. How many lb., oz., dwt., and grains are there in 11697 grains?
5. Find all the prime numbers that are common factors of 1155 and 660.
6. Find the least common denominator of the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.
7. Express $\frac{220}{1875}$ in its lowest terms.
8. Add together $9\frac{1}{2}$ and $7\frac{1}{3}$.
9. Subtract $4\frac{3}{8}$ from $11\frac{1}{2}$.
10. Multiply $2\frac{1}{2}$ by $\frac{1}{3}$.
11. Divide $5\frac{5}{8}$ by $3\frac{1}{2}$.
12. Add together 0.00125, 18.97, 45.946, and 0.1234.
13. Subtract 17.9625 from 20.00043.
14. Multiply 13.217 by 0.04108.
15. Divide 462288 by 96.31.
16. Express $\frac{1}{16}$ as a decimal fraction.
17. Express, in lowest terms, £2 5s. 4½d. as a fraction of £2. 7s. 5½d.
18. Express in hours, minutes, seconds, and the decimal of a second, 0.6123 of a day.
19. Find the dividend on a debt of £641 at 13s. 4½d. in the £.
20. Find the cost of 3 tons 7 cwt. 1 qr. 11 lb. at £4 13s. 4d. per ton.
21. If a man walks 140 miles in 7 days, how many miles would he walk in 4 days at the same rate?
22. If 5 gas-jets can burn 10 days for 5 hours each day at a cost of 4s. 3d., how many jets can burn 4 hours every evening for 15 days for £3. 16s. 6d.?
23. On selling certain goods I receive £87. 9s. 11d. and find that I have lost 17 per cent. on the transaction; what was the price I paid for them?
24. In an army of 24640 men, 2772 are reported ill; what percentage of the whole force does this represent?
25. How many tiles, each measuring 4½ inches by 6 inches, will be required to pave a courtyard 27 feet long by 18 feet broad?

XXIII. CIVIL SERVICE COMMISSION. PRELIMINARY.

1. Multiply £19. 17s. 6 $\frac{3}{4}$ d. by 62.
2. Divide 188 miles 5 fur. 16 po. 5 yds. by 42.
3. Reduce 19 tons 12 $\frac{1}{2}$ cwt. to ounces.
4. How many quarters, bushels, pecks, etc. are there in 1447 pints?
5. Find the g.c.m. of 1334, 2346, and 207.
6. What is the least number which contains 584 and 438?
7. Find a fraction with numerator 391 and equal in value to $\frac{17}{18}$.
8. Add together $17\frac{1}{2}$ and $9\frac{7}{10}$.
9. Subtract $2\frac{1}{2}$ from $5\frac{2}{7}$.
10. Multiply $7\frac{1}{11}$ by $5\frac{1}{2}$.
11. Divide $43\frac{1}{2}$ by $7\frac{1}{4}$.
12. Add together 21·009, 4·09837, 0·5392, and 201·9.
13. Subtract 2·42935 from 5·07.
14. Multiply 2·0905 by 23·04.
15. Divide 6·711265 by ·3025.
16. Express 0·28475 as a vulgar fraction in its lowest terms.
17. What decimal of 21 lb. 11 oz. 0 dwt. 10 grs. is 4 lb. 7 oz. 15 dwt. 5 grs.?
18. What is the value in pounds, shillings, pence, and the decimal of a penny of ·405 of 3 $\frac{1}{2}$ guineas?
19. Find the value of 20375 articles at 17s. 5d. per score.
20. Find the value of 5 qrs. 3 bush. 1 pk. 1 gall. at £1. 13s. 4d. per quarter.
21. If 19 boys earn £2. 7s. 6d., how many boys could be paid at the same rate with £4. 17s. 6d.?
22. Four railway cars with five compartments apiece, each of which carries eight passengers at the rate of 2d. each per mile, earn £20 on a given journey; how much will be earned on the same journey by eleven cars of seven compartments apiece, each carrying ten passengers at 1d. each per mile?
23. A man gains 5 $\frac{1}{2}$ per cent. by selling an article at 17s. 7d.; what did it cost him?
24. At a certain place sunshine was last year (1898) recorded on 305 days; express as a percentage of the whole year the number of days on which the sun was not seen there.
25. How many boards 4 ft. 3 in. long and 9 in. wide will floor a room 17 ft. long and 14 ft. 3 in. wide?

XXIV. CIVIL SERVICE COMMISSION. PRELIMINARY.

1. Multiply £20. 14s. 7d. by 59.
2. Divide 6 qrs. 5 bush 1 pk. 1 gall. by 56.
3. Reduce (i) 1 mile $6\frac{1}{2}$ furlongs to yards; and (ii) 42 kilometres $15\frac{1}{2}$ metres to millimetres.
4. Express 16375 grains in lb., oz., dwt, grs.; and find how many kilograms there are in 12467 grams, and how many grains remain over.
5. Find the G.C.M. of 4284, 1190 and 374.
6. Find the L.C.M. of 1218 and 870.
7. Reduce $\frac{3456}{128}$ to its lowest terms.
8. Add together $13\frac{1}{8}$ and $12\frac{1}{4}$.
9. Subtract $3\frac{1}{2}$ from $7\frac{7}{8}$.
10. Multiply $41\frac{2}{3}$ by $23\frac{1}{2}$.
11. Divide $8\frac{8}{11}$ by $4\frac{4}{7}$.
12. Add together 14.0907, 0.73, 415.6 and 2.00012.
13. Subtract 3.95764 from 41.63.
14. Multiply 33.005 by 0.7644.
15. Divide 0.41004 by 0.031.
16. Express 0.10625 as a vulgar fraction in its lowest terms.
17. What decimal of 3 qrs. 21 lb. is 1 qr. 16 lb. 10 oz.?
18. What is the value in pounds, shillings, pence, and the decimal of a penny of 0.625 of £3. 17s. 6d.?
19. Find the value of 36758 articles at £2. 17s. 9 $\frac{3}{4}$ d. each.
20. Find the value of 3 cwt. 3 qrs. 10 lb. 8 oz. at £1. 16s. 8d. per cwt.
21. If $4\frac{1}{2}$ yards of cloth cost 13s. 4d., how many yards can be bought for £5. 6s. 8d.?
22. If 8 horses can plough 28 acres of land, working 9 hours a day for 4 days, how many days will it take 6 horses to plough 49 acres, working 8 hours a day?
23. A man has an income of £2075; if he saves 23 $\frac{1}{2}$ per cent. of it, how much does he spend?
24. A man has 5000 sheep and sells 400 of them; what per cent. of his flock has he left?
25. How many yards of carpet will be wanted to cover the floor of a room, 16 ft. 6 in. long and 12 ft. 3 in. broad, the width of the carpet being 2 ft. 9 in.?

XXV. CIVIL SERVICE COMMISSION. PRELIMINARY.

1. Multiply 1 week 3 days 7 hours 15 seconds by 68.
2. Divide 70 lb. 7 oz. 16 dwt. by 81.
3. Reduce (i) 3 miles 6 furlongs 30 poles 3 yards to yards; and (ii) 27 kilometres 20 metres to metres.
4. Express 1715 quarts in quarters, bushels, pecks, etc.; and find how many kilolitres there are in 39625 litres, and how many litres remain over.
5. Find the g.c.m. of 1785 and 1683.
6. What is the least number which can be exactly divided by 18, 30, 45 and 48?
7. Reduce $\frac{3300}{8}$ to its lowest terms.
8. Add together $5\frac{2}{3}$ and $7\frac{1}{4}$.
9. Find the difference between $1\frac{3}{4}$ and $1\frac{1}{8}$.
10. Multiply $10\frac{1}{8}$ by $3\frac{3}{8}$.
11. Divide $7\frac{1}{8}$ by $5\frac{2}{7}$.
12. Add together 170·0644, 0·96437, 3·8405, and 25·00673.
13. Subtract 92·64382 from 203·07604.
14. Multiply 300·75 by 2·908.
15. Divide 19·323135 by 0·0603.
16. Express 0·053125 as a vulgar fraction in its lowest terms.
17. What decimal fraction of £3. 17s. is £3. 12s. 2½d.?
18. Find in yards, feet, inches, and the decimal of an inch, the value of 2·0637 poles.
19. What would 79 dozen articles cost at 10s. 7½d. each?
20. Find the value of 4 lb. 6 oz. 8 dwt. 16 grs. at £3. 7s. 6d. per lb.
21. If a tax on £75 is £4. 10s. 7½d., how much should it be on £48?
22. If a man walks 21 miles in 6 hours, how many minutes will he take to walk half a mile, his speed being increased in the ratio of 8 to 7?
23. How many quarts are there in 3¾ per cent. of 20 gallons?
24. If I sell for £30. 16s. what cost me £27. 10s., what is my gain per cent.?
25. A room is 30 feet long, and its width is four-fifths of its length; find the area of the floor in square yards.

ANSWERS.

I. NOTATION. PAGE 5.

(1) 11.	(2) 14.	(3) 41.	(4) 17.	(5) 71.
(6) 63.	(7) 101.	(8) 111.	(9) 110.	(10) 121.
(11) 108.	(12) 801.	(13) 811.	(14) 204.	(15) 214.
(16) 241.	(17) 717.	(18) 707.	(19) 770.	(20) 1700.
(21) 1717.	(22) 1771.	(23) 7007.	(24) 7070.	(25) 7017.
(26) 1700.	(27) 1001.	(28) 1011.	(29) 1111.	(30) 50505.
(31) 70070.	(32) 88818.	(33) 100100.	(34) 300030.	
(35) 909009.	(36) 880080.	(37) 1000001.	(38) 1001000.	
(39) 1100000.	(40) 10010010.			

2. NUMERATION. PAGE 6.

- (1) Eleven. (2) One hundred and one. (3) One hundred and ten.
 (4) Twelve. (5) Twenty-one. (6) One hundred and two.
 (7) Two hundred and one. (8) One hundred and twenty.
 (9) Two hundred and ten. (10) Twenty-three. (11) Thirty two.
 (12) Two hundred and three. (13) Twenty-nine. (14) Ninety-two.
 (15) Two hundred and nine. (16) Nine hundred and two.
 (17) Two hundred and ninety. (18) Nine hundred and twenty.
 (19) Nine thousand and two. (20) Nine thousand and twenty.
 (21) Nine thousand two hundred. (22) Nine thousand and twelve.
 (23) Nine thousand and twenty-one.
 (24) Nineteen hundred and two, or one thousand nine hundred and two.
 (25) Nine thousand nine hundred and ninety.
 (26) Ten thousand and one. (27) Twenty thousand five hundred.
 (28) Seventy thousand and sixty. (29) Seventeen thousand and seven.
 (30) Three hundred thousand and three.
 (31) Three hundred thousand three hundred.
 (32) Three hundred and three thousand and thirteen.
 (33) Four hundred and six thousand and sixty.
 (34) Four hundred and one thousand six hundred and six.
 (35) Five hundred and seven thousand and five.
 (36) Six hundred thousand eight hundred.

- (37) Nine hundred thousand and nine.
 (38) One million twenty thousand and three.
 (39) Two millions two thousand and two.
 (40) Three millions three hundred.

3. SIMPLE ADDITION. PAGE 7.

(1) 87.	(2) 90.	(3) 95.	(4) 83.	(5) 98.
(6) 82.	(7) 134.	(8) 121.	(9) 154.	(10) 171.
(11) 134.	(12) 160.	(13) 851.	(14) 910.	(15) 902.
(16) 1321.	(17) 1181.	(18) 1533.	(19) 9889.	(20) 6683.
(21) 10039.	(22) 10451.	(23) 11676.	(24) 16139.	(25) 18934.
(26) 11196.	(27) 12480.	(28) 13136.	(29) 11456.	(30) 12519.
(31) 97339.	(32) 156546.	(33) 83699.	(34) 87400.	
(35) 173552.	(36) 191633.	(37) 205875.	(38) 114037.	
(39) 156524.	(40) 216582.	(41) 942947.	(42) 576098.	
(43) 1294120.	(44) 962391.	(45) 1084009.	(46) 1321249.	
(47) 1155519.	(48) 1133206.	(49) 1432681.	(50) 1185216.	
(51) 843737.	(52) 212691.	(53) 924397.	(54) 416550.	
(55) 1922117.	(56) 1568080.	(57) 1038848.	(58) 1059343.	
(59) 937930.	(60) 1076915.	(61) 870731.	(62) 769047.	
(63) 752724.	(64) 1216506.	(65) 1458299.	(66) 1793752.	
(67) 182027.	(68) 3987124.	(69) 2447033.	(70) 2453421.	

4. SIMPLE SUBTRACTION. PAGE 11.

(1) 5153.	(2) 5133.	(3) 2265.	(4) 3732.	(5) 434.
(6) 2606.	(7) 491.	(8) 77.	(9) 1197.	(10) 4397.
(11) 1752.	(12) 1343.	(13) 6649.	(14) 2997.	(15) 5611.
(16) 3137.	(17) 2913.	(18) 268.	(19) 19804.	(20) 11959.
(21) 14332.	(22) 8762.	(23) 8104.	(24) 40459.	(25) 44978.
(26) 32867.	(27) 9077.	(28) 76917.	(29) 676422.	(30) 380244.
(31) 285669.	(32) 492232.	(33) 309832.	(34) 589747.	
(35) 579176.	(36) 316734.	(37) 236258.	(38) 305934.	
(39) 496938.	(40) 72684.	(41) 46275.	(42) 21222.	
(43) 58661.	(44) 215364.	(45) 652728.	(46) 255118.	
(47) 301257.	(48) 653017.	(49) 59049.	(50) 508267.	
(51) 355956.	(52) 691138.	(53) 92209.	(54) 132299.	
(55) 571173.	(56) 93594.	(57) 562757.	(58) 32527.	
(59) 3963.	(60) 8875.	(61) 20304.	(62) 2864.	
(63) 78809.	(64) 131875.	(65) 394236.	(66) 768828.	
(67) 460919.	(68) 480185.	(69) 91056.	(70) 399094.	

5. SIMPLE MULTIPLICATION. PAGE 14.

- | | | |
|------------------------|------------------------|----------------------|
| (1) 5690; 8535. | (2) 11871; 15828. | (3) 15456; 19320. |
| (4) 12895; 15474. | (5) 15576; 18172. | (6) 27118; 30992. |
| (7) 29992; 33741. | (8) 21184; 29128. | (9) 26874; 35832. |
| (10) 41635; 45420. | (11) 53814; 80721. | (12) 242076; 322768. |
| (13) 148192; 185240. | (14) 473900; 568680. | (15) 241554; 281813. |
| (16) 180642; 206448. | (17) 474408; 533709. | (18) 278586; 340494. |
| (19) 676170; 737640. | (20) 235340; 564816. | |
| (21) 1276950; 3192375. | (22) 1186192; 2075836. | |
| (23) 4119820; 9063664. | (24) 3200981; 4115547. | |
| (25) 3587643; 1594568. | (26) 4478292; 8210202. | |
| (27) 4433484; 2955656. | (28) 5046096; 1376208. | |
| (29) 4746136; 5339403. | (30) 2918352; 2431960. | |

6. SIMPLE MULTIPLICATION. PAGE 15.

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|-------------------------|--------------------------|
| (1) 254780; 509560. | (2) 982950; 1310600. |
| (3) 1346700; 1616040. | (4) 2449090; 2798960. |
| (5) 4832820; 5369800. | (6) 14490600; 21735900. |
| (7) 9831600; 12289500. | (8) 19606800; 23874600. |
| (9) 33974400; 38221200. | (10) 75179500; 82014000. |

7. SIMPLE MULTIPLICATION. PAGE 16.

- | | | | |
|----------------|----------------|----------------|----------------|
| (1) 7532637. | (2) 30323430 | (3) 20626844. | (4) 49358799. |
| (5) 36315648. | (6) 25444608. | (7) 56642796. | (8) 73356576. |
| (9) 21578208. | (10) 43714608. | (11) 23931648. | (12) 23610123. |
| (13) 33862500. | (14) 12242286. | (15) 53414900. | (16) 84750688. |
| (17) 67000500. | (18) 39285675. | (19) 83486700. | (20) 57091536. |

8. SIMPLE MULTIPLICATION. PAGE 16.

- | | | |
|--------------------------|--------------------------|----------------------|
| (1) 54689; 57906. | (2) 82004; 69056. | (3) 44618; 47805. |
| (4) 153909; 249186. | (5) 367830; 302438. | (6) 505548; 299584. |
| (7) 406463; 342525. | (8) 381416; 295792. | (9) 348170; 263480. |
| (10) 428967; 320023. | (11) 174972; 581157. | (12) 154713; 333228. |
| (13) 2234856; 2750592. | (14) 2890850; 3589575. | |
| (15) 4177872; 4690944. | (16) 5697770; 5072405. | |
| (17) 2848365; 2341989. | (18) 2781942; 3587241. | |
| (19) 4804264; 5626046. | (20) 3034804; 2770908. | |
| (21) 7165307; 4230121. | (22) 2551195; 3601830. | |
| (23) 39614705; 53022759. | (24) 25051804; 27029578. | |

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|------|-----------------------|------|-----------------------|
| (25) | 17837010; 21592170. | (26) | 33041013; 28272207. |
| (27) | 16245162; 25768188. | (28) | 58651385; 35190831. |
| (29) | 18061552; 28042936. | (30) | 65906990; 24937780. |
| (31) | 17892806; 31597934. | (32) | 58613936; 40252944. |
| (33) | 708084; 777504. | (34) | 1527575; 1649781. |
| (35) | 702875; 770351. | (36) | 1577628; 3822714. |
| (37) | 2262835; 3193586. | (38) | 5656338; 2056275. |
| (39) | 4661250; 6965772. | (40) | 2740241; 3872985. |
| (41) | 24854232; 29220516. | (42) | 22822568; 33050975. |
| (43) | 54546176; 42158048. | (44) | 11144325; 36720150. |
| (45) | 43673067; 40297371. | (46) | 27003004; 38165340. |
| (47) | 45300900; 53035200. | (48) | 24113383; 17310615. |
| (49) | 31452300; 38521125. | (50) | 45490416; 57441288. |
| (51) | 235943976; 567026652. | (52) | 352274118; 674645050. |
| (53) | 355642056; 165247824. | (54) | 153524375; 95062293. |
| (55) | 355456352; 221194306. | (56) | 233349106; 101956923. |
| (57) | 853994788; 506782812. | (58) | 498528475; 552978575. |
| (59) | 602444448; 299315496. | (60) | 391985995; 269166375. |

9. SIMPLE MULTIPLICATION. PAGE 17.

- | | | | |
|------|------------------------|------|------------------------|
| (1) | 28841520; 143500700. | (2) | 23824500; 413865600. |
| (3) | 54969920; 289702000. | (4) | 23258500; 103571000. |
| (5) | 65641000; 520325000. | (6) | 28425000; 115216000. |
| (7) | 43449600; 378432000. | (8) | 31953200; 130203000. |
| (9) | 41549400; 156570300. | (10) | 15257000; 85272000. |
| (11) | 79192255; 392047325. | (12) | 62992260; 827780256. |
| (13) | 270960864; 3374204192. | (14) | 221239724; 1891080372. |
| (15) | 44863312; 648576823. | (16) | 233955742; 1382591104. |
| (17) | 204178160; 3645180680. | (18) | 350441280; 2234063160. |
| (19) | 164272056; 1490486800. | (20) | 147288500; 2716050000. |

10. SIMPLE MULTIPLICATION. PAGE 18.

- | | | | | | | | |
|------|------------|------|------------|------|-----------|------|------------|
| (1) | 17874780. | (2) | 22887645. | (3) | 38574564. | (4) | 347256. |
| (5) | 156400. | (6) | 613586 | (7) | 133225. | (8) | 334084. |
| (9) | 481636. | (10) | 565504. | (11) | 755161. | (12) | 2653641. |
| (13) | 6436369. | (14) | 13307904. | (15) | 22363441. | (16) | 34070569. |
| (17) | 15625. | (18) | 46656. | (19) | 103823. | (20) | 195112. |
| (21) | 8615125. | (22) | 28652616. | (23) | 67419143. | (24) | 131096512. |
| (25) | 238328000. | (26) | 389017000. | | | | |

ANSWERS.

V

11. SHORT DIVISION. PAGE 19.

- | | | | | |
|----------------------|----------------------|----------------------|------------|------------|
| (1) 23492. | (2) 43781. | (3) 23192. | (4) 32485. | (5) 14567. |
| (6) 24089. | (7) 13579 | (8) 9567. | (9) 14678. | (10) 9583. |
| (11) 12485. | (12) 7398. | (13) 8579. | (14) 4687. | (15) 3468. |
| (16) 5792. | (17) 2653. | (18) 3789. | (19) 2563. | (20) 4792. |
| (21) 365217; rem. 1. | (22) 416892; rem. 1. | (23) 253971; rem. 1. | | |
| (24) 37854; rem. 2. | (25) 25937; rem. 1. | (26) 37894; rem. 3. | | |
| (27) 56932; rem. 2. | (28) 76398; rem. 3. | (29) 61457; rem. 4. | | |
| (30) 78528, rem. 1. | (31) 49357; rem. 3. | (32) 68492, rem. 3. | | |
| (33) 35746; rem. 1. | (34) 52873; rem. 3. | (35) 31982; rem. 1. | | |
| (36) 49876; rem. 3. | (37) 25317; rem. 2. | (38) 36945; rem. 3. | | |
| (39) 29873; rem. 3. | (40) 37546, rem. 5. | | | |

12. SHORT DIVISION. PAGE 20.

- | | | | | |
|------------|------------|------------|------------|------------|
| (1) 4829. | (2) 3738. | (3) 2649. | (4) 2328. | (5) 3453. |
| (6) 3674. | (7) 3875. | (8) 2984. | (9) 1379. | (10) 1829. |
| (11) 1734. | (12) 1652. | (13) 1564. | (14) 1476. | (15) 1385. |
| (16) 1294. | (17) 1187. | (18) 1276. | (19) 1368. | (20) 1459. |
| (21) 637. | (22) 574. | (23) 788. | (24) 647 | |

13. SHORT DIVISION (FACTORS). PAGE 21.

- | | | |
|----------------------|----------------------|----------------------|
| (1) 10823; rem. 2. | (2) 10518; rem. 9. | (3) 7642; rem. 4. |
| (4) 9423; rem. 41. | (5) 14822; rem. 7. | (6) 14208; rem. 8. |
| (7) 10778; rem. 29. | (8) 10664; rem. 53. | (9) 19347; rem. 15. |
| (10) 19696, rem. 1. | (11) 11700, rem. 11. | (12) 11991, rem. 26. |
| (13) 21874; rem. 18. | (14) 17557; rem. 7. | (15) 12417; rem. 8. |
| (16) 12088; rem. 25. | (17) 22419; rem. 8. | (18) 18081; rem. 31. |
| (19) 14097; rem. 5. | (20) 11959; rem. 66. | (21) 6753; rem. 40. |
| (22) 7757; rem. 55. | (23) 4837; rem. 14. | (24) 2761; rem. 97. |

14. SHORT DIVISION. PAGE 21.

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|-------------------------|----------------------|
| (1) (i) 623475; rem. 1. | (ii) 62347, rem. 51. |
| (2) (i) 729863; rem. 2. | (ii) 72986; rem. 32. |
| (3) (i) 498073; rem. 6. | (ii) 49807; rem. 36. |
| (4) (i) 329987; rem. 5. | (ii) 32938; rem. 75. |
| (5) (i) 501689; rem. 3. | (ii) 50168; rem. 93. |
| (6) (i) 856407; rem. 8. | (ii) 85640; rem. 78. |
| (7) (i) 921485; rem. 7. | (ii) 92146; rem. 57. |
| (8) (i) 689007; rem. 9. | (ii) 68900; rem. 79. |
| (9) (i) 387205; rem. 8. | (ii) 38720; rem. 58. |

15. SHORT DIVISION. PAGE 22.

(1) 45867; rem. 5.	(2) 28390; rem. 9.	(3) 19879; rem. 4.
(4) 12790; rem. 28.	(5) 13807; rem. 16	(6) 10898; rem. 31.
(7) 7578; rem. 15.	(8) 6006; rem. 79.	(9) 3197; rem. 127.
(10) 1913; rem. 82.	(11) 1043; rem. 146.	(12) 703; rem. 292.
(13) 1234; rem. 218.	(14) 977; rem. 173.	(15) 660; rem. 437.
(16) 514; rem. 263.	(17) 572; rem. 192.	(18) 819; rem. 695.

16. LONG DIVISION. PAGE 23.

(1) 324.	(2) 324.	(3) 234.	(4) 234.	(5) 156.
(6) 156.	(7) 156.	(8) 195	(9) 184.	(10) 157.
(11) 149.	(12) 457.	(13) 269.	(14) 258.	(15) 247.
(16) 174.	(17) 185.	(18) 395.	(19) 187.	(20) 148.
(21) 189.	(22) 265.	(23) 147.	(24) 176.	(25) 9639.
(26) 8547.	(27) 9368.	(28) 8592.	(29) 8492.	(30) 7583.
(31) 5679.	(32) 9587.	(33) 8938.	(34) 7840.	(35) 8678.
(36) 7987.				

17. LONG DIVISION. PAGE 23.

(1) 9405.	(2) 8027.	(3) 7509.	(4) 6047.	(5) 5628.
(6) 4347.	(7) 3925.	(8) 2469.	(9) 1907.	(10) 9005.
(11) 8670.	(12) 7890.	(13) 6192.	(14) 5387.	(15) 4576.
(16) 3462.	(17) 2905.	(18) 1890.	(19) 6072.	(20) 5009.
(21) 4500.	(22) 3405.	(23) 2350.	(24) 1298.	(25) 987.
(26) 976.	(27) 965.	(28) 873.	(29) 924.	(30) 895.
(31) 764.	(32) 692.	(33) 587.	(34) 485.	(35) 397.
(36) 276.				

18. MISCELLANEOUS (SIMPLE RULES). PAGE 24.

(1) 1478.	(2) 465.	(3) 2616.	(4) Tom; 103.
(5) 699.	(6) 23328.	(7) 91500.	(8) 35532.
(9) 4524.	(10) 78.	(11) 308.	(12) 1518.
(13) 260.	(14) 849.	(15) 90.	(16) 180.
(17) 195.	(18) 161.	(19) 520.	(20) 29952.
(21) 1558.	(22) 209061.	(23) 7505760.	(24) 4148384.
(25) 203670.	(26) 178464.	(27) 99991.	(28) 230480.
(29) 63300.	(30) 416422.	(31) 1435.	(32) 2191.
(33) 517.	(34) 77456.	(35) 515.	(36) 207.
(37) 365.	(38) 470.	(39) 1090.	(40) 3207.

- (41) 500410. (42) 409. (43) 5767839. (44) 42258; rem. 681.
 (45) 293631. (46) 5562981. (47) 28. (48) 29942.
 (49) 1324. (50) 169; rem. 29.

19. REDUCTION OF MONEY. PAGE 28.

(1) 95.	(2) 247.	(3) 518.	(4) 1004.
(5) 150.	(6) 305.	(7) 628.	(8) 1393.
(9) 464.	(10) 853.	(11) 1750.	(12) 4075.
(13) 144.	(14) 41.	(15) 110.	(16) 215.
(17) 120.	(18) 168.	(19) 257.	(20) 362.
(21) 403.	(22) 485.	(23) 606.	(24) 647.
(25) 150.	(26) 211.	(27) 322.	(28) 463.
(29) 524.	(30) 655.	(31) 706.	(32) 757.
(33) 828.	(34) 939.	(35) 1002.	(36) 2131.
(37) 3619.	(38) 4696.	(39) 5201.	(40) 8816.
(41) 36681.	(42) 63747.	(43) 90061.	(44) 2657.
(45) 5149.	(46) 12423.	(47) 18210.	(48) 72333.
(49) 127462.	(50) 6269.	(51) 9583.	(52) 12189.
(53) 16186.	(54) 17847.	(55) 24715.	(56) 31646.
(57) 139710.	(58) 240758.	(59) 360595.	(60) 567163.

20. REDUCTION OF MONEY. PAGE 29.

- (1) $93\frac{1}{2}d.$; $1543\frac{1}{2}d.$; $15582d$; $153369d.$
 (2) $35s\ 7d.$; $274s.\ 9d.$; $5819s.\ 4d.$; $34467s.\ 5d.$
 (3) $\pounds 16.\ 15s.$; $\pounds 308\ 14s.$; $\pounds 1648.\ 7s.$; $\pounds 34684.\ 16s.$
 (4) $\pounds 1.\ 16s.\ 5d.$; $\pounds 9.\ 7s.\ 7\frac{1}{2}d.$; $\pounds 84.\ 7s.\ 6\frac{1}{2}d.$; $\pounds 1079.\ 19s.\ 7d.$
 (5) $\pounds 1.\ 0s.\ 7d.$; $\pounds 7.\ 2s.\ 9d.$; $\pounds 13.\ 12s.\ 9\frac{3}{4}d.$; $\pounds 111.\ 11s.\ 11\frac{3}{4}d.$
 (6) $\pounds 106.\ 5s.$; $\pounds 516.\ 15s.$; $\pounds 2690.\ 10s.$; $\pounds 47844.$
 (7) $\pounds 56.\ 2s.$; $\pounds 136.\ 6s.$; $\pounds 2963.\ 18s.$; $\pounds 28028.\ 10s.$
 (8) $\pounds 87.\ 5s.$; $\pounds 637.\ 7s.\ 6d.$; $\pounds 5341.\ 17s.\ 6d.$; $\pounds 47399.\ 12s.\ 6d.$
 (9) $\pounds 17\ 16s.$; $\pounds 139$; $\pounds 726.\ 16s.\ 6d.$; $\pounds 11700.\ 6s.\ 6d.$
 (10) $\pounds 10.\ 8s.$; $\pounds 13.\ 13s.\ 3d.$; $\pounds 587.\ 5s.\ 6d.$; $\pounds 6974.\ 0s.\ 9d.$

21. ADDITION OF MONEY. PAGE 30.

	£	s.	d.		£	s.	d.		£	s.	d.
(1)		2	$4\frac{3}{4}$	(2)		3	$2\frac{3}{4}$	(3)		3	4
(4)		2	10	(5)		3	$3\frac{1}{2}$	(6)	2	14	$7\frac{1}{2}$
(7)	2	16	$7\frac{3}{4}$	(8)	3	13	2	(9)	3	1	$6\frac{3}{4}$
(10)	2	15	$2\frac{1}{4}$	(11)	15	13	2	(12)	21	3	3
(13)	84	18	5	(14)	35	13	3	(15)	32	16	4

	£	s.	d.		£	s.	d.		£	s.	d.
(16)	46	8	8	(17)	45	17	3	(18)	45	19	10 $\frac{1}{2}$
(19)	46	4	11	(20)	43	11	2	(21)	40	12	5 $\frac{1}{2}$
(22)	49	15	10 $\frac{1}{2}$	(23)	63	11	10 $\frac{1}{2}$	(24)	64	3	6 $\frac{1}{2}$
(25)	59	8	8 $\frac{1}{2}$	(26)	54	15	9 $\frac{1}{2}$	(27)	63	3	0 $\frac{1}{2}$
(28)	59	6	11	(29)	66	16	10 $\frac{1}{2}$	(30)	67	14	6
(31)	184	9	2	(32)	235	5	0 $\frac{1}{2}$	(33)	259	16	4 $\frac{1}{2}$
(34)	241	17	6	(35)	157	0	11	(36)	156	3	7 $\frac{1}{2}$
(37)	224	12	8 $\frac{1}{2}$	(38)	232	8	1	(39)	2318	19	1
(40)	2077	6	3 $\frac{1}{2}$	(41)	2141	15	8 $\frac{1}{2}$	(42)	1968	18	4 $\frac{1}{2}$
(43)	3003	5	7	(44)	2029	7	1 $\frac{1}{2}$	(45)	3169	11	1 $\frac{1}{2}$
(46)	1980	19	4	(47)	2135	16	7 $\frac{1}{2}$	(48)	2406	1	10 $\frac{1}{2}$
(49)	3383	14	10 $\frac{1}{2}$	(50)	1923	5	9 $\frac{1}{2}$				

22. SUBTRACTION OF MONEY. PAGE 33.

	£	s.	d.		£	s.	d.		£	s.	d.
(1)	3	6	3	(2)	3	3	3	(3)	6	8	3
(4)	14	1	7	(5)	13	5	2	(6)	1	1	9
(7)	3	2	10	(8)	4	6	10	(9)	1	18	7
(10)	3	16	6	(11)	6	0	9 $\frac{1}{2}$	(12)	19	4	11 $\frac{1}{2}$
(13)	23	19	3 $\frac{1}{2}$	(14)	27	19	3 $\frac{1}{2}$	(15)	27	14	3 $\frac{1}{2}$
(16)	24	17	2 $\frac{1}{2}$	(17)	63	15	9 $\frac{1}{2}$	(18)	39	18	9 $\frac{1}{2}$
(19)	5	19	9 $\frac{1}{2}$	(20)	9	6	10 $\frac{1}{2}$	(21)	7	19	4 $\frac{1}{2}$
(22)	67	10	4 $\frac{1}{2}$	(23)	11	9	8 $\frac{1}{2}$	(24)	27	17	4 $\frac{1}{2}$
(25)	64	16	7 $\frac{1}{2}$	(26)	33	16	2 $\frac{1}{2}$	(27)	81	2	5 $\frac{1}{2}$
(28)	56	18	8 $\frac{1}{2}$	(29)	30	10	5 $\frac{1}{2}$	(30)	59	2	10 $\frac{1}{2}$
(31)	289	0	2	(32)	861	2	10 $\frac{1}{2}$	(33)	94	3	4 $\frac{1}{2}$
(34)	688	18	10 $\frac{1}{2}$	(35)	803	19	2 $\frac{1}{2}$	(36)	694	8	2 $\frac{1}{2}$
(37)	810	10	10 $\frac{1}{2}$	(38)	488	17	10 $\frac{1}{2}$	(39)	222	7	7 $\frac{1}{2}$
(40)	122	17	10 $\frac{1}{2}$	(41)	22	14	11 $\frac{1}{2}$	(42)	95	17	8 $\frac{1}{2}$
(43)	569	16	10 $\frac{1}{2}$	(44)	6	18	10 $\frac{1}{2}$	(45)	481	2	5 $\frac{1}{2}$
(46)	27	18	11	(47)	249	2	9	(48)	249	8	10
(49)	114	7	3	(50)	95	1	7				

23. MULTIPLICATION OF MONEY. PAGE 35.

	£	s.	d.		£	s.	d.		£	s.	d.
(1)	6	15	0	(2)	12	17	3	(3)	21	17	4
	10	2	6		17	3	0		27	6	8
(4)	32	2	1	(5)	43	5	6	(6)	59	11	3 $\frac{1}{2}$
	38	10	6		50	9	9		68	1	6

ANSWERS.

ix

	£	s.	d.		£	s.	d.		£	s.	d.
(7)	77	3	0	(8)	79	0	0 $\frac{1}{2}$	(9)	76	13	9
	86	15	10 $\frac{1}{2}$		87	15	7 $\frac{1}{2}$		84	7	1 $\frac{1}{2}$
(10)	129	13	2 $\frac{1}{2}$	(11)	57	19	8 $\frac{1}{2}$	(12)	59	13	5
	222	5	6		104	7	5 $\frac{1}{2}$		164	1	10 $\frac{1}{2}$
(13)	53	9	9 $\frac{1}{2}$	(14)	119	3	8	(15)	155	15	1 $\frac{1}{2}$
	178	6	0 $\frac{1}{2}$		178	14	10 $\frac{1}{2}$		519	3	9
(16)	258	14	1	(17)	383	18	7 $\frac{1}{2}$	(18)	497	7	4 $\frac{1}{2}$
	582	1	8 $\frac{1}{2}$		614	5	10		911	16	10 $\frac{1}{2}$
(19)	601	18	6 $\frac{1}{2}$	(20)	748	12	10				
	1031	17	6		935	16	0 $\frac{1}{2}$				

24. MULTIPLICATION OF MONEY. PAGE 35.

	£	s.	d.		£	s.	d.		£	s.	d.
(1)	221	13	7 $\frac{1}{2}$	(2)	430	4	8	(3)	680	1	7 $\frac{1}{2}$
	395	17	2 $\frac{1}{2}$		806	13	9		1209	0	8
(4)	974	10	10	(5)	1148	1	8 $\frac{1}{2}$	(6)	1670	15	0
	2046	10	9		2186	15	10		3480	14	7
(7)	597	14	0 $\frac{1}{2}$	(8)	1497	7	7 $\frac{1}{2}$	(9)	603	4	6
	876	12	7 $\frac{1}{2}$		3465	7	8 $\frac{1}{2}$		1163	7	3
(10)	845	17	9	(11)	1427	10	6	(12)	2101	15	0
	1315	16	6		1946	12	6		2889	18	1 $\frac{1}{2}$
(13)	3191	0	0	(14)	3647	9	6	(15)	5259	10	0
	3490	3	1 $\frac{1}{2}$		3900	15	5 $\frac{1}{2}$		5522	9	6
(16)	6492	9	6	(17)	4765	8	0	(18)	6105	9	9
	6640	0	7 $\frac{1}{2}$		4963	19	2		6218	11	0 $\frac{1}{2}$
(19)	4148	5	0	(20)	5172	12	5 $\frac{1}{2}$				
	4563	1	6		6155	17	0				

25. MULTIPLICATION OF MONEY. PAGE 36.

	£	s.	d.		£	s.	d.		£	s.	d.
(1)	87	5	3	(2)	132	0	8	(3)	168	16	0 $\frac{1}{2}$
	154	7	9		225	4	8		257	12	11 $\frac{1}{2}$
(4)	858	13	4 $\frac{1}{2}$	(5)	435	14	2 $\frac{1}{2}$	(6)	540	2	10 $\frac{1}{2}$
	630	1	10 $\frac{1}{2}$		924	11	1 $\frac{1}{2}$		1193	6	9 $\frac{1}{2}$
(7)	742	5	1 $\frac{3}{4}$	(8)	932	8	6	(9)	1270	19	8 $\frac{1}{2}$
	1468	14	5 $\frac{1}{2}$		1595	17	7 $\frac{1}{2}$		1422	14	8 $\frac{1}{2}$
(10)	1424	12	8 $\frac{1}{2}$	(11)	905	13	10 $\frac{1}{2}$	(12)	1202	7	6 $\frac{1}{2}$
	1507	4	5 $\frac{1}{2}$		2159	14	7 $\frac{1}{2}$		5021	13	11 $\frac{1}{2}$
(13)	490	17	5 $\frac{1}{2}$	(14)	1231	9	7 $\frac{3}{4}$	(15)	2077	15	9 $\frac{1}{2}$
	2351	0	5 $\frac{1}{2}$		3480	5	6 $\frac{1}{2}$		5588	10	9
(16)	3056	9	10 $\frac{1}{2}$	(17)	1688	3	3 $\frac{1}{2}$	(18)	1473	10	2 $\frac{1}{2}$
	7641	4	8 $\frac{1}{2}$		3604	9	1 $\frac{3}{4}$		3664	17	7 $\frac{1}{2}$

	£	s.	d.
(19)	3795	2	11½
	8793	12	8½
(22)	1341	13	4
	2928	6	8
(25)	2306	17	0
	5397	8	9
(28)	2760	19	4½
	11200	3	1½
(31)	12896	1	10½
	18467	4	1½
(34)	93288	5	7½
	48404	5	11½
(37)	68303	12	6
	35976	3	2¾
(40)	8358	19	6
	23683	15	3

	£	s.	d.
(20)	2732	4	0½
	5655	0	5½
(23)	2203	1	0
	4084	0	4
(26)	2565	4	8
	7398	10	2
(29)	2912	6	0½
	13344	19	7
(32)	15942	16	6
	26940	8	5½
(35)	39178	19	0
	56019	0	6
(38)	37813	16	5½
	35438	1	8½

	£	s.	d.
(21)	1548	2	6
	2115	15	5
(24)	2534	7	0
	4567	8	0
(27)	2128	1	11½
	9601	3	7½
(30)	3481	1	7½
	17426	4	11
(33)	31907	9	11
	39126	15	9½
(36)	38181	12	0
	43550	17	9
(39)	20725	19	3
	49614	18	10½

26. DIVISION OF MONEY. PAGE 37.

	£	s.	d.
(1)	12	8½	
(4)	2	3	8
(7)	3	12	6½
(10)	5	10	4½
(13)	25	13	10½
(16)	37	14	2½
(19)	736	9	8
(22)	207	10	3½
(25)	452	13	6½
(28)	375	15	7½

	£	s.	d.
(2)	16	9½	
(5)	2	14	9
(8)	4	11	7½
(11)	5	14	5¾
(14)	16	17	9¾
(17)	39	15	8½
(20)	405	9	7
(23)	170	12	4½
(26)	503	14	3¾
(29)	281	16	2½

	£	s.	d.
(3)	13	10¾	
(6)	3	15	10
(9)	4	13	8¾
(12)	6	17	9½
(15)	26	18	3½
(18)	37	5	9½
(21)	550	9	5
(24)	308	11	5½
(27)	370	10	10½
(30)	293	17	11½

27. DIVISION OF MONEY. PAGE 38.

	£	s.	d.
(1)	3	6	8
(4)	6	13	11½
(7)	9	10	6½
(10)	7	19	4½
(13)	21	0	5½
(16)	35	10	0¾
(19)	40	15	10½
(22)	10	10	10½
(25)	438	12	3½
(28)	412	13	3½

	£	s.	d.
(2)	4	12	10
(5)	7	14	6½
(8)	10	12	5¾
(11)	4	16	5½
(14)	32	6	0¾
(17)	40	0	10½
(20)	32	10	0½
(23)	11	11	11½
(26)	639	7	8½
(29)	1139	18	6½

	£	s.	d.
(3)	5	10	9
(6)	8	15	10¾
(9)	11	11	7½
(12)	5	17	10¾
(15)	40	7	6½
(18)	15	9	0½
(21)	10	5	6½
(24)	12	12	6½
(27)	861	11	9½
(30)	129	17	10¾

28. DIVISION OF MONEY. PAGE 39.

£	s.	d.	£	s.	d.	£	s.	d.
(1)	7	6½	(2)	10	7½	(3)	15	4½
(4)	16	10½	(5)	19	5½	(6)	6	5 8½
(7)	7	8 9½	(8)	5	11 5½	(9)	8	11 9½
(10)	2	15 4½	(11)	5	5 6½	(12)	17	12 10½
(13)	18	14 5½	(14)	89	16 6	(15)	41	5 7½
(16)	29	2 2½	(17)	64	19 11½	(18)	13	16 8½
(19)	790	19 10½	(20)	432	6 2½	(21)	681	7 3½
(22)	731	19 11½	(23)	130	18 9½	(24)	1100	11 9½
(25)	83	15 3½	(26)	85	16 4½	(27)	49	16 8½
(28)	59	5 7½	(29)	34	18 3½	(30)	64	18 5½
(31)	33	17 6½	(32)	58	7 3½	(33)	23	15 6½
(34)	49	16 8½	(35)	2169	15 6½	(36)	399	15 2½
(37)	896	15 0½	(38)	132	1 3½	(39)	1854	7 7½
(40)	401	11 4½						

29. DIVISION OF MONEY. PAGE 40.

(1) 11.	(2) 3.	(3) 12.	(4) 234.	(5) 14.
(6) 36.	(7) 40.	(8) 13.	(9) 57.	(10) 19.

30. MISCELLANEOUS (MONEY). PAGE 41.

(1) 670.	(2) 2748.	(3) 7320.	(4) 398.	(5) 67.
(6) £62. 10s.	(7) £27. 10s. 8d.	(8) £337. 10s.		
(9) £7. 5s. 6½d.	(10) £23. 6s. 3d.	(11) £1. 5s. 9d.		
(12) £13 15s. 1d.	(13) £130. 15s. 4d.	(14) £984. 13s. 2d.		
(15) £3331. 6s. 9d.	(16) £1040. 6s. 9d.	(17) 9s. 10½d.		
(18) £1. 13s. 6½d.	(19) £12. 12s. 6d.	(20) £16. 19s. 8d.		
(21) £21. 10s. 7d.	(22) £117. 14s. 8d.	(23) £73. 14s. 5d.		
(24) 18s. 11d.	(25) £14.	(26) £54. 17s. 8d.		
(27) £20. 5s. 2d.	(28) £11. 3s. 11d.	(29) £2. 19s. 9d too much.		
(30) 10s. too much.	(31) £76. 5s. 4½d.	(32) £65. 7s. 10d.		
(33) £44. 0s. 5d.	(34) £1. 11s. 1d.	(35) £11.		
(36) £20. 6s. 6d.	(37) £7. 8s. 9d.	(38) £17. 18s. 3d.		
(39) £23. 7s. 2d.	(40) £6. 7s 6½d.	(41) 17s 9½d.		
(42) £2. 6s.	(43) £2. 13s. 11½d.	(44) £35. 11s. 8d.		
(45) £36. 3s. 3d.	(46) £174. 15s. 2d.	(47) £56. 12s. 6d.		
(48) £2. 5s. 4d.	(49) £4. 10s. 9d.	(50) £19. 7s. 4d.		

31. REDUCTION OF WEIGHTS AND MEASURES. PAGE 48.

(1) 4656.	(2) 2716.	(3) 1763	(4) 4240.
(5) 512768.	(6) 227584.	(7) 17108.	(8) 57223.
(9) 154096.	(10) 2293504.	(11) 3936.	(12) 84480.
(13) 44568.	(14) 13179.	(15) 120.	(16) 32796.
(17) 45780.	(18) 14532.	(19) 151092.	(20) 4257.
(21) 236940.	(22) 529.	(23) 23255.	(24) 27104.
(25) 373599.	(26) 22120560.	(27) 175201.	(28) 19304.
(29) 615187.	(30) 1232.	(31) 1270.	(32) 31016.
(33) 1752.	(34) 20346.	(35) 4584.	(36) 2181240.

32. REDUCTION OF WEIGHTS AND MEASURES. PAGE 51.

(1) 142 lb. 11 oz. 12 drs.	(2) 24531 cwt.
(3) 18 cwt. 8 lb. 7 oz.	(4) 4 tons 15 cwt. 3 qrs. 5 lb.
(5) 417 tons 13 cwt. 1 qr. 19 lb.	(6) 18 tons 1 cwt. 3 qrs. 4 lb. 7 oz.
(7) 54 tons 11 cwt. 3 qrs. 20 lb. 6 oz.	
(8) 261 tons 7 cwt. 2 qrs. 12 lb. 3 oz.	
(9) 11 tons 4 cwt. 1 qr. 24 lb. 5 oz.	(10) 162 tons 4 cwt. 13 lb.
(11) 65 lb. 7 oz. 16 dwt. 18 grs.	(12) 564 lb. 6 oz. 10 dwt. 3 grs.
(13) 175 lb (troy).	(14) 90000 lb. (avoir.).
(15) 2064 yds. 6 in.	(16) 33 m. 2 fur. 39 po. 4 yds. 1 ft. 6 in.
(17) 20 m. 6 fur. 4 po. 5 yds.	(18) 2 m. 7 fur. 1 yd. 1 ft. 6 in.
(19) 25 sq. yds. 1 ft. 23 in.	(20) 22 ac. 1346 sq. yds.
(21) 174 ac. 1 ro. 12 per. 5 sq. yds. 6 ft.	
(22) 6 ac. 2 ro. 5 per. 25 sq. yds. 2 ft. 49 in.	(23) 1502 cub. yds. 6 ft.
(24) 2 cub. yds. 26 ft. 57 in.	(25) 4 cub. yds. 6 ft. 1640 in.
(26) 10 cub. yds. 10 ft. 10 in.	(27) 35 gall.
(28) 577 pks. 1 qt.	(29) 37456 qrs.
(30) 156 qrs. 2 bush.	(31) 297 bush. 2 pks. 1 gall. 1 qt.
(32) 7147 qrs. 7 bush.	(33) 3 hrs. 35 min. 28 sec.
(34) 17 days 23 hrs. 44 min. 57 sec.	(35) 223 weeks 1 day 12 hrs.
(36) 5 weeks 5 days 7 hrs. 51 min. 14 sec.	

33. ADDITION OF WEIGHTS AND MEASURES. PAGE 52.

(1) 1 qr. 12 lb. 13 oz. 3 drs.	(2) 3 qrs. 2 lb. 2 oz. 8 drs.
(3) 2 tons 17 cwt. 1 qr. 10 lb.	(4) 119 tons 14 cwt. 2 qrs. 26 lb.
(5) 117 tons 14 cwt. 1 qr. 7 lb.	(6) 94 tons 6 cwt. 2 qrs. 24 lb.
(7) 3 lb. 14 dwt. 13 grs.	(8) 4 lb. 16 dwt. 7 grs.
(9) 29 lb. 11 oz. 1 dwt. 21 grs.	(10) 186 lb. 10 oz. 10 dwt. 7 grs.
(11) 94 yds. 2 ft. 8 in.	(12) 138 m. 1 fur. 23 po.

- (13) 35 ac. 3 ro. 19 per. (14) 126 ac. 24 per. 17 sq. yds. 2 ft. 86 in.
 (15) 35 cub. yds. 3 ft. 1068 in. (16) 127 cub. yds. 13 ft. 648 in.
 (17) 243 gall. 3 qts. 1 pt. (18) 184 qrs. 1 bush. 1 gall.
 (19) 254 days 11 hrs. 48 min. 58 sec.
 (20) 13 weeks 6 days 17 hrs. 56 min. 8 sec.

34. SUBTRACTION OF WEIGHTS AND MEASURES. PAGE 53.

- (1) 3 cwt. 2 qrs. 15 lb. (2) 5 cwt. 2 qrs. 26 lb. 13 oz.
 (3) 3 tons 17 cwt. 1 qr. 25 lb. (4) 14 tons 17 cwt. 2 qrs. 19 lb. 2 oz.
 (5) 1 ton 16 cwt. 2 qrs. 25 lb. 8 oz. (6) 10 lb. 10 oz. 11 dra.
 (7) 4 dwt. 12 grs. (8) 17 dwt. 9 grs.
 (9) 9 oz. 14 dwt. 23 grs. (10) 17 yds. 1 ft. 6 in.
 (11) 34 m. 5 fur. 6 yds. (12) 2 m. 539 yds. 2 ft.
 (13) 6 ac. 2 ro. 38 per. (14) 2 sq. yds. 7 ft. 114 in.
 (15) 190 cub. yds. 15 ft. 1426 in. (16) 66 cub. yds. 14 ft. 759 in.
 (17) 1 qr. 1 bush. 2 pks. 1 gall. (18) 1 bush. 3 pks. 1 gall. 2 qts.
 (19) 19 days 17 hrs. 35 min. 39 sec.
 (20) 1 week 6 days 3 hrs. 52 min. 57 sec.

35. MULTIPLICATION OF WEIGHTS AND MEASURES. PAGE 55.

- (1) 15 lb. 11 oz. 10 dra. (2) 7 tons 9 cwt.
 (3) 96 tons 11 cwt. 1 qr. 24 lb. (4) 34 tons 19 cwt. 1 qr. 20 lb.
 (5) 66 tons 6 cwt. 3 qrs. 3 oz. (6) 1 ton 1 cwt. 26 lb. 15 oz.
 (7) 27 tons 6 cwt. 3 qrs. 1 lb. 8 oz. (8) 141 tons 16 cwt. 8 oz.
 (9) 111 tons 6 cwt. 2 qrs. 24 lb. (10) 933 tons 9 cwt. 2 qrs. 1 lb.
 (11) 69 lb. 7 oz. 8 dwt. 21 grs. (12) 116 lb. 8 oz. 12 dwt.
 (13) 27 lb. 9 oz. 17 dwt. 3 grs. (14) 70 yds. 1 ft.
 (15) 743 yds. 1 ft. 7 in. (16) 34 m. 2 fur. 208 yds.
 (17) 1020 m. 2 fur. 10 po. 4 yds. 1 ft. 6 in. (18) 96 ac. 3 ro. 25 per.
 (19) 32798 ac. 1 ro. 10 per. (20) 931 sq. yds. 6 ft.
 (21) 80 ac. 2 ro. 1 per. 7 sq. yds. 4 ft. 72 in.
 (22) 686 cub. yds. 26 ft. 1026 in. (23) 568 cub. yds. 360 in.
 (24) 107 cub. yds. 1 ft. 1088 in. (25) 2 qrs. 3 bush. 3 pks. 3 qts.
 (26) 16 qrs. 1 bush. 3 pks. 1 gall. 2 qts. (27) 145 qrs. 1 pk.
 (28) 1 year 1 month 3 weeks 4 days.
 (29) 105 days 9 hrs. 22 min. 24 sec.
 (30) 226 days 13 hrs. 37 min. 50 sec.

36. DIVISION OF WEIGHTS AND MEASURES. PAGE 56.

- (1) 1 ton 4 cwt. 2 qrs. 12 lb. (2) 1 ton 18 cwt. 3 qrs. 11 lb.
 (3) 3 tons 14 cwt. 3 qrs. 18 lb. 4 oz.
 (4) 4 tons 15 cwt. 2 qrs. 17 lb. 13 oz. (5) 1 qr. 1 lb. 8 oz. 10 dra.
 (6) 1 ton 10 cwt. 1 qr. 8 lb. 7 oz. (7) 2 qrs. 3 lb. 13 oz.
 (8) 6 cwt. 2 qrs. 9 lb. (9) 13 cwt. 3 qrs. 14 lb. 15 oz.
 (10) 3 lb. 10 oz. 8 dwt. 16 gra. (11) 3 lb. 7 oz. 15 dwt. 9 gra.
 (12) 1 oz. 3 dwt. 8 gra. (13) 4 yds. 2 ft. 11 in.
 (14) 10 yds. 1 ft. 7 in. (15) 25 yds. 1 ft. 11 in.
 (16) 3 fur. 27 po. 3 yds. (17) 1 m. 1 po. 1 ft.
 (18) 2 fur. 3 po. 4 yds. 2 ft. (19) 1 ac. 1 ro. 25 per.
 (20) 3 ac. 2 ro. 10 per. 5 sq. yds. 54 in. (21) 1 ro. 27 per. 2 sq. yds.
 (22) 19 cub. ft. 888 in. (23) 16 cub. yds. 48 in.
 (24) 1 cub. yd. 14 ft. 747 in. (25) 3 pks. 3 qts.
 (26) 2 qrs. 1 bush. 3 pks. 1 gall. 1 qt.
 (27) 1 bush. 3 pks. 1 gall. 3 qts. 1 pt.
 (28) 4 days 14 hrs. 21 min. 10 sec. (29) 4 days 4 hrs. 8 min. 15 sec.
 (30) 5 days 23 hrs. 48 min. 9 sec.

37. DIVISION OF WEIGHTS AND MEASURES. PAGE 57.

- (1) 30. (2) 7. (3) 237. (4) 64. (5) 37440.
 (6) 8000. (7) 54. (8) 288. (9) 400. (10) 23.
 (11) 358. (12) 47.

38. MISCELLANEOUS (WEIGHTS AND MEASURES). PAGE 58.

- (1) 4 tons 6 cwt. 3 qrs. 26 lb. (2) 2 cwt. 12 lb.
 (3) 18 cwt. 3 qrs. 6 lb. (4) 11 tons 2 cwt. 7 lb.
 (5) 4 tons 18 cwt. 19 lb. (6) 15 cwt. 10 lb. (7) 281; 9 lb.
 (8) 23 tons 18 cwt. 2 qrs. 8 lb. (9) 180. (10) 1 oz. 6 dwt. 10 gra.
 (11) 1202. (12) 37440. (13) 55 yds. (14) 110 yds.
 (15) 1 m. 7 fur. 16 po. 2 yds. (16) 114 ac. 2 ro.
 (17) 1386 ac. 3 ro. 25 per. (18) 3 ro. 28 sq. yds.
 (19) 4 cub. ft. 851 in. (20) 2880. (21) 30.
 (22) 1203 qrs. 3 pks. (23) 4632. (24) 156. (25) 13.
 (26) 986. (27) 27 days 7 hrs. 43 min. 11 sec. (28) 319 days 9 hrs.
 (29) 1 hr. 39 min. 15 sec. (30) 1152.

39. PRIME FACTORS. PAGE 61.

- (1) $2^2 \cdot 3 \cdot 5$. (2) $2 \cdot 3 \cdot 13$. (3) $2 \cdot 3 \cdot 17$. (4) $2 \cdot 3 \cdot 5 \cdot 7$.
 (5) $2^2 \cdot 3^2 \cdot 7$. (6) $3^2 \cdot 5 \cdot 7$. (7) $8 \cdot 5^2 \cdot 7$. (8) $2^2 \cdot 3^2 \cdot 17$.

ANSWERS.

XV

- | | | |
|--|---|--|
| (9) 5.11.13. | (10) 2.7.61. | (11) 11 ² . |
| (12) 2.5.11.13. | (13) 2 ⁴ .7.13. | (14) 3 ² .5.7.11. |
| (15) 3.5 ³ .7 ² . | (16) 2 ³ .3 ⁴ .7. | (17) 2 ² .3.5.7.11. |
| (18) 3 ³ .7 ³ .11. | (19) 2.3.5 ³ .7. | (20) 3 ³ .7.11 ² . |

40. GREATEST COMMON MEASURE. PAGE 62.

- | | | | | |
|------------|-----------|-----------|----------|----------|
| (1) 9. | (2) 9. | (3) 18. | (4) 33. | (5) 24. |
| (6) 9. | (7) 6. | (8) 240. | (9) 117. | (10) 39. |
| (11) 3. | (12) 672. | (13) 87. | (14) 39. | (15) 53. |
| (16) 5544. | (17) 649. | (18) 121. | | |

41. G.C.M. PAGE 63.

- | | | | |
|---------|---------|----------|--------|
| (1) 3. | (2) 81. | (3) 2. | (4) 2. |
| (5) 12. | (6) 75. | (7) 442. | (8) 8. |

42. LEAST COMMON MULTIPLE. PAGE 64.

- | | | | | |
|------------|------------|------------|------------|------------|
| (1) 120. | (2) 252. | (3) 180. | (4) 360. | (5) 1620. |
| (6) 12600. | (7) 360. | (8) 5040. | (9) 1716. | (10) 3360. |
| (11) 1320. | (12) 7056. | (13) 476. | (14) 240. | (15) 180. |
| (16) 1008. | (17) 1260. | (18) 1080. | (19) 2520. | (20) 7920. |

43. FRACTIONS (INTRODUCTORY). PAGE 65.

- | | | |
|-----------------------------------|------------------------------------|--------------------|
| (1) £8. 15s. 10d. | (2) £5. 15s. 2d. | (3) £7. 3s. 10½d. |
| (4) £8. 2s. 4d. | (5) £21. 11s. 9¾d. | (6) £17. 5s. 10d. |
| (7) £3. 9s. 10½d. | (8) £26. 19s. 7d. | (9) £50. 5s. 9¾d. |
| (10) £21. 18s. 6½d. | (11) 5 oz. | (12) 1 qr. 26 lb. |
| (13) 2 qrs. 20 lb. | (14) 1 qr. 20 lb. | (15) 3 cwt. 14 lb. |
| (16) 1 cwt. 1 qr. 20 lb. | (17) 10 lb. 8 oz. | (18) 5 cwt. 3 qrs. |
| (19) 2 tons 10 cwt. 2 qrs. 14 lb. | (20) 21 tons 13 cwt. 2 qrs. 13 lb. | |

44 a. REDUCTION TO LOWEST TERMS. PAGE 66.

- | | | | | | |
|---------|---------|---------|---------|---------|---------|
| (1) ⅔. | (2) ⅔. | (3) ⅔. | (4) ⅔. | (5) ⅔. | (6) ⅔. |
| (7) ⅔. | (8) ⅔. | (9) ⅔. | (10) ⅔. | (11) ⅔. | (12) ⅔. |
| (13) ⅔. | (14) ⅔. | (15) ⅔. | (16) ⅔. | (17) ⅔. | (18) ⅔. |
| (19) ⅔. | (20) ⅔. | | | | |

44 b. REDUCTION TO LOWEST TERMS. PAGE 66.

- | | | | | |
|--------|--------|--------|--------|---------|
| (1) ⅔. | (2) ⅔. | (3) ⅔. | (4) ⅔. | (5) ⅔. |
| (6) ⅔. | (7) ⅔. | (8) ⅔. | (9) ⅔. | (10) ⅔. |

- (11) $\frac{1}{2}$. (12) $\frac{1}{3}$. (13) $\frac{1}{4}$. (14) $\frac{1}{5}$. (15) $\frac{1}{6}$.
 (16) $\frac{1}{7}$. (17) $\frac{1}{8}$. (18) $\frac{1}{9}$. (19) $\frac{1}{10}$. (20) $\frac{1}{11}$.
 (21) $\frac{1}{12}$. (22) $\frac{1}{13}$. (23) $\frac{1}{14}$. (24) $\frac{1}{15}$. (25) $\frac{1}{16}$.
 (26) $\frac{1}{17}$. (27) $\frac{1}{18}$. (28) $\frac{1}{19}$.

45. COMPARISON OF FRACTIONS. PAGE 67.

- | | | | | | |
|----------|----------|----------|----------|----------|----------|
| (1) 1. | (2) 2. | (3) 3. | (4) 4. | (5) 5. | (6) 6. |
| (7) 7. | (8) 8. | (9) 9. | (10) 10. | (11) 11. | (12) 12. |
| (13) 13. | (14) 14. | (15) 15. | (16) 16. | (17) 17. | (18) 18. |
| (19) 19. | (20) 20. | (21) 21. | (22) 22. | (23) 23. | (24) 24. |

46. COMPARISON OF FRACTIONS. PAGE 67.

- (1) $\frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \frac{1}{30}, \frac{1}{40}, \frac{1}{60}, \frac{1}{80}, \frac{1}{120}$.
- (2) $\frac{1}{40}, \frac{1}{60}, \frac{1}{80}, \frac{1}{120}, \frac{1}{160}, \frac{1}{240}, \frac{1}{320}, \frac{1}{480}$.
- (3) $\frac{1}{20}, \frac{1}{30}, \frac{1}{40}, \frac{1}{60}, \frac{1}{80}, \frac{1}{120}, \frac{1}{160}, \frac{1}{240}$.
- (4) $\frac{1}{12}, \frac{1}{18}, \frac{1}{24}, \frac{1}{36}, \frac{1}{48}, \frac{1}{72}, \frac{1}{96}, \frac{1}{144}$.
- (5) $\frac{1}{30}, \frac{1}{45}, \frac{1}{60}, \frac{1}{90}, \frac{1}{120}, \frac{1}{180}, \frac{1}{240}, \frac{1}{360}$.
- (6) $\frac{1}{40}, \frac{1}{60}, \frac{1}{80}, \frac{1}{120}, \frac{1}{160}, \frac{1}{240}, \frac{1}{320}, \frac{1}{480}$.
- (7) $\frac{1}{20}, \frac{1}{30}, \frac{1}{40}, \frac{1}{60}, \frac{1}{80}, \frac{1}{120}, \frac{1}{160}, \frac{1}{240}$.
- (8) $\frac{1}{12}, \frac{1}{18}, \frac{1}{24}, \frac{1}{36}, \frac{1}{48}, \frac{1}{72}, \frac{1}{96}, \frac{1}{144}$.
- (9) $\frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \frac{1}{30}, \frac{1}{40}, \frac{1}{60}, \frac{1}{80}, \frac{1}{120}$.
- (10) $\frac{1}{20}, \frac{1}{30}, \frac{1}{40}, \frac{1}{60}, \frac{1}{80}, \frac{1}{120}, \frac{1}{160}, \frac{1}{240}$.
- (11) $\frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \frac{1}{30}, \frac{1}{40}, \frac{1}{60}, \frac{1}{80}, \frac{1}{120}$.
- (12) $\frac{1}{12}, \frac{1}{18}, \frac{1}{24}, \frac{1}{36}, \frac{1}{48}, \frac{1}{72}, \frac{1}{96}, \frac{1}{144}$.
- (13) $\frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \frac{1}{30}, \frac{1}{40}, \frac{1}{60}, \frac{1}{80}, \frac{1}{120}$.
- (14) $\frac{1}{12}, \frac{1}{18}, \frac{1}{24}, \frac{1}{36}, \frac{1}{48}, \frac{1}{72}, \frac{1}{96}, \frac{1}{144}$.
- (15) $\frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \frac{1}{30}, \frac{1}{40}, \frac{1}{60}, \frac{1}{80}, \frac{1}{120}$.

47. MIXED NUMBERS. PAGE 68.

- | | | | | |
|-------------------------|---------------------------|--------------------------|---------------------------|---------------------------|
| (1) 1 $\frac{1}{2}$. | (2) 8 $\frac{1}{2}$. | (3) 7 $\frac{1}{2}$. | (4) 7. | (5) 10 $\frac{3}{4}$. |
| (6) 20 $\frac{1}{2}$. | (7) 9. | (8) 13 $\frac{3}{4}$. | (9) 17 $\frac{1}{4}$. | (10) 11 $\frac{5}{16}$. |
| (11) 17 $\frac{1}{2}$. | (12) 1044 $\frac{1}{2}$. | (13) 121 $\frac{1}{2}$. | (14) 481 $\frac{5}{16}$. | (15) 122 $\frac{1}{16}$. |
| (16) 18 $\frac{3}{4}$. | (17) 16 $\frac{1}{2}$. | (18) 15 $\frac{1}{4}$. | (19) 12 $\frac{3}{4}$. | (20) 10 $\frac{1}{4}$. |
| (21) 17 $\frac{1}{4}$. | (22) 18 $\frac{1}{16}$. | (23) 18 $\frac{1}{16}$. | (24) 14 $\frac{1}{16}$. | (25) 15 $\frac{1}{16}$. |

48. IMPROPER FRACTIONS. PAGE 69.

- | | | | | |
|--|---|-------------------------|--------------------------|---------------------------|
| (1) $\frac{8}{5}$. | (2) $\frac{8}{5}$. | (3) $\frac{3}{7}$. | (4) $\frac{7}{11}$. | (5) $\frac{7}{4}$. |
| (6) $\frac{127}{10}$. | (7) $\frac{47}{8}$. | (8) $\frac{7}{6}$. | (9) $\frac{119}{10}$. | (10) $\frac{191}{10}$. |
| (11) $\frac{259}{10}$. | (12) $\frac{396}{7}$. | (13) $\frac{363}{10}$. | (14) $\frac{787}{12}$. | (15) $\frac{547}{11}$. |
| (16) $\frac{374}{5}$. | (17) $\frac{312}{11}$. | (18) $\frac{521}{5}$. | (19) $\frac{617}{4}$. | (20) $\frac{1437}{3}$. |
| (21) $\frac{1338}{5}$. | (22) $\frac{1537}{11}$. | (23) $\frac{433}{33}$. | (24) $\frac{7219}{11}$. | (25) $\frac{10134}{11}$. |
| (26) $\frac{3}{7}, \frac{3}{7}, \frac{3}{7}, \frac{3}{7}, \frac{3}{7}$. | (27) $\frac{4}{11}, \frac{4}{11}, \frac{4}{11}, \frac{4}{11}, \frac{4}{11}$. | | | |
| (28) $\frac{95}{18}, \frac{113}{18}, \frac{113}{18}, \frac{113}{18}, \frac{113}{18}$. | (29) $\frac{4}{15}, \frac{10}{15}, \frac{10}{15}, \frac{10}{15}, \frac{10}{15}$. | | | |
| (30) $\frac{9}{17}, \frac{139}{17}, \frac{179}{17}, \frac{294}{17}, \frac{356}{17}$. | | | | |

49. ADDITION OF FRACTIONS. PAGE 70.

- | | | | | |
|------------------------|------------------------|-------------------------|-------------------------|------------------------|
| (1) $1\frac{1}{2}$. | (2) $1\frac{1}{4}$. | (3) $1\frac{1}{11}$. | (4) $1\frac{1}{4}$. | (5) $\frac{3}{2}$. |
| (6) $2\frac{11}{11}$. | (7) $2\frac{17}{11}$. | (8) $2\frac{1}{2}$. | (9) $2\frac{1}{11}$. | (10) $2\frac{1}{11}$. |
| (11) $2\frac{7}{11}$. | (12) $1\frac{1}{11}$. | (13) $1\frac{47}{11}$. | (14) $1\frac{23}{11}$. | (15) $\frac{1}{11}$. |
| (16) $1\frac{1}{11}$. | (17) $2\frac{1}{11}$. | (18) $2\frac{1}{11}$. | (19) $3\frac{1}{11}$. | (20) $1\frac{7}{11}$. |
| (21) $1\frac{1}{11}$. | (22) $\frac{7}{11}$. | (23) $1\frac{63}{11}$. | (24) $\frac{3}{11}$. | (25) $1\frac{1}{11}$. |
| (26) $\frac{2}{11}$. | (27) $1\frac{1}{11}$. | (28) $1\frac{2}{11}$. | (29) $1\frac{1}{11}$. | (30) $1\frac{1}{11}$. |

50. ADDITION OF FRACTIONS. PAGE 70.

- | | | | | |
|-------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| (1) $18\frac{1}{11}$. | (2) $17\frac{7}{11}$. | (3) $47\frac{1}{11}$. | (4) $10\frac{1}{11}$. | (5) $21\frac{1}{11}$. |
| (6) $14\frac{1}{11}$. | (7) $18\frac{13}{11}$. | (8) $12\frac{1}{11}$. | (9) $7\frac{1}{11}$. | (10) $61\frac{1}{11}$. |
| (11) $24\frac{1}{11}$. | (12) $4\frac{1}{11}$. | (13) $18\frac{13}{11}$. | (14) $53\frac{10}{11}$. | (15) $62\frac{1}{11}$. |
| (16) $6\frac{1}{11}$. | (17) $15\frac{1}{11}$. | (18) $14\frac{1}{11}$. | (19) $60\frac{10}{11}$. | (20) $18\frac{17}{11}$. |
| (21) $19\frac{1}{11}$. | (22) $29\frac{1}{11}$. | (23) $33\frac{1}{11}$. | (24) $27\frac{1}{11}$. | (25) $18\frac{1}{11}$. |
| (26) $28\frac{1}{11}$. | (27) $32\frac{1}{11}$. | (28) $28\frac{1}{11}$. | (29) $18\frac{1}{11}$. | (30) $25\frac{1}{11}$. |
| (31) $12\frac{1}{11}$. | (32) $12\frac{1}{11}$. | (33) $12\frac{1}{11}$. | (34) $19\frac{1}{11}$. | (35) $12\frac{1}{11}$. |
| (36) $12\frac{1}{11}$. | (37) $24\frac{10}{11}$. | (38) $13\frac{1}{11}$. | (39) $22\frac{1}{11}$. | (40) $14\frac{1}{11}$. |
| (41) $34\frac{1}{11}$. | (42) $22\frac{1}{11}$. | (43) $12\frac{1}{11}$. | (44) $19\frac{1}{11}$. | (45) $13\frac{1}{11}$. |
| (46) $8\frac{1}{11}$. | (47) $16\frac{1}{11}$. | (48) $16\frac{1}{11}$. | (49) $19\frac{1}{11}$. | (50) $35\frac{1}{11}$. |

51. SUBTRACTION OF FRACTIONS. PAGE 71.

- | | | | | | |
|------------------------|------------------------|-------------------------|------------------------|------------------------|----------------------|
| (1) $\frac{1}{11}$. | (2) $\frac{1}{11}$. | (3) $\frac{1}{11}$. | (4) $\frac{1}{11}$. | (5) $\frac{1}{11}$. | (6) $\frac{1}{11}$. |
| (7) $\frac{1}{11}$. | (8) $\frac{1}{11}$. | (9) $\frac{1}{11}$. | (10) $1\frac{1}{11}$. | (11) $3\frac{1}{11}$. | |
| (12) $1\frac{1}{11}$. | (13) $2\frac{1}{11}$. | (14) $10\frac{1}{11}$. | (15) $5\frac{1}{11}$. | (16) $5\frac{1}{11}$. | |

52. SUBTRACTION OF FRACTIONS. PAGE 71.

- | | | | | |
|------------------------|------------------------|------------------------|-------------------------|-------------------------|
| (1) $11\frac{1}{11}$. | (2) $8\frac{1}{11}$. | (3) $\frac{1}{11}$. | (4) $61\frac{1}{11}$. | (5) $4\frac{1}{11}$. |
| (6) $1\frac{1}{11}$. | (7) $3\frac{1}{11}$. | (8) $5\frac{1}{11}$. | (9) $12\frac{1}{11}$. | (10) $9\frac{1}{11}$. |
| (11) $4\frac{1}{11}$. | (12) $7\frac{1}{11}$. | (13) $9\frac{1}{11}$. | (14) $11\frac{1}{11}$. | (15) $12\frac{1}{11}$. |

- (16) $6\frac{4}{7}$. (17) $\frac{1}{2}$. (18) $2\frac{2}{3}$. (19) $\frac{4}{7}$. (20) $4\frac{3}{8}$.
 (21) $8\frac{1}{2}$. (22) $10\frac{2}{3}$. (23) $3\frac{2}{3}$. (24) $3\frac{1}{2}$. (25) $17\frac{3}{8}$.
 (26) $5\frac{7}{8}$. (27) $4\frac{3}{8}$. (28) $11\frac{7}{8}$. (29) $13\frac{2}{3}$. (30) $16\frac{1}{2}$.
 (31) $7\frac{1}{2}$. (32) $6\frac{1}{2}$. (33) $6\frac{5}{8}$. (34) $45\frac{1}{8}$. (35) $3\frac{9}{8}$.
 (36) $16\frac{1}{8}$. (37) $1\frac{1}{10}$. (38) $24\frac{1}{8}$. (39) $17\frac{3}{8}$. (40) $14\frac{3}{8}$.

53. MIXED EXAMPLES. PAGE 72.

- (1) $3\frac{1}{4}$. (2) $7\frac{2}{3}$. (3) $2\frac{2}{3}$. (4) $5\frac{5}{8}$. (5) $1\frac{7}{8}$.
 (6) $5\frac{5}{4}$. (7) $2\frac{8}{3}$. (8) $1\frac{4}{9}$. (9) $39\frac{1}{8}$. (10) $3\frac{1}{2}$.
 (11) $13\frac{1}{2}$. (12) $\frac{1}{9}$. (13) 5. (14) $9\frac{1}{8}$. (15) $5\frac{1}{2}$.
 (16) $10\frac{7}{8}$. (17) $5\frac{1}{10}$. (18) $5\frac{7}{10}$. (19) $4\frac{1}{10}$. (20) 10.
 (21) 12. (22) $3\frac{1}{2}$. (23) 0. (24) 0. (25) 2.

54. MULTIPLICATION OF FRACTIONS. PAGE 73.

- (1) $\frac{4}{11}$; $\frac{6}{11}$. (2) $\frac{2}{3}$; $1\frac{7}{8}$. (3) $4\frac{7}{8}$; $2\frac{1}{2}$.
 (4) $4\frac{9}{11}$; $6\frac{6}{11}$. (5) $\frac{3}{11}$; $2\frac{2}{3}$. (6) $1\frac{2}{7}$; $2\frac{4}{7}$.
 (7) $1\frac{1}{8}$; $3\frac{1}{2}$. (8) $2\frac{2}{7}$; $5\frac{2}{3}$. (9) $11\frac{2}{7}$; $11\frac{2}{3}$.
 (10) 10; $12\frac{1}{2}$. (11) $32\frac{2}{3}$; $51\frac{1}{3}$. (12) $20\frac{2}{3}$; $81\frac{2}{3}$.
 (13) $29\frac{1}{7}$; $109\frac{2}{7}$. (14) $108\frac{2}{4}$; $158\frac{1}{4}$. (15) $7\frac{6}{11}$; $41\frac{1}{2}$.
 (16) $20\frac{1}{7}$; $176\frac{1}{2}$. (17) $37\frac{2}{3}$; $42\frac{5}{7}$. (18) $80\frac{1}{2}$; $240\frac{2}{3}$.

55. DIVISION OF FRACTIONS. PAGE 74.

- (1) $\frac{3}{20}$; $\frac{3}{28}$; $\frac{3}{35}$; $\frac{3}{40}$. (2) $\frac{2}{27}$; $\frac{2}{45}$; $\frac{2}{63}$; $\frac{2}{81}$.
 (3) $\frac{1}{18}$; $\frac{1}{18}$; $\frac{1}{18}$; $\frac{1}{18}$. (4) $\frac{1}{18}$; $\frac{1}{18}$; $\frac{1}{36}$; $\frac{1}{72}$.
 (5) $\frac{5}{18}$; $\frac{1}{6}$; $\frac{1}{36}$; $\frac{1}{4}$. (6) $\frac{1}{40}$; $\frac{1}{80}$; $\frac{1}{160}$; $\frac{1}{40}$.
 (7) $1\frac{1}{2}$; $1\frac{5}{2}$; $\frac{5}{8}$. (8) $1\frac{5}{6}$; $1\frac{5}{2}$; $1\frac{5}{4}$. (9) $3\frac{1}{8}$; $2\frac{8}{21}$; $1\frac{1}{6}$.
 (10) $\frac{1}{6}$; $1\frac{1}{6}$; $\frac{1}{2}$. (11) $\frac{2}{3}$; $\frac{1}{3}$; $\frac{1}{4}$. (12) $\frac{2}{3}$; $\frac{2}{3}$; $1\frac{1}{2}$.

56. MULTIPLICATION OF FRACTIONS. PAGE 75.

- (1) $\frac{1}{8}$; $\frac{1}{8}$; $\frac{7}{8}$. (2) $\frac{7}{8}$; $1\frac{3}{8}$; $\frac{9}{8}$. (3) $\frac{1}{8}$; $1\frac{2}{7}$; $2\frac{7}{8}$.
 (4) $3\frac{2}{3}$; $1\frac{7}{8}$; 4. (5) $10\frac{2}{3}$; $3\frac{2}{3}$; 4. (6) $9\frac{2}{3}$; $19\frac{1}{3}$; $28\frac{2}{3}$.
 (7) $13\frac{2}{3}$; $33\frac{2}{3}$; $28\frac{1}{2}$. (8) $19\frac{1}{2}$; $14\frac{2}{3}$; $28\frac{7}{8}$. (9) $55\frac{1}{2}$; $27\frac{2}{3}$; $23\frac{5}{6}$.
 (10) $2\frac{1}{3}$; $19\frac{1}{3}$; $7\frac{1}{3}$. (11) 18. (12) 9. (13) 60. (14) 29.
 (15) 6. (16) 66. (17) 16. (18) 12. (19) 40.
 (20) 54. (21) $2\frac{2}{3}$. (22) $7\frac{1}{2}$. (23) $17\frac{1}{2}$. (24) $22\frac{1}{2}$.
 (25) $98\frac{1}{2}$. (26) $25\frac{3}{8}$. (27) $18\frac{2}{3}$. (28) $192\frac{1}{2}$. (29) $6\frac{2}{3}$.
 (30) $4\frac{1}{8}$. (31) $31\frac{1}{2}$. (32) $18\frac{2}{3}$. (33) $16\frac{1}{2}$. (34) $80\frac{1}{2}$.
 (35) $\frac{7}{8}$. (36) 3. (37) $4\frac{2}{3}$. (38) $32\frac{2}{3}$. (39) $3\frac{1}{2}$.
 (40) $82\frac{2}{3}$. (41) $1\frac{7}{8}$. (42) $33\frac{1}{2}$. (43) 13. (44) $3\frac{1}{3}$.
 (45) $105\frac{1}{3}$. (46) $16\frac{1}{2}$. (47) 27. (48) $22\frac{1}{2}$. (49) $2\frac{2}{3}$.

- (50) $5\frac{6}{11}$. (51) $291\frac{1}{2}$. (52) $10\frac{1}{2}$. (53) 22. (54) $\frac{1}{2}$.
 (55) 5. (56) $2\frac{1}{2}$. (57) $38\frac{1}{2}$. (58) $1\frac{1}{2}$. (59) $7\frac{1}{2}$. (60) $13\frac{1}{2}$.

57. DIVISION OF FRACTIONS. PAGE 76.

- (1) $\frac{2}{3}$; $\frac{11}{12}$; $1\frac{1}{2}$; $1\frac{1}{3}$. (2) $1\frac{1}{2}$; $4\frac{1}{2}$; $1\frac{1}{10}$; $1\frac{1}{12}$.
 (3) $1\frac{1}{2}$; $\frac{2}{3}$; 2; $1\frac{7}{8}$. (4) $\frac{1}{10}$; $1\frac{1}{4}$; $1\frac{1}{2}$; $1\frac{1}{12}$.
 (5) $162\frac{1}{2}$; 120; $212\frac{1}{2}$; $95\frac{1}{11}$. (6) 120; $300\frac{1}{2}$; $116\frac{1}{2}$; $247\frac{1}{2}$.
 (7) $\frac{3}{4}$; $\frac{2}{3}$; $\frac{5}{8}$; $1\frac{3}{8}$. (8) $\frac{1}{2}$; $1\frac{1}{2}$; $\frac{1}{3}$; $1\frac{2}{3}$.
 (9) $\frac{2}{3}$. (10) $\frac{1}{2}$. (11) $3\frac{1}{2}$. (12) $3\frac{1}{2}$. (13) $11\frac{1}{2}$.
 (14) $\frac{1}{11}$. (15) $\frac{1}{10}$. (16) 4. (17) $1\frac{1}{12}$. (18) $3\frac{1}{2}$.
 (19) $\frac{2}{3}$. (20) $1\frac{1}{2}$. (21) $2\frac{1}{2}$. (22) $4\frac{1}{2}$. (23) $1\frac{1}{2}$.
 (24) $\frac{1}{11}$. (25) $2\frac{1}{2}$. (26) $\frac{1}{2}$. (27) $\frac{1}{11}$. (28) $\frac{1}{2}$.
 (29) 8. (30) $13\frac{1}{10}$. (31) $2\frac{1}{10}$. (32) $7\frac{1}{2}$. (33) $1\frac{1}{2}$.
 (34) $\frac{1}{2}$. (35) $1\frac{1}{11}$. (36) $\frac{2}{3}$. (37) $1\frac{1}{11}$. (38) 7.
 (39) $1\frac{2}{3}$. (40) $2\frac{1}{2}$. (41) $\frac{1}{2}$. (42) $1\frac{1}{2}$. (43) $\frac{1}{2}$.
 (44) $\frac{1}{11}$. (45) $\frac{1}{11}$. (46) $\frac{1}{11}$. (47) $8\frac{1}{2}$. (48) $2\frac{1}{11}$.

58. MIXED EXAMPLES. PAGE 77.

- (1) $\frac{1}{2}$. (2) $14\frac{1}{2}$. (3) $\frac{1}{2}$. (4) $4\frac{1}{2}$. (5) $13\frac{1}{2}$.
 (6) $46\frac{1}{2}$. (7) $17\frac{1}{2}$. (8) 10. (9) $\frac{1}{2}$. (10) $4\frac{1}{2}$.

59. MIXED EXAMPLES. PAGE 78.

- (1) $1\frac{1}{2}$. (2) $7\frac{1}{2}$. (3) $1\frac{1}{2}$. (4) $\frac{1}{10}$. (5) $\frac{1}{11}$. (6) 3.
 (7) $\frac{1}{2}$. (8) $1\frac{1}{2}$. (9) 44. (10) $2\frac{1}{10}$. (11) $3\frac{1}{2}$. (12) $18\frac{1}{2}$.

60. MIXED EXAMPLES. PAGE 78.

- (1) $20\frac{1}{11}$. (2) $\frac{1}{10}$. (3) $\frac{1}{11}$. (4) $3\frac{1}{10}$. (5) $7\frac{1}{2}$.
 (6) $4\frac{1}{11}$. (7) $3\frac{1}{11}$. (8) $15\frac{1}{11}$. (9) $8\frac{1}{11}$. (10) $5\frac{1}{2}$.
 (11) 9. (12) $1\frac{1}{10}$. (13) $116\frac{1}{11}$. (14) $15\frac{1}{11}$. (15) $15\frac{1}{11}$.
 (16) $14\frac{1}{11}$. (17) $13\frac{1}{11}$. (18) $\frac{1}{11}$. (19) $6\frac{1}{2}$. (20) $2\frac{1}{10}$.
 (21) $1\frac{1}{11}$. (22) $\frac{1}{2}$. (23) $1\frac{1}{10}$. (24) $1\frac{1}{10}$. (25) 16.
 (26) $\frac{1}{11}$. (27) $\frac{1}{10}$. (28) $\frac{1}{11}$. (29) 2. (30) $6\frac{1}{10}$.

61. FRACTIONS OF CONCRETE QUANTITIES. PAGE 79.

- (1) £15. 2s. $2\frac{1}{2}d$. (2) £1. 16s. 10d. (3) £61. 7s. $5\frac{1}{2}d$.
 (4) £14. 0s. $1\frac{1}{2}d$. (5) £1. 7s. $8\frac{1}{2}d$. (6) £21. 8s. $9\frac{1}{2}d$.
 (7) £250. 5s. $2\frac{1}{2}d$. (8) £17. 17s. $8\frac{1}{2}d$. (9) £1. 10s.
 (10) 2s. $9\frac{1}{2}d$. (11) £51. 1s. $10\frac{1}{2}d$. (12) £794. 3s. $1\frac{1}{2}d$.
 (13) 6 cwt. 2 qrs. (14) 1 qr. 7 oz. 18 drs. (15) 13 cwt. 24 lb. 8 oz.

- (16) 2 tons 9 cwt. 7 st. 2 lb. (17) 18 tons 7 cwt. 3 lb. 8 oz.
 (18) 4 cwt. 3 qrs. 14 lb. (19) 1 ton 2 cwt. 3 qrs. 3 lb. 8 oz.
 (20) 1 ton 20 lb. (21) 2 yds. 1 ft. 7 in. (22) 3 qrs. 3 bush. 2 pks.
 (23) 32 days 23 hrs. 46 min. 30 sec. (24) 6 days 19 hrs. 26 min. 15 sec.

62. FRACTIONS OF CONCRETE QUANTITIES. PAGE 80.

- (1) 8s. 3d. (2) £1. 9s. (3) £1. 5s. 8½d. (4) £2. 9s. 2d.
 (5) £2. (6) £3. 11s. 9½d. (7) £3. 5s. (8) £8. 18s. 3d.
 (9) £1. 16s. 11½d. (10) £3. 6s. 9½d. (11) 7 lb. 4 oz. 10 drs.
 (12) 1 ton 17 cwt. 3 qrs. 12 lb. (13) 8 cwt. 1 qr. 22 lb.
 (14) 18 cwt. 3 qrs. 7 lb. (15) 1 qr. 24 lb. 8 oz.
 (16) 2 fur. 20 yds. 1 ft. 1½ in. (17) 3 ro. 10 per.
 (18) 4 ao. 2 ro. 5 per. 4 sq. yds. (19) 2 ao. 2 ro. 35 per.
 (20) 2 days 4 hrs. 12 min. 44 sec.

63. FRACTIONS OF CONCRETE QUANTITIES. PAGE 81.

- (1) $\frac{1}{2}$. (2) $\frac{3}{4}$. (3) $\frac{1}{3}$. (4) $\frac{1}{4}$. (5) $\frac{1}{5}$. (6) $\frac{1}{6}$.
 (7) $\frac{1}{7}$. (8) $\frac{1}{8}$. (9) $\frac{1}{9}$. (10) $\frac{1}{10}$. (11) $\frac{1}{11}$. (12) $\frac{1}{12}$.
 (13) $\frac{1}{13}$. (14) $\frac{1}{14}$. (15) $\frac{1}{15}$. (16) $\frac{1}{16}$. (17) $\frac{1}{17}$. (18) $\frac{1}{18}$.
 (19) $\frac{1}{19}$. (20) $\frac{1}{20}$. (21) $\frac{1}{21}$. (22) $\frac{1}{22}$. (23) $\frac{1}{23}$. (24) $\frac{1}{24}$.
 (25) $\frac{1}{25}$. (26) $\frac{1}{26}$. (27) $\frac{1}{27}$. (28) $\frac{1}{28}$. (29) $\frac{1}{29}$. (30) $\frac{1}{30}$.
 (31) $\frac{1}{31}$. (32) $\frac{1}{32}$. (33) $\frac{1}{33}$. (34) $\frac{1}{34}$. (35) $\frac{1}{35}$. (36) $\frac{1}{36}$.
 (37) $\frac{1}{37}$. (38) $\frac{1}{38}$. (39) $\frac{1}{39}$. (40) $\frac{1}{40}$.

64. COMPLEX FRACTIONS. PAGE 83.

- (1) $8\frac{3}{4}$. (2) $25\frac{1}{2}$. (3) 4. (4) $\frac{57}{112}$. (5) $\frac{1}{2}$.
 (6) $1\frac{1}{2}$. (7) 48. (8) $1\frac{1}{2}$. (9) 16. (10) $2\frac{3}{4}$.
 (11) $2\frac{1}{4}$. (12) $103\frac{1}{2}$. (13) 6. (14) $1\frac{1}{2}$. (15) $\frac{1}{2}$.
 (16) $\frac{5}{16}$. (17) $17\frac{1}{2}$. (18) $\frac{3}{16}$. (19) 2. (20) $\frac{1}{16}$.
 (21) $1\frac{1}{4}$. (22) $\frac{1}{2}$. (23) $\frac{3}{8}$. (24) 1. (25) 1.
 (26) $\frac{1}{2}$. (27) $\frac{3}{16}$. (28) 1. (29) $\frac{3}{8}$. (30) $1\frac{1}{4}$.
 (31) 2. (32) $\frac{1}{2}$. (33) $\frac{3}{16}$. (34) 9. (35) $\frac{1}{2}$.
 (36) $2\frac{1}{2}$. (37) 2. (38) 1. (39) $1\frac{1}{4}$. (40) $\frac{1}{16}$.
 (41) $\frac{1}{2}$. (42) $\frac{1}{4}$. (43) $\frac{1}{8}$. (44) $\frac{3}{16}$. (45) $\frac{5}{16}$.

65. MISCELLANEOUS (FRACTIONS). PAGE 85.

- (1) $\frac{1}{2}$. (2) $\frac{1}{3}$. (3) $\frac{2}{3}$. (4) $\frac{1}{3}$. (5) $\frac{1}{3}$.
 (6) $\frac{2}{3}$. (7) 8s. (8) 10s. 5d. (9) 2s. 4d.; 2s.; 4d.

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- (10) $1\frac{1}{4}$. (11) $2\frac{1}{4}$ days. (12) $1\frac{1}{4}$ hrs. (13) $1\frac{1}{4}$ hrs.
 (14) $17\frac{1}{2}$ min. (15) $2\frac{1}{2}$ min. (16) 5. (17) 17. (18) 480
 (19) 7. (20) $\frac{1}{2}$. (21) $3\frac{1}{2}$. (22) $24\frac{1}{2}$. (23) 448.
 (24) $62\frac{1}{2}$. (25) $1\frac{1}{2}$ ft.; $\frac{2}{3}$ of the pole.

66. DECIMAL NOTATION. PAGE 87.

- (1) .4. (2) .7. (3) .14. (4) .75. (5) .03. (6) .795.
 (7) .052. (8) .008. (9) .0583. (10) .0029. (11) .0005.
 (12) 9.5. (13) 1.7. (14) 30.6. (15) 8.53. (16) 2.005.

67. MULTIPLICATION OF DECIMALS. PAGE 87.

- (1) 324.25. (2) .25. (3) 580.2. (4) 50.02.
 (5) .002. (6) 4.5. (7) 45. (8) .45.
 (9) 424.5. (10) 6.275. (11) .025. (12) 5.25.
 (13) 1010. (14) .1. (15) 11. (16) 1001.
 (17) 37275.1. (18) 5267.5. (19) 52427.8. (20) 47.25.
 (21) 1. (22) 7.5. (23) 15625. (24) .05.

68. DIVISION OF DECIMALS. PAGE 88.

- (1) 1.6375. (2) .065. (3) .0352. (4) 1.35.
 (5) 12.5025. (6) .001. (7) .011. (8) .0011.
 (9) .4775. (10) 3.24005. (11) 3.652. (12) .000465.
 (13) .004. (14) .0005. (15) .00006. (16) .000005.
 (17) 1.00265. (18) .365275. (19) .02556. (20) .005825.
 (21) .0005. (22) .000025. (23) .0000035. (24) .00004.

69. CONVERSION TO VULGAR FRACTIONS. PAGE 88.

- (1) $\frac{1}{10}$; $\frac{1}{10}$. (2) $\frac{1}{10}$; $\frac{1}{10}$. (3) $\frac{1}{100}$; $\frac{1}{100}$.
 (4) $\frac{1}{100}$; $\frac{1}{100}$. (5) $\frac{1}{100}$; $\frac{1}{100}$. (6) $\frac{1}{1000}$; $\frac{1}{1000}$.
 (7) $\frac{1}{1000}$; $\frac{1}{1000}$. (8) $\frac{1}{1000}$; $\frac{1}{1000}$. (9) $\frac{1}{1000}$; $\frac{1}{1000}$.
 (10) $\frac{1}{1000}$; $\frac{1}{1000}$. (11) $\frac{1}{1000}$; $\frac{1}{1000}$. (12) $\frac{1}{1000}$; $\frac{1}{1000}$.
 (13) $\frac{1}{1000}$; $\frac{1}{1000}$. (14) $\frac{4}{1000}$; $\frac{4}{1000}$. (15) $\frac{5}{1000}$; $\frac{5}{1000}$.
 (16) $\frac{1}{1000}$; $\frac{1}{1000}$. (17) $\frac{1}{1000}$; $\frac{1}{1000}$. (18) $\frac{1}{1000}$; $\frac{1}{1000}$.
 (19) $\frac{1}{1000}$; $\frac{1}{1000}$. (20) $\frac{6}{1000}$; $\frac{6}{1000}$. (21) $\frac{7}{1000}$; $\frac{7}{1000}$.
 (22) $\frac{5}{1000}$; $\frac{5}{1000}$. (23) $\frac{2}{1000}$; $\frac{2}{1000}$. (24) $\frac{4}{1000}$; $\frac{4}{1000}$.
 (25) $13\frac{1}{1000}$; $13\frac{1}{1000}$. (26) $21\frac{1}{1000}$; $21\frac{1}{1000}$.
 (27) $15\frac{1}{1000}$; $15\frac{1}{1000}$. (28) $29\frac{1}{1000}$; $29\frac{1}{1000}$.
 (29) $82\frac{1}{1000}$; $82\frac{1}{1000}$. (30) $90\frac{1}{1000}$; $90\frac{1}{1000}$.

70. ADDITION OF DECIMALS. PAGE 89.

- | | | | |
|------------------|------------------|-------------------|--------------|
| (1) 182.1999. | (2) 515.7578. | (3) 10.39976. | (4) .0113. |
| (5) 20.11201. | (6) 16.83165. | (7) 5.0232. | (8) 748.195. |
| (9) 365.055291. | (10) 590.50254. | (11) 493.511606. | |
| (12) 525.710841. | (13) 250.093876. | (14) 619.62482. | |
| (15) 93.639977. | (16) 445.012456. | (17) 570.6444478. | |
| (18) 1259.71947. | (19) 5725.2492. | (20) 6568.57854. | |
| (21) 4108.3528. | (22) 119.102907. | (23) 199.876. | |
| (24) 716.15477. | (25) 1056.23193. | | |

71. SUBTRACTION OF DECIMALS. PAGE 90.

- | | | | |
|-----------------|-----------------|------------------|---------------|
| (1) 2.075. | (2) .071. | (3) .999. | (4) .99. |
| (5) 6.491. | (6) 9.325. | (7) 2.5239. | (8) 1.097. |
| (9) 86.273. | (10) .0966. | (11) 6.124. | (12) 1.13995. |
| (13) 5.449788. | (14) 10.831893. | (15) 100.732687. | |
| (16) 12.499897. | (17) 41.6922. | (18) 12.06908. | |
| (19) 10.64402. | (20) 27.255035. | (21) 23.064848. | |
| (22) 60.788291. | (23) 4.65863. | (24) 110.43222. | |
| (25) 2.501246. | (26) 80.18131. | (27) 7.7257. | |
| (28) 5.7288. | (29) 22.28775. | (30) 9.2721. | |

72. MULTIPLICATION OF DECIMALS. PAGE 91.

- (1) 1; 1.5; 2.5; 4.5; 5. (2) .25; .375; .5; 1; .625; 3.125; 15.625.
 (3) .45; .6; .75; .9; 1.2; 1.35; 1.65.
 (4) .4; .7; .9; 1.05; 1.25.
 (5) 12.96; 22.68; 181.44; 155.52; 194.4.
 (6) 9.5034; 142.551; 199.5714; 354.7936; 342.1224.
 (7) .03; .0525; .072; .225; 1.65.
 (8) 90.15; 135.225; 171.285; 616.025; 3756.25.
 (9) 2297.25; 2756.7; 3216.15; 18378; 52071.
 (10) 2550.25; 3315.325; 3621.355; 22952.25; 77527.6.

73. MULTIPLICATION OF DECIMALS. PAGE 92.

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|--------------------|---------------------|--------------------|------------------|
| (1) 142.2. | (2) 146.6621. | (3) 211.06. | (4) 23.408. |
| (5) .380716. | (6) .01546875. | (7) 3200. | (8) .0097565. |
| (9) 1.092347. | (10) .0001292. | (11) .00011355. | (12) .000038665. |
| (13) 874.581. | (14) 6.384512. | (15) 24.5400662. | (16) .010157427. |
| (17) .01431025914. | (18) 364.406442. | (19) 1.593629862. | |
| (20) 1706.870035. | (21) 19.8219661336. | (22) 430.68561447. | |
| (23) 72.7093511. | (24) 327.28160968. | (25) 196.7252176. | |

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| (26) 5·312700445. | (27) ·0520556. | (28) ·000006321782 |
| (29) ·001700784372. | (30) 129·0737915144. | (31) ·0110523. |
| (32) ·0000028561. | (33) ·000001. | (34) ·00001111. |
| (35) ·0806. | (36) ·103125. | |

74. DIVISION OF DECIMALS. PAGE 93.

- | | |
|--|-----------------------------------|
| (1) 8·5; 5·1; 1·7; 1·5; ·17; 15 | |
| (2) ·085; ·051; ·0075; ·00425; ·0025; ·001. | |
| (3) 1·2096; ·5184; ·4032; ·0576; ·0288; ·0192. | |
| (4) 2·49; ·15; ·075; ·0375; ·03. | |
| (5) 128·947; 59·514; 9·919; 4·578; ·763. | |
| (6) ·0017; ·00085; ·0000425. | (7) 13·2545; ·2705; ·13525. |
| (8) 3·249; 1·088; 1·6245. | (9) 31·34; 6·268; 1·2536; ·12536. |
| (10) ·00001108; ·00000554; ·00000277. | (11) ·1947. |
| (12) 2·7643. | (13) ·005976. |
| (14) ·00375. | (15) ·000817. |
| (16) ·00038. | (17) ·00004. |
| (18) ·0000068. | (19) ·000001. |
| (20) ·00000016. | |

75. DIVISION OF DECIMALS. PAGE 94.

- | | | | |
|-----------------|------------------|----------------|------------------|
| (1) 116·214. | (2) ·008. | (3) 3636 4. | (4) 7416400. |
| (5) ·00756. | (6) 49·22 | (7) 3000. | (8) 300300. |
| (9) 63. | (10) 1200. | (11) 8·1. | (12) 15. |
| (13) 605000. | (14) 19·04. | (15) 3122. | (16) ·0059. |
| (17) 408000. | (18) ·06624. | (19) ·00501. | (20) ·0000152. |
| (21) 47050. | (22) 3·041. | (23) 5080. | (24) ·01023. |
| (25) 8·17. | (26) ·54. | (27) 436. | (28) ·08462. |
| (29) 19·1. | (30) 26·4. | (31) 20 5. | (32) 320·45. |
| (33) ·000961. | (34) 800·576. | (35) 7·70071. | (36) 23780000. |
| (37) 3750. | (38) 125000. | (39) 12500. | (40) 21180000. |
| (41) 4024·7457. | (42) ·0054. | (43) ·0129. | (44) 20·4148. |
| (45) 4119·2218 | (46) 12302·6315. | (47) ·0012. | (48) ·0386. |
| (49) 25·4417. | (50) 1·5791. | (51) 1 1371. | (52) ·2218. |
| (53) 18·6478. | (54) ·2254. | (55) ·8642. | (56) ·0021. |
| (57) 33·0619. | (58) 55 8836. | (59) 148 2799. | (60) 12302 6315. |

76. CONVERSION INTO DECIMALS. PAGE 95.

- | | | | | |
|--------------|-----------------|-----------------|------------------|---------------|
| (1) ·15. | (2) 35. | (3) ·45 | (4) ·72. | (5) ·076. |
| (6) ·888. | (7) 4·48. | (8) 10·425. | (9) 5·3125. | (10) 8 078125 |
| (11) ·01875. | (12) ·01171875. | (13) 1 5625. | (14) ·609375. | |
| (15) ·09375. | (16) 8·6428571. | (17) 9 2692307. | (18) 24·3529411. | |

- (19) 4·5238095. (20) 2·0370370. (21) 8·4411764. (22) 10·3095238.
 (23) 7·3260869. (24) 3·3854166. (25) 9·0462962. (26) 12·1315789.
 (27) 14·2678571. (28) 16·37190088. (29) 18·8046875. (30) 7·9179687.

77. COMPLEX DECIMALS. PAGE 96.

- (1) ·0575. (2) ·00001. (3) 4·255. (4) ·000125.
 (5) 2070000. (6) 42. (7) 30518·59. (8) 4·8.
 (9) 10·125. (10) 562·1.

78. RECURRING DECIMALS. PAGE 97.

- (1) ·26̇. (2) ·107̇. (3) ·135̇. (4) ·38̇. (5) ·56̇.
 (6) ·093̇. (7) ·590̇. (8) ·10̇. (9) ·0227̇. (10) ·02439̇.
 (11) 1·42̇. (12) 3·583̇. (13) 4·42̇. (14) 5·05̇. (15) 6·153846̇.
 (16) 10·190476̇. (17) 12·24̇. (18) 14·629̇. (19) 15·9583̇.
 (20) 16·138̇. (21) 21·81̇. (22) 32·857142̇. (23) 43·615384̇.
 (24) 54·5370̇. (25) 60·1714285̇.

79. RECURRING DECIMALS. PAGE 98.

- (1) $\frac{1}{3}$. (2) $\frac{14}{33}$. (3) $\frac{4}{37}$. (4) $\frac{27}{37}$. (5) $\frac{12}{101}$.
 (6) $\frac{11}{11}$. (7) $\frac{28}{498}$. (8) $\frac{31}{198}$. (9) $\frac{1}{800}$. (10) $15\frac{1}{11}$.
 (11) $\frac{1}{88}$. (12) $\frac{1}{270}$. (13) $\frac{1}{2400}$. (14) $3\frac{2}{35}$. (15) $5\frac{2}{36}$.
 (16) $10\frac{17}{88}$. (17) $3\frac{37}{600}$. (18) $6\frac{47}{408}$. (19) $12\frac{266}{378}$. (20) $2\frac{1}{17}$.
 (21) $2\frac{5}{78}$. (22) $5\frac{327}{2020}$. (23) $2\frac{23}{880}$. (24) $8\frac{283}{880}$. (25) $1\frac{26819}{133200}$.
 (26) ·01. (27) ·001. (28) ·2. (29) ·32. (30) ·063.
 (31) ·0245. (32) ·7. (33) 10. (34) 100. (35) 100·9.

80. ADDITION AND SUBTRACTION OF RECURRING DECIMALS. PAGE 99.

- (1) 1·12̇. (2) 1·0189̇. (3) ·6116̇. (4) 8·7194̇.
 (5) 5·43862̇. (6) 94·2124̇. (7) 5·11095̇. (8) 1·67670̇.
 (9) 34·01̇. (10) 3·06̇. (11) ·8̇. (12) 31·081̇.
 (13) 5·62576̇. (14) 6·9725̇. (15) 42·65924̇. (16) 15·343̇.
 (17) 7·5. (18) ·5115̇. (19) ·425. (20) ·8.

81. MULTIPLICATION AND DIVISION OF RECURRING DECIMALS. PAGE 100.

- (1) 395·01̇. (2) 4125·785̇. (3) 5·87723̇. (4) 6·382798̇.
 (5) ·6627̇. (6) ·00188922̇. (7) ·047619̇. (8) 7·417̇.
 (9) ·013302̇. (10) 1522·778̇. (11) ·72̇. (12) ·0001999̇.

ANSWERS.

XXV

- | | | | |
|-----------------|-----------------|---------------|----------------|
| (13) .1656. | (14) 1.469083. | (15) .005527. | (16) 99.0383. |
| (17) 92.597371. | (18) .22222477. | (19) 10.4. | (20) 830.5. |
| (21) 4080.5. | (22) .4783. | (23) .00062. | (24) 1.714285. |

82. MULTIPLICATION AND DIVISION OF RECURRING DECIMALS. PAGE 101.

- | | | | |
|-----------|-------------|------------|-------------|
| (1) 4.3. | (2) 2.45. | (3) .0006. | (4) .0026. |
| (5) .007. | (6) .00999. | (7) 6.6. | (8) 5.3162. |
| (9) .12. | (10) .23. | (11) 20.5. | (12) .4. |

83. DECIMALS OF CONCRETE QUANTITIES. PAGE 101.

- | | | | |
|-----------------|-----------------|---------------|----------------|
| (1) .9375. | (2) 971875. | (3) 3.23125. | (4) .778125. |
| (5) .0684375. | (6) .0588125. | (7) .00625. | (8) .8375. |
| (9) .375. | (10) .01171875. | (11) .04375. | (12) .1390625. |
| (13) .24140625. | (14) .009375. | (15) .01875. | (16) .1375. |
| (17) .39625. | (18) .4375. | (19) .003125. | (20) .6875. |

84. DECIMALS OF CONCRETE QUANTITIES. PAGE 102.

- | | | | | |
|-------------|------------|------------|------------|-------------|
| (1) .75. | (2) .625. | (3) .625. | (4) .15. | (5) .625. |
| (6) .04. | (7) .384. | (8) 2.25. | (9) .005. | (10) 0.66. |
| (11) 3.275. | (12) 6.75. | (13) 1.08. | (14) .375. | (15) 1.155. |
| (16) .6305. | (17) .2. | (18) .06. | (19) .305. | (20) .6. |

85 a. DECIMAL VALUES. PAGE 103.

- | | | | | |
|------------|-------------|--------------|--------------------|-------------|
| (1) 6d. | (2) 60d. | (3) 13d. | (4) 9 half-crowns. | (5) 7 lb. |
| (6) 63 lb. | (7) 12 drs. | (8) 572 yds. | (9) 462 hrs. | (10) 8 cwt. |

85 b. DECIMAL VALUES. PAGE 103.

- | | | | |
|---|---------------------------|---------------------------|-------------------|
| (1) 1s. 3d. | (2) 18s. 9d. | (3) £3. 12s. 3d. | (4) £26. 19s. 6d. |
| (5) £50. 18s. 6½d. | (6) 2 cwt. 3 qrs. 7 lb. | (7) 18 tons 4 cwt. 2 qrs. | |
| (8) 16 cwt 3 qrs. 10 lb. 8 oz. | (9) 1 bush 2 pk. 1 gall. | | |
| (10) 3 pk. 1 gall. 1 pt. | (11) 52 days 18 min. | (12) 6 fur. 12 po. | |
| (13) 2 ro. 14 per. | (14) £1. 18s. 5½d. | (15) £48. 8s. 1½d. | |
| (16) £5. 6s. 9d. | (17) £1 11s. 6¾d. | (18) £1 10s. 5¾d. | |
| (19) 1 ac. 1 ro. 17 per. 15 yds. 1 ft. 18 in. | (20) 1 cwt. 3 qrs. 18 lb. | | |
| (21) 228 days 6 hrs. 45 min. | (22) 5 ac. 3 ro. 15 per. | | |
| (23) 94 miles 7 fur. 36 po. | (24) 2 ac. 2 ro. | | |
| (25) 7 bush. 3 pk. 1 gall. | (26) 4s. 5d. | (27) 15s. 9d. | |
| (28) £1. 1s. 7½d. | (29) £2. 9s. 2d. | (30) £45. 4s. 9d. | |

86. DECIMAL VALUES. PAGE 104.

- (1) 9 $\frac{1}{2}$ d. (2) 7s. 6d. (3) £1. 4s. 1 $\frac{1}{2}$ d. (4) £4. 17s. 8 $\frac{1}{2}$ d.
 (5) £414. (6) 3s. 11d. (7) £11 6s. 8 $\frac{1}{2}$ d. (8) 10s. 11d.
 (9) 18s. 5 $\frac{1}{2}$ d. (10) £2. 13s. 3 $\frac{1}{2}$ d. (11) 14 cwt. 7 lb.
 (12) 3 qrs. 15 lb. 5 oz. (13) 2 tons 3 cwt. 3 qrs. 13 lb. (14) 1 lb.
 (15) 232 days 10 hrs. 50 min. 1 $\frac{1}{2}$ sec. (16) £8. 8s. (17) 13s.
 (18) 13s. 10 $\frac{1}{2}$ d. (19) £20. 11s. 0 $\frac{1}{2}$ d. (20) 6 fur. 18 po. 5 yds

87. MISCELLANEOUS (DECIMALS). PAGE 104.

- (1) .163; .163; .036; .005868. (2) 19.7136; 12078.229801.
 (3) .00091; 100000. (4) 11000; .0011. (5) .300847.
 (6) 4.44444; .01001. (7) 92959.677; .00009375. (8) 2.3081.
 (9) .036742. (10) .8675. (11) 1005.6979. (12) 103.9888.
 (13) 1.02515; 1.00485; .01030225; 100.
 (14) 16.21987; 1621.987; 162198.7; .1621987. (15) £19. 1s. 7 $\frac{1}{2}$ d.
 (16) 3.875; 14.53125; 3.75. (17) £13. 12s. 3d.
 (18) .03125; .0025. (19) 2s. 3d.; .45. (20) .567.

88 a. SIMPLE PRACTICE. PAGE 106.

- (1) £87. 10s. (2) £96. 15s. (3) £53. 16s.
 (4) £79. 10s. (5) £48. 8s. (6) £52. 7s. 6d.
 (7) £45. 6s. 8d. (8) £37. 16s. 8d. (9) £43. 16s. 8d.
 (10) £46. 9s. 4d. (11) £53. 6s. 3d. (12) £15. 7s. 6d.
 (13) £9. 4s. 8d. (14) £12. 8s. 6d. (15) £6. 12s. 9d.
 (16) £6. 19s. 8d. (17) £6. 6s. 6d. (18) £5. 3s. 4 $\frac{1}{2}$ d.
 (19) £2. 18s. 2d. (20) £1. 16s. 1 $\frac{1}{2}$ d. (21) £621. 15s.
 (22) £229. 7s. 6d. (23) £271. 16s. (24) £186. 13s. 4d.
 (25) £306 16s. 8d. (26) £350. 7s. (27) £245. 14s.
 (28) £278. 12s. 6d. (29) £89. 6s. 6d. (30) £133. 18s. 8d.
 (31) £178. 17s. 6d. (32) £84. 5s. 9d. (33) £96. 13s. 4d.
 (34) £97. 1s. 9d. (35) £97. 15s. 4d. (36) £34. 8s. 6d.
 (37) £48 9s. 6d. (38) £41. 18s. 1 $\frac{1}{2}$ d. (39) £44. 11s. 7d.
 (40) £240. 19s. 6d. (41) £415. 2s. (42) £563. 6s. 8d.
 (43) £355. 19s. (44) £313. 19s. (45) £109. 7s. 6d.

88 b. SIMPLE PRACTICE. PAGE 107.

- (1) £9520. 10s. (2) £16625. 5s. (3) £12396. 16s.
 (4) £32215. 15s. (5) £54252. (6) £75119. 8s.
 (7) £38994. 15s. (8) £49853. 8s. 4d. (9) £30020. 2s.
 (10) £7524. (11) £13928. 8s. (12) £22572. 13s. 4d.

ANSWERS.

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|------------------------|------------------------|-----------------------|
| (13) £39082. 10s. | (14) £48249. 5s. | (15) £83018. 3s. |
| (16) £60540. 4s. | (17) £40704 7s. 6d. | (18) £24667. 4s. 6d. |
| (19) £6840. 11s. 8d. | (20) £119662. 6s 2d. | (21) £6775. 2s. 2½d. |
| •(22) £42385. 8s. 3¼d. | (23) £10473. 18s. 2½d. | |
| (24) £6189. 5s. 7½d. | (25) £15812. 8s. 1½d. | |
| (26) £19019. 19s. 7½d. | (27) £44173. 19s. | (28) £45197. 17s. 9d. |
| (29) £35360. 6s. 5d. | (30) £69339. 19s. 3¼d. | (31) £5858. 6s. |
| (32) £5554. 10s. | (33) £3621. 12s. | (34) £2969. 5s. |
| (35) £3348. 16s. 10½d. | (36) £623. 11s. 10½d. | |
| (37) £1379. 15s. 5d. | (38) £1093. 10s. | |
| (39) £2350. 5s. 2½d. | (40) £2721. 2s. 10½d. | |

89. SIMPLE PRACTICE. PAGE 108.

- | | | |
|------------------------|------------------------|----------------------|
| (1) £30. 15s. 10d. | (2) £53. 10s. 7½d. | (3) £100. 16s. 6¾d. |
| (4) £49. 9s. 4d. | (5) £97. 0s. 9d. | (6) £73. 11s. 11d. |
| (7) £70. 17s. 11d. | (8) £143. 4s. 9¼d. | (9) £97. 19s. 11½d. |
| (10) £75. 6s. 11¾d. | (11) £10865. 6s. 10d. | (12) £4011. 6s. 6¾d. |
| (13) £8533. 8s. 9d. | (14) £1927. 16s. 10½d. | |
| (15) £5695. 9s. 7d. | (16) £21571. 9s. 5½d. | |
| (17) £19005. 15s. 2½d. | (18) £12290. 17s. 8½d. | |
| (19) £20955. 13s. 2½d. | (20) £11410. 17s. 7½d. | |

90. SIMPLE PRACTICE. PAGE 108.

- | | | |
|------------------------|-----------------------|-------------------------|
| (1) £5934. 1s. 10½d. | (2) £39478. 1s. 6d. | (3) £2144. 13s. 4½d. |
| (4) £4549. 19s. 5¼d. | (5) £780. 2s. 7½d. | (6) £16072. 1s. 9d. |
| (7) £69. | (8) £22388. 19s. 8¼d. | (9) £11. 16s. 3d. |
| (10) £1197. 17s. 3d. | (11) £9064. 19s. 5¼d. | (12) £28516. 16s. |
| (13) £41137. 13s. 3¾d. | (14) £5576. 18s. 8¼d. | (15) £177864. 12s. 11d. |

91 a. COMPOUND PRACTICE. PAGE 109.

- | | | |
|-------------------|--------------------|--------------------|
| (1) £24. 7s. 6d. | (2) 12s. 10d. | (3) £1. 11s. 6d. |
| (4) £11. 13s. 4d. | (5) £8. 4s. 6d. | (6) £4. 19s. 9d. |
| (7) £18 16s. 10d. | (8) 15s. 11½d. | (9) 18s. 4d. |
| (10) £39. 4s. | (11) £5. 18s. 9d. | (12) £6. 6s. |
| (13) 12s. 10d. | (14) 4s. 9½d. | (15) £16. 10s. |
| (16) £13. | | |
| (17) 3s. 6d. | (18) 5s. 6½d. | (19) 9s 2¼d. |
| (20) £7. 3s. | | |
| (21) 1s. 2½d. | (22) 1s. 10½d. | (23) 7s 10½d. |
| (24) 9s. 4½d. | | |
| (25) 4s. 9¾d. | (26) 6s. 7¾d. | (27) 11s. 0¾d. |
| (28) 6s. 1½d. | | |
| (29) 12s. 6½d. | (30) £56. 2s. 4½d. | (31) £32. 9s. 0¾d. |
| (32) £1. 1s 4½d. | (33) £22. 13s. 3d. | (34) 1s. 11d. |
| (35) £21. 5s. 6d. | (36) £18. 4s. 2d. | |

91 b. COMPOUND PRACTICE. PAGE 109.

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|---------------------|-----------------------|---------------------|
| (1) £16. 5s. 1½d. | (2) £13. 10s. 5d. | (3) £1. 12s. 9¾d. |
| (4) £2. 1s. 1½d. | (5) £13. 9s. 0¾d. | (6) £40. 4s. 2d. |
| (7) £39. 8s. 3d. | (8) £53. 5s. 2¼d. | (9) £48. 18s. 6½d. |
| (10) £211. 13s. 4d. | (11) £116. 19s. 10¾d. | (12) £87. 0s. 9d. |
| (13) £228. 13s. 6d. | (14) 3s. 2¾d. | (15) £1. 6s. 9d. |
| (16) £71. 17s. 2¼d. | (17) £33. 16s. 7¼d. | (18) £2. 5s. 10d. |
| (19) £21. 18s. 9¾d. | (20) £20. 6s. 2¼d. | (21) £253. 0s. 4d. |
| (22) £94. 2s. 8½d. | (23) £26. 17s. 10½d. | (24) £28. 18s. 6d. |
| (25) £28. 4s. 8d. | (26) £148. 16s. 8d. | (27) £38. 16s. 3d. |
| (28) £33. 0s. 4½d. | (29) £22. 11s. 8¼d. | (30) 9s. 8¼d. |
| (31) £6. 14s. 5½d. | (32) £21. 10s. 1d. | (33) £3. 7s. 0¾d. |
| (34) £19. 11s. | (35) £29. 12s. 9¾d. | (36) £45. 17s. 9¾d. |

92. MISCELLANEOUS (PRACTICE). PAGE 110.

- | | | |
|------------------------|---------------------|---------------------|
| (1) £35. 6s. 9¼d. | (2) £3266. 4s. 2¾d. | (3) £270. 13s. 4d. |
| (4) £208. 8s. 11¼d. | (5) £1784. 11s. 9d. | (6) £1569. 17s. 4d. |
| (7) £42. 14s. 3½d. | (8) £1355. 9s. 6d. | (9) £149. 19s. 6d. |
| (10) £524. 13s. 9d. | (11) £5. 6s. 8d. | (12) £400. |
| (13) £62327. 6s. 10½d. | (14) £123. 4s. | (15) £468. 15s. |
| (16) £60500. 3s. 6d. | (17) £1. 6s. | (18) £10. 12s. 4¾d. |
| (19) £43. 0s. 0¾d. | (20) £5. 5s. | |

93. INVOICES. PAGE 112.

- (1) 15s. + 2s. 3d. + 2s. 8d. + 13s. 4d. + 2s. 1d. = £1. 15s. 4d.
- (2) 11s. 1d. + 6s. 3d. + 8s. 6d. + 6s. + 8s. 3d. = £2. 0s. 1d.
- (3) 1s. 10d. + 7s. 4d. + 1s. 10½d. + 3s. 3d. + 2s. 11d. = 17s. 2½d.
- (4) £4. 8s. + 9s. 7d. + £1. 2s. 6d. + £2. 10s. 3¾d. + £1. 0s. 9d.
= £9. 11s. 1¾d.
- (5) £7. 10s. 9d. + £1. 2s. 9d. + £7. 2s. 6d. + £8. 8s. 7d. + 6s. 6¾d.
= £24. 11s. 1¾d.
- (6) 19s. 1½d. + £1. 8s. 1½d. + £5. 15s. + £13 + £6. 15s. = £27. 17s. 3d.
- (7) £5. 19s. 7d. + £1. 8s. 1½d. + 8s. 4d. + 9s. 11d. + £11. 5s.
= £19. 10s. 11½d.
- (8) 9s. 9d. + 7s. 6d. + 18s. 8d. + 7s. 1d. + 5s. 10d. = £2. 8s. 10d.
- (9) £3. 9s. + £5. 18s. + £3. 9s. + 13s. + £2. 13s. 9d. = £16. 2s. 9d.
- (10) 10s. + £1. 5s. 6d. + £1. 2s. + £3. 3s. + £11. 5s. = £17. 5s. 6d.
- (11) £1. 1s. 1¾d. + 16s. 11½d. + 10s. + 8s. 9d. + 10s. 3¼d. = £3. 7s. 1½d.

- (12) $19s. 6\frac{1}{2}d. + £6. 19s. 1\frac{1}{2}d. + £6. 15s. 7\frac{1}{2}d. + £2 + £1. 2s. 11d.$
 $= £17. 17s. 2\frac{1}{2}d.$
- (13) $5s. 8\frac{1}{4}d. + 8s. 0\frac{1}{4}d. + 8s. 7\frac{1}{2}d. + 8s. 2d. + 6s. 3d. = £1. 16s. 9d.$
- (14) $£1. 0s. 10d. + 8s. 5\frac{3}{4}d. + 17s. 5\frac{1}{4}d. + 18s. 5\frac{1}{4}d. + 17s. 9\frac{3}{4}d. = £4. 3s.$
- (15) $1s. 7\frac{1}{2}d. + 10s. + £3. 2s. 6d. + 5s. 6d. + 3s. 6d. = £4. 3s. 1\frac{1}{2}d.$
- (16) $£1. 7s. 6\frac{3}{4}d. + £1. 11s. 3d. + £5. 1s. 6d. + 17s. 7\frac{1}{2}d. + £3. 11s. 3d.$
 $= £12. 9s. 2\frac{1}{4}d.$
- (17) $£1. 1s. 4\frac{1}{2}d. + 18s. 9d. + £4. 18s. + £1. 15s. + 18s. 8d. = £9. 11s. 9\frac{1}{2}d.$
- (18) $11s. 8\frac{1}{4}d. + £4. 17s. 6d. + 9s. 5\frac{3}{4}d. + 2s. + £5. 3s. 1\frac{1}{2}d. = £11. 3s. 9\frac{1}{2}d.$
- (19) $1s. 10\frac{1}{2}d. + £6. 18s. 0\frac{1}{4}d. + £1. 10s. 0\frac{3}{4}d. + 17s. 11\frac{1}{4}d. + £9. 12s.$
 $= £18. 19s. 10\frac{3}{4}d.$
- (20) $6s. 3d. + £6. 10s. 8d. + £1. 18s. 3d. + £1. 16s. 8d. + £1. 7s. 6d.$
 $= £11. 19s. 4d.$
- (21) $3s. + 19s. 6d. + £2. 6s. 9d. + £11. 13s. 4d. + £2. 19s. 4\frac{1}{2}d.$
 $= £18. 1s. 11\frac{1}{2}d.$
- (22) $1s. 8d. + £5. 5s. + £13 + £1. 6s. 3d. + £1. 4s. 5\frac{1}{4}d. = £20. 17s. 4\frac{1}{4}d.$
- (23) $£1. 3s. 7\frac{1}{2}d. + £1. 15s. 9d. + 7s. 3\frac{1}{2}d. + 14s. 8\frac{1}{4}d. + £3. 14s. 8d.$
 $= £7. 16s. 0\frac{1}{4}d.$
- (24) $1s. 8\frac{1}{4}d. + 13s. 1\frac{1}{2}d. + £5. 5s. + £2. 6s. 9d. + £1. 0s. 3d. = £9. 6s. 9\frac{3}{4}d.$
- (25) $9s. 7\frac{1}{2}d. + £8. 4s. 6d. + £2. 10s. + 9s. 5\frac{3}{4}d. + 7s. 2\frac{1}{4}d. = £12. 0s. 9\frac{1}{2}d.$
- (26) $15s. + £1. 7s. 1d. + £4. 1s. + 13s. + £3. 5s. 10\frac{1}{2}d. = £10. 1s. 11\frac{1}{2}d.$
- (27) $8s. 7\frac{1}{2}d. + 2s. 9\frac{1}{4}d. + £2. 4s. 7\frac{1}{2}d. + 12s. 10d. + 4s. 4\frac{1}{2}d.$
 $= £3. 13s. 2\frac{3}{4}d.$
- (28) $8s. 4\frac{3}{4}d. + 18s. 8d. + 2s. 11d. + 12s. 10d. + £3. 3s. = £5. 5s. 9\frac{3}{4}d.$
- (29) $16s. 3\frac{1}{2}d. + 5s. 5\frac{1}{4}d. + £1. 18s. 6\frac{1}{2}d. + £13. 5s. 6d. + £1$
 $= £17. 5s. 9\frac{1}{4}d.$
- (30) $12s. 6\frac{1}{2}d. + £10. 8s. 4d. + 5s. + £2. 6s. 9d. + 19s. 6d. = £14. 12s. 1\frac{1}{2}d.$

94. SIMPLE PROPORTION. PAGE 114.

- | | | | |
|----------------------------------|--------------------|------------------------------|------------------------------|
| (1) 1s. 6d. | (2) 6s. 3d. | (3) £10. 10s. | (4) £1. 10s. 3d. |
| (5) £10. 17s. 4 $\frac{1}{2}$ d. | (6) £23. 6s. 8d. | (7) £6. 9s. 6d. | |
| (8) £35. 3s. | (9) £7. 19s. 6d. | (10) £115. 4s. | |
| (11) £211. 9s. 2d. | (12) £22. 1s. | (13) £1. 14s. | (14) 8s. 9d. |
| (15) £4. 0s. 10d. | (16) £84. 10s. 6d. | (17) £1. 3s. 8d. | |
| (18) £6. 18s. 10d. | (19) £56. | (20) £2. 5s. 9d. | (21) 14s. 1 $\frac{1}{2}$ d. |
| (22) £10. 7s. 10d. | (23) 306. | (24) 36. | (25) 60. |
| (26) 17. | (27) 365. | (28) 79 qrs. 6 bush. | (29) 140. |
| (30) 5. | (31) 3312. | (32) 14s. 0 $\frac{3}{4}$ d. | (33) 10s. 5d. |
| (34) 2s. 8 $\frac{1}{2}$ d. | (35) 40. | (36) 4. | (37) 108. |
| (38) 112. | | | |
| (39) 15. | (40) 12. | (41) 12. | (42) 24. |
| (43) 16. | | | |
| (44) 12. | (45) £100. | (46) £360. | (47) £480. |
| | | | (48) £250. |

95. SIMPLE PROPORTION. PAGE 118.

- | | | | |
|-------------------|----------------------|-----------------------|-------------|
| (1) 2s. 1d. | (2) 1s. 4d. | (3) £1. 5s. 8d. | (4) £1. 7s. |
| (5) £1. 10s. | (6) £210. | (7) 4s. 11½d. | (8) 3s. 9d. |
| (9) 3s. 9d. | (10) £13. 0s. 7½d. | (11) £2191. 3s. 8d. | |
| (12) £7700. | (13) £33. 18s. 4d. | (14) £24059. 17s. 9d. | |
| (15) 65. | (16) 41½. | (17) 4½. | (18) 5½. |
| (19) £1. 2s. 8½d. | (20) £450. 15s. 7½d. | (21) £33. 13s. 6d. | |
| (22) 6s. 9½d. | (23) 2½d. | (24) £14. 8s. | |

96. COMPOUND PROPORTION. PAGE 119.

- | | | | | |
|----------------------|----------|--------------------|-----------------|--------------|
| (1) 165. | (2) 21. | (3) 704. | (4) 8. | (5) 81. |
| (6) 21. | (7) 30. | (8) £7. 10s. | (9) 21. | (10) 10. |
| (11) £109. 14s. 9½d. | (12) 5. | (13) 660. | (14) 10. | (15) ½ hour. |
| (16) £21. 7s. 6d. | (17) 30. | (18) 23½. | (19) 11. | |
| (20) 12. | (21) 12. | (22) £16. 13s. 4d. | (23) £955. 10s. | |
| (24) 4s. 6d. | (25) 40. | (26) 7. | (27) 8. | (28) 31. |
| (29) £75. | (30) 9. | | | |

97. PROPORTIONATE DIVISION. PAGE 122.

- | | | |
|---|----------------------------|--|
| (1) £4. 4s. 4½d.; £1. 8s. 1½d. | (2) £335. 4s. | (3) £955. 4s.; £636. 16s. |
| (4) £5. 17s. 6½d.; £11. 15s. 0½d.; £17. 12s. 6½d. | | |
| (5) £2. 13s. 4d.; £3. 6s. 8d.; £4. | | |
| (6) 115; 69; 23. | (7) £1899; £1266; £633. | |
| (8) £55. 12s. 8d.; £27. 16s. 4d.; £13. 18s. 2d. | | |
| (9) Man's share = £1 10s.; woman's share = 10s. | (10) £2447; £2602. | |
| (11) £46. 10s. 8½d.; £49. 4s. 10½d. | (12) £6; £6. 17s.; £8. 3s. | |
| (13) 978. | (14) 111; 222. | (15) 7 sov.; 21 shillings; 28 pennies. |
| (16) 22 pennies, 44 shillings; 66 crowns. | | |
| (17) 42 half-crowns; 126 shillings; 84 florins. | (18) 12. | |
| (19) Man's share = 18 lb.; woman's share = 9 lb. | (20) 3 lb.; 1½ lb. | |

98. BANKRUPTCIES. PAGE 124.

- | | | |
|--------------------------------------|--------------------|------------------------------------|
| (1) 9s. 8½d. | (2) 5s. | (3) 6s. 8d.; £249. 10s. 7½d. |
| (4) £2184. 10s. | (5) £675. 18s. 9d. | (6) £1871. 3s. 4d. |
| (7) 15s. 3d. | (8) 15s. | (9) £46. 5s.; £75. 5s.; £105. 10s. |
| (10) £39. 18s.; £63. 12s.; £97. 10s. | | |

99. RATES AND TAXES. PAGE 125.

- | | | |
|---------------------|------------------|----------------|
| (1) 1s. 0½d. | (2) 2s. 7½d. | (3) 3d. |
| (4) £379. 14s. 8½d. | (5) £12. 8s. 4d. | (6) £631. 10s. |
| (7) £70. | (8) £500. | (9) £65. |
| | | (10) £2. 18s. |

100. PERCENTAGES. PAGE 126.

- (1) $\frac{1}{100}$. (2) $\frac{1}{100}$. (3) $\frac{1}{100}$. (4) $\frac{1}{100}$. (5) $\frac{1}{100}$.
 (6) $\frac{1}{100}$. (7) $\frac{1}{100}$. (8) $\frac{1}{100}$. (9) $\frac{1}{100}$. (10) $\frac{1}{100}$.

101. PERCENTAGES. PAGE 126.

- (1) 50. (2) 20. (3) 5. (4) 4. (5) 2.
 (6) $8\frac{1}{2}$. (7) $3\frac{1}{2}$. (8) 60. (9) 70. (10) 75.

102. PERCENTAGES. PAGE 126.

- (1) £1; £3. 10s.; £7. 10s. (2) £1. 10s.; £4. 10s.; £6.
 (3) 5s.; £1; £1. 10s. (4) £4; £8; £6. (5) £2; £8; £17. 10s.
 (6) £2; £10; £30. (7) £57. 10s.; £93. 15s. (8) £3. 14s.
 (9) 3 oz. 12 drs. (10) 3 cwt. 3 qrs. 12 lb. 4 oz. (11) 4 lb. 6 oz.
 (12) 5 cwt. 2 qrs. 23 lb. (13) 9d. (14) £1. 2s. $5\frac{1}{2}$ d.
 (15) £3. 18s. 9d. (16) £1. 4s. $5\frac{1}{2}$ d. (17) £4. 9s.
 (18) £1. 18s. (19) £2. 19s. (20) £10. 16s. (21) £13. 17s. 6d.
 (22) £1. 9s. 3d. (23) £2. 15s. 6d. (24) £5. 0s. $1\frac{1}{2}$ d.

103. PERCENTAGES. PAGE 127.

- (1) 50. (2) 20. (3) 60. (4) $66\frac{2}{3}$. (5) 5.
 (6) $3\frac{1}{2}$. (7) $11\frac{1}{2}$. (8) 3. (9) $\frac{1}{2}$. (10) 30.
 (11) 15. (12) 12. (13) $3\frac{1}{2}$. (14) $6\frac{1}{2}$. (15) $16\frac{2}{3}$.
 (16) 25. (17) $2\frac{1}{2}$. (18) 5. (19) $12\frac{1}{2}$. (20) $16\frac{2}{3}$.

104. PERCENTAGES. PAGE 128.

- (1) £1600 (2) £2880. (3) £2400 (4) £1040.
 (5) £3532. 10s. (6) £13750. (7) £402. (8) £36291. 13s. 4d.
 (9) £2950. (10) £890.

105. MISCELLANEOUS (PERCENTAGES). PAGE 128.

- (1) £1425. (2) £1940. (3) 7374. (4) $33\frac{1}{3}$. (5) $7\frac{1}{2}$.
 (6) $14\frac{1}{2}$. (7) £1240. (8) 9600. (9) 15. (10) £277. 11s. $10\frac{1}{2}$ d.
 (11) 340. (12) 400. (13) $8\frac{1}{2}$. (14) 40. (15) 4148 tons.
 (16) $16\frac{2}{3}$. (17) £81. 10s. $8\frac{1}{2}$ d. (18) 66. (19) $2\frac{1}{17}$. (20) $12\frac{1}{2}$.

106. COMMISSION, ETC. PAGE 130.

- (1) £50. (2) £12. 0s. 9d. (3) £98. 5s. (4) £202. 10s.
 (5) £23. (6) £8650. (7) £765. (8) £1600.
 (9) (i) £2. 6s. 9d.; (ii) £4. 19s. $10\frac{1}{2}$ d.; (iii) £9. 0s. $7\frac{1}{2}$ d.
 (10) 1s. $0\frac{1}{2}$ d. too much.

107. PROFIT AND LOSS. PAGE 131.

- | | | |
|------------------------------|-------------------------------|-----------------------------|
| (1) 25 % gain. | (2) 20 % gain. | (3) 50 % gain. |
| (4) $44\frac{4}{5}$ % gain. | (5) 40 % gain. | (6) $37\frac{1}{2}$ % loss. |
| (7) 25 % gain. | (8) $22\frac{6}{7}$ % gain. | (9) 20 % gain. |
| (10) $29\frac{3}{8}$ % gain. | (11) $18\frac{1}{11}$ % gain. | (12) $9\frac{3}{8}$ % gain. |

108. PROFIT AND LOSS. PAGE 132.

- | | | | |
|------------------|---------------|----------------|-------------------|
| (1) £510; £4760. | (2) 1s. 10½d. | (3) £58. 10s. | (4) £41. 17s. |
| (5) £1. 6s. | (6) 7½d. | (7) £1191. 9s. | (8) £109. 2s. 6d. |
| (9) 6d. | (10) 18s. | (11) 1s. 3d. | (12) 2s. 3d. |

109. PROFIT AND LOSS. PAGE 133.

- | | | | |
|-----------|-------------------|--------------------|---------------|
| (1) £26. | (2) £1812. 10s. | (3) £24. | (4) £120. |
| (5) £240. | (6) 16s. 10½d. | (7) £1. 13s. 4d. | (8) £3. 10s. |
| (9) £290. | (10) £1. 5s. 6¼d. | (11) £26. 15s. 5d. | (12) 3s. 2¼d. |

110. PROFIT AND LOSS. PAGE 134.

- | | | |
|------------------------------|-----------------------------|----------------------------|
| (1) 5 % gain. | (2) $31\frac{3}{4}$ % gain. | (3) $6\frac{1}{2}$ % loss. |
| (4) 5 % loss. | (5) 8 % gain. | (6) $3\frac{3}{4}$ % gain. |
| (7) $15\frac{1}{2}$ % gain. | (8) $3\frac{1}{3}$ % gain. | (9) 40 % gain. |
| (10) $15\frac{5}{8}$ % loss. | | |

111. PROFIT AND LOSS. PAGE 135.

- | | | | |
|--------------|------------------|------------------|-------------|
| (1) £214. | (2) £16. 16s. | (3) £13. 6s. 8d. | (4) £3. 3s. |
| (5) 18s. 9d. | (6) £10. 5s. | (7) 9s. 9d. | (8) 4s. 8d. |
| (9) 8d. | (10) £6. 9s. 2d. | | |

112. MISCELLANEOUS (PROFIT AND LOSS). PAGE 136.

- | | | | |
|------------------|------------------|--------------|-----------------------------------|
| (1) £1. 10s. | (2) 29½. | (3) 183 gu. | (4) $10\frac{1}{17}\frac{2}{9}$. |
| (5) £2. 16s. 9d. | (6) 28½. | (7) 3s. 3½d. | (8) 1s. 7d.; $5\frac{5}{19}$. |
| (9) 36 % loss. | (10) £1687. 10s. | | |

113. SIMPLE INTEREST. PAGE 137.

- | | | |
|-----------------------|---------------------|---------------------|
| (1) £32. 10s. | (2) £18. 7s. 6d. | (3) £13. 8s. 9d. |
| (4) £49. 16s. 9d. | (5) £89. 15s. 9d. | (6) £118. 3s. 3d. |
| (7) £14. 14s. | (8) £26. 14s. | (9) £91. 17s. 6d. |
| (10) £259. 4s. | (11) £32. 11s. | (12) £154. 18s. 3d. |
| (13) £197. 0s. 2d. | (14) £339. 18s. | (15) £621. 13s. 6d. |
| (16) £371. 14s. 10½d. | (17) £248. 11s. 9d. | (18) £257. 5s. 3d. |
| (19) £22. 7s. 2d. | (20) £80. 14s. 8d. | (21) £124. 6s. 3d. |
| (22) £288. 8s. | (23) £82. 9s. | (24) £1. 5s. |

114. SIMPLE INTEREST. PAGE 138.

- (1) £3. 4s. (2) £129. 12s. 9d. (3) £2. 12s. (4) £4. 8s.
 (5) £5. 12s. 6d. (6) £4. 11s. (7) £12. 10s. (8) £28. 7s. 2½d.
 (9) £2. 12s. 6d. (10) £9. 7s. 6d.

115. SIMPLE INTEREST. PAGE 139.

- (1) £350. (2) £720. (3) £175. (4) £370.
 (5) £6307. 15s. (6) £3245. 10s. (7) £5768. 15s. (8) £3562. 10s.
 (9) £850. (10) £2160. 12s. 6d.

116. SIMPLE INTEREST. PAGE 140.

- (1) 3 years. (2) 2 years. (3) 5 years. (4) 3½ years.
 (5) 4½ years. (6) 7½ years. (7) 7½ years. (8) 4½ years.
 (9) 3½ years. (10) 7 years

117. SIMPLE INTEREST. PAGE 140.

- (1) 3½. (2) 10½. (3) 8½. (4) 9½. (5) 10. (6) 2½.
 (7) 5. (8) 4. (9) 5½. (10) 3½.

118. COMPOUND INTEREST. PAGE 142.

- (1) £1428. 10s. 2d.; £194. 10s. 2d. (2) £689. 12s. 3d.; £64. 2s. 3d.
 (3) £1069. 10s.; £145. 12s. 6d. (4) £2525. 4s. 1d.; £800. 9s. 1d.
 (5) £3978. 3s. 6d.; £1184. 3s. 6d. (6) £3727. 18s. 8d.; £1109. 13s. 8d.
 (7) £8695. 17s. 10d.; £620 17s. 10d.
 (8) £4251. 10s. 6d.; £155. 10s. 6d.
 (9) £1071. 6s. 5d.; £228. 13s. 11d. (10) £478. 11s. 2d.; £101. 14s. 7d.

119. COMPOUND INTEREST. PAGE 143.

- (1) £9819. 4s. 9d.; £219. 4s. 9d. (2) £4058. 3s. 8d.; £806. 3s. 3d.
 (3) £7699. 13s. 10d.; £854. 13s. 10d.
 (4) £5359. 11s. 5d.; £859. 11s. 5d.
 (5) £4285. 19s. 7d.; £635. 19s. 7d.
 (6) £4125. 16s. 4d.; £430. 16s. 4d.
 (7) £7155. 19s. 11d.; £655. 19s. 11d.
 (8) £1216. 10s. 9d.; £166. 10s. 9d.
 (9) £1169. 17s. 2d.; £169. 17s. 2d. (10) £11. 3s. 6d.; £1. 8s. 6d.
 (11) £273. 5s. 5d.; £23. 5s. 5d. (12) £959. 14s. 10d.; £79. 14s. 10d.

120. MISCELLANEOUS (INTEREST). PAGE 143.

- (1) £750. (2) 10%. (3) 8½ yrs. (4) £2000. (5) 10½%.
 (6) £475; £74. 16s. 3d. (7) £33. 2s. 3d. (8) £212. 7s. 3½d.
 (9) £7. 16s. 3d. (10) £9. 5s.

121. TRUE DISCOUNT. PAGE 145.

- | | |
|----------------------------------|-----------------------------------|
| (1) £168. 16s. 8d.; £8. 8s. 10d. | (2) £5600; £147. |
| (3) £216. 17s. 6d.; £8. 13s. 6d. | (4) £4000. 8s. 4d.; £120. 0s. 3d. |
| (5) £1820; £22. 15s. | (6) £356. 5s.; £21. 7s. 6d. |
| (7) £524. 8s. 11d.; £26. 4s. 5d. | (8) £241. 10s. 1d.; £7. 10s. 11d. |
| (9) £150. 12s. 6d.; £2. 0s. 2d. | (10) £270. 16s. 8d.; £5. 13s. 9d. |

122. BANKER'S DISCOUNT. PAGE 146.

- | | | |
|------------------|------------------|---------------------|
| (1) £25. 10s. | (2) £8. 11s. 3d. | (3) £2. 1s. 4d. |
| (4) £6. 0s. 2d. | (5) £13. 4s. 9d. | (6) £1893. 7s. 2d. |
| (7) £12. 1s. 4d. | (8) £24. 0s. 5d. | (9) £150. 14s. 10d. |
| (10) £307. 14s. | | |

123. STOCKS AND SHARES. PAGE 148.

- | | | | | |
|--------------------|--------------------|-------------------|---------------|---------------|
| (1) £165. | (2) £171. | (3) £28. | (4) £323. 5s. | (5) £344. 5s. |
| (6) £205. 12s. 6d. | (7) £194. 10s. 8d. | (8) £108. 3s. 3d. | | |
| (9) £768. | (10) £1406. 5s. | | | |

124. STOCKS AND SHARES. PAGE 149.

- | | | | |
|------------|-------------|------------|-------------|
| (1) £5000. | (2) £2280. | (3) £4200. | (4) £14400. |
| (5) £3725. | (6) £3600. | (7) £900. | (8) £1000. |
| (9) £1300. | (10) £3610. | | |

125. STOCKS AND SHARES. PAGE 150.

- | | | | |
|----------------|-----------------|-----------------|-----------------|
| (1) £1350. | (2) £4095. | (3) £5062. 10s. | (4) £1710. |
| (5) £3279. 4s. | (6) £8308. 15s. | (7) £1045. 16s. | (8) £6909. 18s. |
| (9) £1845. | (10) £4209. 5s. | | |

126. STOCKS AND SHARES. PAGE 150.

- | | | | |
|-------------------|----------------|----------------|--------------------|
| (1) £583. 6s. 8d. | (2) £49. | (3) £864. 10s. | (4) £126. |
| (5) £240. | (6) £159. 12s. | (7) £140. | (8) £541. 13s. 4d. |
| (9) £500. | (10) £252. | | |

127. STOCKS AND SHARES. PAGE 151.

- | | | | |
|--------------------|----------------------|-------------|-------------|
| (1) £3220. | (2) £801. 2s. 6d. | (3) £24000. | (4) £25935. |
| (5) £4311. 8s. 9d. | (6) £1100. | (7) £7200. | (8) £3591. |
| (9) £869. 5s. | (10) £2666. 13s. 4d. | | |

128. STOCKS AND SHARES. PAGE 151.

- | | | | |
|-----------------|------------------|--------------------|-----------------|
| (1) The former. | (2) The latter. | (3) The latter. | (4) The latter. |
| (5) The latter. | (6) The latter. | (7) No difference. | (8) The latter. |
| (9) The former. | (10) The former. | | |

129. STOCKS AND SHARES. PAGE 152.

- (1) $3\frac{2}{7}$. (2) $3\frac{1}{3}$. (3) $3\frac{1}{3}$. (4) $2\frac{1}{2}$. (5) $3\frac{1}{2}$.
 (6) $4\frac{1}{3}$. (7) $3\frac{1}{2}$. (8) $4\frac{1}{3}$. (9) $3\frac{1}{3}$. (10) $3\frac{1}{2}$.

130. STOCKS AND SHARES. PAGE 153.

- (1) £4262. 10s. (2) £8201. 8s 1½d. (3) £2440. 15s. (4) £324.
 (5) £31833. 6s. 8d. (6) 4½. (7) £127. 10s. (8) £448.
 (9) No change; £96 from each investment. (10) £18 11s. 3d. increase.

131. MISCELLANEOUS (STOCKS AND SHARES). PAGE 154.

- (1) £107. (2) £2. 1s. 8d. (3) £50. (4) The former.
 (5) £1250; £2. 10s. decrease. (6) £1. 12s. increase.
 (7) £18 increase. (8) £1660. (9) 10½. (10) £130.
 (11) £5 loss. (12) £1300, £4 loss. (13) £3600, £126.
 (14) £19508. 4s.; $3\frac{1}{3}$. (15) £41. 13s. 4d.
 (16) The latter; £136762. 10s. (17) £15000. (18) £8400.
 (19) £225. (20) £11000.

132. SQUARE ROOT (INTEGERS). PAGE 157.

- (1) 19. (2) 59. (3) 83. (4) 98. (5) 149.
 (6) 119. (7) 253. (8) 476. (9) 503. (10) 739.
 (11) 9852. (12) 6304. (13) 8027. (14) 8254. (15) 4701.
 (16) 5083. (17) 20501. (18) 30507. (19) 26004. (20) 30842.

133. SQUARE ROOT (DECIMALS). PAGE 158.

- (1) 1.4. (2) 9.3. (3) 5.37. (4) 18.2.
 (5) 31.87. (6) 18.05. (7) 10.043. (8) 26.004.
 (9) .207. (10) .0031. (11) .0101. (12) .00043.
 (13) .0074. (14) .0319. (15) .00072. (16) .00148.
 (17) .00251. (18) .01809. (19) .0119. (20) 90.909.

134. SQUARE ROOT (FRACTIONS). PAGE 159.

- (1) $\frac{1}{3}$. (2) $\frac{1}{3}$. (3) $6\frac{1}{2}$. (4) $14\frac{1}{2}$. (5) $6\frac{1}{2}$.
 (6) $8\frac{1}{2}$. (7) $2\frac{1}{7}$. (8) $3\frac{1}{5}$. (9) $2\frac{1}{3}$. (10) $22\frac{1}{3}$.
 (11) $22\frac{1}{3}$. (12) $1\frac{1}{3}$. (13) $48\frac{1}{2}$. (14) $6\frac{1}{5}$. (15) 4.1509....
 (16) 9354. (17) .8528. (18) .5477.... (19) .7559....
 (20) 2.0586....

135. MISCELLANEOUS (SQUARE ROOT). PAGE 159.

- (1) 38. (2) 144; 144. (3) 70 yards. (4) 770 yards. (5) 8030.
 (6) £35. 15s. (7) 45½. (8) £10. 10s. (9) $4\frac{1}{2}$ min. (10) Twice.

136. MENSURATION OF RECTANGLES. PAGE 161.

- | | |
|-----------------------------|--|
| (1) 170 sq. ft. 6 sq. in. | (2) 529 sq. ft. 4 sq. in. |
| (3) 190 sq. ft. 132 sq. in. | (4) 208 sq. ft. 48 sq. in. |
| (5) 293 sq. ft. 70 sq. in. | (6) 98 sq. ft. $15\frac{1}{2}$ sq. in. |
| (7) 258 sq. ft. 39 sq. in. | (8) 322 sq. ft. 126 sq. in. |
| (9) 2892 sq. ft. 72 sq. in. | (10) 920 sq. ft. 126 sq. in. |
| (11) 140 sq. yds. | (12) 105. |
| (13) 16. | (14) £47. 16s. 3d. |
| (15) £13. 10s. | (16) 10000; £208. 6s. 8d. |
| (17) 864; £233. 17s. 9d. | (18) £72. 6s. 3d. |

137. CARPETING FLOORS. PAGE 162.

- | | |
|--|--|
| (1) $23\frac{1}{2}$ yds.; £6. 8s. 4d. | (2) 87 yds.; £21. 7s. 9d. |
| (3) 70 yds.; £21. 17s. 6d. | (4) 25 yds.; £3. 13s. $11\frac{1}{2}$ d. |
| (5) $52\frac{2}{3}$ yds.; £8. 18s. 9d. | (6) $25\frac{1}{2}$ yds.; £5. 14s. 9d. |
| (7) $65\frac{1}{2}$ yds.; £17. 19s. 4d. | (8) $32\frac{1}{2}$ yds.; £7. 10s. $3\frac{1}{2}$ d. |
| (9) $58\frac{1}{2}$ yds.; £16. 3s. $1\frac{1}{2}$ d. | (10) $41\frac{1}{2}$ yds.; £10. 16s. 8d. |

138. AREA OF WALLS. PAGE 164.

- | | | |
|------------------------|------------------------|------------------|
| (1) 1260 sq. ft. | (2) 858 sq. ft. | (3) 800 sq. ft. |
| (4) 941 sq. ft. 60 in. | (5) 876 sq. ft. 18 in. | (6) 504 sq. ft. |
| (7) 897 sq. ft. | (8) 1143 sq. ft. | (9) 1078 sq. ft. |
| (10) 928 sq. ft. | | |

139. PAPERING AND PAINTING WALLS. PAGE 164.

- | | | |
|---|-----------------|---|
| (1) $97\frac{1}{2}$ yds. | (2) 216 yds. | (3) 115 yds.; £2. 3s. $1\frac{1}{2}$ d. |
| (4) £1. 16s. 3d. | (5) £4. 5s. 3d. | (6) £2. 8s. 9d. |
| (7) £1. 5s. $3\frac{1}{2}$ d. | (8) £3. 12s. | (9) £20. 2s. 6d. |
| (10) £7. 8s. $8\frac{1}{2}$ d. | | |
| (11) Walls, £9. 2s. $4\frac{1}{2}$ d.; Ceiling, 15s. $8\frac{1}{2}$ d.; Total, £9. 18s. $0\frac{1}{2}$ d. | | |
| (12) Walls and doors, £5. 3s. 9d.; ceiling, £1. 6s. 3d.; total £6. 10s. | | |

140. MENSURATION OF RECTANGULAR SOLIDS. PAGE 166.

- | | | |
|--------------------------|--------------------------|-------------------------|
| (1) 217 cu. ft. | (2) 58 cu. ft. 162 in. | (3) 139 cu. ft. 519 in. |
| (4) 12 cu. ft. 1304 in. | (5) 35 cu. ft. 720 in. | |
| (6) 283 cu. ft. 1176 in. | (7) 2809 cu. ft. 198 in. | |
| (8) 14629 cu. ft. 54 in. | (9) 12 ft. | (10) $1\frac{1}{2}$ in. |
| (11) £2. 16s. | (12) 30000. | (13) 10 ft. 5 in. |
| (14) 4 cwt. 1 qr. 24 lb. | | |
| (15) £60. 0s. 6d. | (16) 28. | (17) 36. |
| | | (18) 300. |

141. THE FRENCH COINAGE. PAGE 167.

(1) 850.1 fr.	(2) 31 61 fr.	(3) 40.35 fr.	(4) 62.34 fr.
(5) 156.01 fr.	(6) 340.01 fr.	(7) .15 fr.	(8) .57 fr.
(9) .04 fr.	(10) .03 fr.	(11) .01 fr.	(12) .025 fr.
(13) 5634 c.	(14) 4895 c.	(15) 415 c.	(16) 382 c.
(17) 509 c.	(18) 2503 c.	(19) 3600 c.	(20) 5000 c.
(21) 6200 c.	(22) 12000 c.	(23) 20500 c.	(24) 40000 c.

142. EXCHANGE. PAGE 168.

(1) 6211 fr. 70 c.	(2) 6499 fr. 99 c.	(3) 8442 l.	(4) 11601 l. 20 c.
(5) 9287 dr.	(6) 3769 dr. 70 l.	(7) 2160 ps.	(8) 2468 ps.
(9) 1849 milr. 500 reis.	(10) 2333 milr. 450 reis.	(11) 12877 mk. 70 pf.	
(12) 15330 mk. 36 pf.	(13) 3692 fl. 80 kr.	(14) 8625 fl. 80 kr.	
(15) 10200 fl.	(16) 2648 fl. 46 c.	(17) 22712 kr. 50 öre.	
(18) 65118 kr. 28 öre.	(19) \$760. 42 c.	(20) \$2352. 24½ c.	
(21) 2312 r. 50 co.	(22) 4160 r. 25 co.	(23) Rs. 1905. 12 as.	
(24) Rs. 2859. 12 as.			

143. FOREIGN BILLS PAGE 170.

(1) £2569. 9s. 5d.	(2) £4174. 15s. 2d.	(3) £4320. 19s. 9d.
(4) £5637. 6s. 1d.	(5) £4076. 1s. 9d.	(6) £2329. 18s. 4d.
(7) £8412. 7s. 5d.	(8) £501. 17s. 4d.	(9) £982. 16s. 3d.
(10) £1236 11s. 8d.	(11) £491. 15s. 10d.	(12) £465. 13s. 4d.
(13) 15s. 6d.	(14) 6s. 11½d.	(15) The former.

144. THE METRIC SYSTEM. PAGE 172.

(1) 75.3452 Km.	(2) .344825 Km.	(3) .352565 Km.
(4) .420075 Km.	(5) .03524256 Km.	(6) .0045030275 Km.
(7) 63246000 m.	(8) 732450 m.	(9) 4236900 m.
(10) 752.98 m.	(11) 9345.2 m.	(12) 96.452 m.
(13) 524756500 cm.	(14) 401306.35 cm.	(15) 325007.5 cm.
(16) 4275.23 cm.	(17) .275 cm.	(18) 7 5 cm.
(19) 1.7562 mm.	(20) 38572520 mm.	(21) 52756.5 mm.
(22) 74256.75 mm.	(23) 3.75 mm.	(24) 543757.5 mm.

145. THE METRIC SYSTEM. PAGE 173.

(1) 890904½.	(2) 326.589.	(3) 633.6.	(4) 144018½.
(5) 8046 538.	(6) 6.66225 fr.	(7) 282 fr. 51 c.	(8) 9s. 3¾d approx.
(9) 36 approx.	(10) £2. 2s. 1d. approx.		

EXAMINATION PAPER I. PAGE 176.

- (1) 5046065; 336404 $\frac{1}{2}$. (2) 1932986. (3) 20892.
 (4) £15. 11s. 9d.; 283164. (5) 894640. (6) £36360. 8s. 0 $\frac{3}{4}$ d.
 (7) 2s. (8) 1 $\frac{1}{11}$; $\frac{24, 35, 36, 26}{56}$. (9) $\frac{25}{303}$; $\frac{75}{404}$.
 (10) 6s. 1 $\frac{1}{2}$ d. + 16s. 9 $\frac{1}{2}$ d. + 18s. 4d. + 15s. 7d. + 6s. 10 $\frac{1}{2}$ d. + £1. 5s. 11 $\frac{3}{4}$ d.
 = £4. 9s. 8d.; change = £5. 10s. 4d.

EXAMINATION PAPER II. PAGE 176.

- (1) 4409995. (2) See Arts. 63 and 71; 3001. (3) 12026.
 (4) 20 lb. (5) 83. (6) 27328. (7) £38052.
 (8) 1 $\frac{1}{4}$; $\frac{8}{315}$. (9) $\frac{1}{2}$.
 (10) £16. 11s. 10d. + £1. 6s. 7d. + £1. 3s. 3d. + £1. 16s. 2 $\frac{1}{2}$ d. + £2. 7s. 1 $\frac{1}{4}$ d.
 = £23. 4s. 11 $\frac{3}{4}$ d.

EXAMINATION PAPER III. PAGE 177.

- (1) Seven hundred and one thousand and eleven; seventy thousand one hundred and one.
 (2) 1572904. (3) 156; 57096. (4) £955. 10s. (5) 324.
 (6) 2475. (7) 812. (8) 2659680. (9) $\frac{4}{5}$; $\frac{1}{5}$.
 (10) £2. 12s. 1 $\frac{1}{2}$ d. + £14. 5s. 7 $\frac{1}{2}$ d. + £1. 1s. 1 $\frac{1}{2}$ d. + £29. 15s. 4d.
 = £47. 14s. 2 $\frac{1}{2}$ d.

EXAMINATION PAPER IV. PAGE 177.

- (1) 99099099; nine hundred thousand nine hundred and one.
 (2) 21773; rem. 387. (3) 641.
 (4) 4910 tons 14 cwt. 1 qr. 4 lb.; 1 ac. 1 ro. 29 per. 17 sq. yds. 1 ft. 108 in.
 (5) 18 miles. (6) 1287 = 3². 11. 13; 2145 = 3. 5. 11. 13; 429; 1512.
 (7) 29 $\frac{641}{220}$; 1 $\frac{1}{42}$. (8) $\frac{1}{7}$. (9) £360. (10) £136. 18s. 3 $\frac{3}{4}$ d.

EXAMINATION PAPER V. PAGE 178.

- (1) 20351; 751 rem. 34. (2) £3154944. 3s. 6 $\frac{3}{4}$ d. (3) 13; 1058.
 (4) 3759399. (5) 3 tons 7 cwt. 3 qrs. 2lb. 12 oz.
 (6) 45 $\frac{4}{5}$; 9 $\frac{1}{4}$; 487 $\frac{3}{4}$. (7) 1 $\frac{3}{8}$. (8) £521. 9s. 4 $\frac{1}{2}$ d. (9) 336.
 (10) 70 m. 4 fur. 15 po.

EXAMINATION PAPER VI. PAGE 178.

- (1) 57111104051. (2) 3 tons 4 cwt. 1 qr. 17 lb.; 50000.
 (3) £9541. 9s. 2d.; £100. 2s. 6d.; £1. 2s. 6d. (4) 4s. 2d.
 (5) £3740. 13s. (6) 45 weeks. (7) 2223; 22680.
 (8) 33 $\frac{1}{10}$; 1 $\frac{7}{8}$. (9) (i) 14 $\frac{73}{100}$; (ii) $\frac{1}{11}$. (10) 4.39 p.m.

ANSWERS.

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EXAMINATION PAPER VII. PAGE 179.

- (1) Four hundred and twenty millions seven hundred and twenty-nine thousand two hundred and forty-three.
 (2) 153503. (3) £3. 6s. + £1. 2s. $11\frac{1}{2}d$ + £1. 5s. $8\frac{3}{4}d$. - £5. 14s. $8\frac{1}{2}d$.
 (4) 20 tons 10 cwt. 2 qrs. 19 lb. 14 oz. (5) $33\frac{1}{2}$. (6) 8.
 (7) 6·4140625; 48 fr. 79 c. (8) 17s. 4d. + 16s. 6d. + £4. 4s. = £5. 17s. 10d.
 (9) £2660. (10) $3\frac{1}{3}$.

EXAMINATION PAPER VIII. PAGE 179

- (1) Eighty-five millions three hundred and seventy-eight thousand and nine.
 (2) 11447. (3) 7s. $10\frac{1}{2}d$. + 9s. $2\frac{1}{2}d$. + 2s. $7\frac{1}{2}d$ + 6s. $4\frac{1}{2}d$. - £1. 6s. $0\frac{3}{4}d$.
 (4) 111111 oz. (5) $2\frac{1}{2}$ (6) 5. (7) 725·42463.
 (8) £1. 1s. $6\frac{1}{4}d$. ; -359375. (9) 9378 fr. 75 c.
 (10) £147. 7s. $1\frac{1}{2}d$.

EXAMINATION PAPER IX. PAGE 180.

- (1) Six hundred and sixty-seven thousand three hundred and thirty-four.
 (2) 13 po. 3 yds. (3) $4\frac{1}{2}$. (4) ·78125. (5) ·127506.
 (6) $\frac{1}{12}$. (7) £64. 4s.

EXAMINATION PAPER X. PAGE 180

- (1) Eleven thousand and eleven. (2) 4 tons 14 cwt. 2 qrs.
 (3) $\frac{1}{10}$. (4) (i) $1\frac{1}{2}$; (ii) $8\frac{1}{2}$. (5) 19·03. (6) £11. 0s. 10d.
 (7) £20. 12s. 6d. (8) £72. 2s.

EXAMINATION PAPER XI. PAGE 181.

- (1) $\frac{7}{8}$; 7. (2) $\frac{3}{25}$; $\frac{3\frac{1}{2}}{25}$; $8\frac{1}{2}d$. (3) 143. (4) £12. 0s. $2\frac{1}{2}d$.
 (5) £4. 5s. (6) £318. 15s. (7) ·8685. (8) 5 miles.
 (9) 13s. 4d. (10) £11747. 5s.

EXAMINATION PAPER XII. PAGE 181.

- (1) $\frac{17}{2}$; 0. (2) 05; 1 344 lb; ·012. (3) 17·6; 1s. 2d
 (4) £18. (5) £510. (6) 300. (7) £4. 8s. (8) 90 miles
 (9) £5000. (10) 1718750.

EXAMINATION PAPER XIII. PAGE 182.

- (1) 192 tons 61 lb. (2) $\frac{1}{16}$ (3) £40. 19s. 6d (4) 7 fr. 75 c.
 (5) 154·54 (6) £2. 16s. (7) £2. 11s. $8\frac{1}{2}d$. (8) 768.
 (9) 1s. $8\frac{1}{2}d$. (10) $3\frac{1}{2}$

EXAMINATION PAPER XIV. PAGE 182.

- (1) 631900375263. (2) £166. 15s. (3) 21·009207; 107·695.
 (4) 46143 fr. 75 c. (5) £1455. 19s. 4½d. (6) £113. 13s. 0¾d.
 (7) 1½; 153 yds. 1 ft. 11½ in. (8) 42 half-crs., 126 fl.; 168s. (9) £15. 9s. 8d.
 (10) 44. (11) 5½ min.; A will be 1½ yards behind B. (12) £50. 3s.

EXAMINATION PAPER XV. PAGE 183.

- (1) 4 hrs. 3 min. 52 sec. (2) £10. 15s. 7½d. (3) 713.
 (4) ·028112. (5) (i) 19·551; (ii) 19½. (6) £373. 15s. 11½d.
 (7) 306 ft. 9 in. (8) 45. (9) £2412. 15s. 9d. (10) £3000.
 (11) ·002222004.... (12) ·125 cu. metre.
 (13) 9 horses, 9 cows, 18 pigs, 36 lots of poultry.

EXAMINATION PAPER XVI. PAGE 184.

- (1) £3342. 1s. 6d. (2) 3916+1=3917. (3) 8294400.
 (4) ⅜. (5) 4·84. (6) 25·6681. (7) £57. 19s. 11d.
 (8) £89511. 12s. 11¼d. (9) 123·454321. (10) 222·22.
 (11) 71. (12) £220000000; 38720000000. (13) £1. 10s.; 7s. 6d.; 3s.

EXAMINATION PAPER XVII. PAGE 185.

- (1) 99904; rem. 53. (2) 50 tons 18 cwt. 46 lb. 10 oz.
 (3) 15 fur. 10 po. 1 yd. 2 ft. (4) 5. (5) 1½.
 (6) 13·75; 23·0625; ·00005. (7) 115 lb. (8) £121. 19s. 0¾d.
 (9) £2. 0s. 3d. + 3s. 2½d. + 11s. 6¼d. + 2s. 4d. = £2. 17s. 3¾d.
 (10) 1720. (11) 10s. 6d. (12) half-cr. (13) £10. 6s. 2¾d.
 (14) 2·875. (15) £1946. 13s. 4d. (16) £312. 10s.
 (17) £1. 2s. 9d. (18) 23½ days. (19) 10.

EXAMINATION PAPER XVIII. PAGE 186.

- (1) £7. 14s. 3d. (2) 40 m. 7 fur. 8 yds. 1 ft. 5 in.
 (3) 31719 bush. 3 pks. (4) 75. (5) 9 cwt. 0 qr. 5⅞ lb.
 (6) ·0015; 12. (7) ·975625. (8) £18. 9s. 7¼d.
 (9) £3. 11s. + £1. 10s. 0¾d. + 3s. 3d. + £2. 7s. 3d. = £7. 11s. 6¾d.
 (10) 140535. (11) £36. 13s. 9d. to one and £73. 7s. 6d. to each
 of two others. (12) 672 in. (13) 648.
 (14) £13. 10s. 10d. (15) 150. (16) £575.
 (17) 10500 foot-pounds. (18) 59 yds. 9½ in.

EXAMINATION PAPER XIX. PAGE 187.

- (1) Quot. 2266; rem. 4. (2) 252 m. 6 fur. 27 po. $1\frac{1}{2}$ yds.
 (3) 7 tons 13 cwt. 23 lb. 9 oz. (4) 7s. $1\frac{1}{2}$ d. (5) 168 lb.
 (6) $2\frac{1}{2}\frac{1}{2}$. (7) £11. 10s. $2\frac{1}{2}$ d. (8) 4-2. (9) 108 men.
 (10) 1 lb. (11) £16. 10s. 9d. (12) £27. 6s. 9d. (13) £3. 9s. 9d.
 (14) $91\frac{1}{2}$. (15) 90 gallons.

EXAMINATION PAPER XX. PAGE 188

- (1) Loss = £31. 2s. (2) £58. 0s. $11\frac{1}{2}$ d. (3) $30\frac{1}{2}\frac{1}{2}$. (4) .03125; .0025.
 (5) 19-36. (6) A, 24 hectares 80 ares; B, 22 hectares 4 ares;
 C, 33 hectares 16 ares. (7) $68\frac{1}{2}$.

EXAMINATION PAPER XXI. PAGE 188.

- (1) 5957356944; 13056. (2) 163 tons 11 cwt. 1 qr. 19 lb.
 (3) £21780. (4) 4 67. (5) $59\frac{1}{2}$ metres. (6) £245. 18s. $11\frac{1}{2}$ d.
 (7) 3s. 4d.

EXAMINATION PAPER XXII. PAGE 189.

- (1) 283 m. 4 fur. 214 yds. (2) 1 ton 11 cwt. 35 lb. (3) 1618.
 (4) 2 lb. 7 dwt. 9 gra. (5) 3, 5 and 11. (6) 390 (7) $\frac{5}{11}$.
 (8) $17\frac{1}{2}$. (9) $6\frac{1}{11}$. (10) 1 (11) $1\frac{1}{2}$. (12) 65.04065.
 (13) 2.03793. (14) .54295436. (15) 4800. (16) .6875.
 (17) $\frac{1}{2}$. (18) 19 hrs. 29 min. 42.72 sec. (19) £428. 0s. $0\frac{1}{2}$ d.
 (20) £15. 14s. $3\frac{1}{2}$ d. (21) 80. (22) 75. (23) £105. 8s. 4d.
 (24) $11\frac{1}{2}$. (25) 2592.

EXAMINATION PAPER XXIII. PAGE 190.

- (1) £1232. 8s. $10\frac{1}{2}$ d (2) 4 m 3 fur. 37 po. 3 yds. (3) 703360 oz.
 (4) 2 qrs. 6 bush. 2 pks. 3 qts. 1 pt. (5) 23. (6) 1752.
 (7) $\frac{1}{2}\frac{1}{2}$. (8) $27\frac{1}{2}$. (9) $2\frac{5}{8}$. (10) 42. (11) $5\frac{1}{2}$.
 (12) 227.54657. (13) 2.64065. (14) 48.16512. (15) 22.186.
 (16) $1\frac{1}{2}$. (17) .212. (18) £1. 9s 9 21d. (19) £887. 3s. $2\frac{1}{2}$ d.
 (20) £9. 0s. $8\frac{1}{2}$ d. (21) 39 (22) £48. 2s. 6d. (23) 16s. 8d.
 (24) $16\frac{1}{2}$. (25) 76.

EXAMINATION PAPER XXIV. PAGE 191.

- (1) £1754. 0s. 5d. (2) 3 pks 1 gall. 2 qts. 1 pt.
 (3) (i) 3190 yards; (ii) 42015500 millimetres.
 (4) (i) 2 lb. 10 oz. 2 dwt. 7 gra.; (ii) 12 kilos 467 grama. (5) 34.

- (6) 6090. (7) $\frac{3}{8}$. (8) $26\frac{1}{8}$. (9) $3\frac{1}{4}$. (10) $965\frac{1}{4}$.
 (11) $2\frac{2}{3}$. (12) 432·42082. (13) 37·67236. (14) 25·196017.
 (15) 12·06. (16) $\frac{17}{180}$. (17) ·425. (18) £2. 8s. 5·25d.
 (19) £106239. 2s. 9 $\frac{3}{4}$ d. (20) £7. 0s. 11 $\frac{1}{4}$ d. (21) 36. (22) $10\frac{1}{2}$.
 (23) £1582. 3s. 9d. (24) 92. (25) $24\frac{1}{2}$.

EXAMINATION PAPER XXV. PAGE 192.

- (1) 99 weeks 6 days 20 hrs. 17 min. (2) 10 oz. 9 dwt. 8 gra.
 (3) (i) 6768 yards ; (ii) 27020 metres.
 (4) (i) 6 qrs. 5 bush. 2 pks. 3 qts. ; (ii) 39 Kl. 625 l. (5) 51.
 (6) 720. (7) $\frac{3}{4}$. (8) $13\frac{1}{4}$. (9) $\frac{1}{8}$. (10) 40.
 (11) $1\frac{1}{2}$. (12) 199·876. (13) 110·43222. (14) 874·581.
 (15) 320·45. (16) $\frac{17}{180}$. (17) ·9375. (18) 11 yds. 1 ft. 0·6126 in.
 (19) £503. 12s. 6d. (20) £15. 6s. 2 $\frac{1}{4}$ d. (21) £2. 18s.
 (22) $7\frac{1}{2}$. (23) 3. (24) 12. (25) 80 square yards.

13
22
23







